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NATURALLY STRAINED YIELD SURFACE SHAPE ESTIMATED UNDER PRE-DEFORMATION OF TENSION AND TORSION

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The purpose of this study is to investigate the shape of yield surfaces formed under large pre-deformations, based on the Natural Strain theory, and to make clear the mechanism of the development of anisotropy of yield surfaces under large deformations. In particular the shape of yield surfaces obtained after applying the pre-deformation of tension and shear with proportional loading states is examined. Using pure-copper test pieces already pre-deformed, proportional loading tests were carried out again changing the ratio of tension and torsion to determine yield stress in each direction of the stress space. The yield stress was estimated by examining the slope of tangent for the principal deviatoric stress-deviatoric strain curve in proportional loading tests. It was revealed that the shape of the yield surface becomes convex at the side of pre-deformation and becomes flat in the opposite direction of the pre-deformation. Moreover, the shape of the estimated yield surface was compared with the shape of conventional proof stress. Consequently, it became clear that the shapes of both yield surfaces almost coincide at the side of pre-deformation, but on the opposite side, the yield surface by proof stress becomes smaller compared with the estimated yield surface.

Keywords: Finite deformation, Elasto-plastic analysis, Subsequent yield surface, Proportional loading, Anisotropy, Ductile materials.

1 INTRODUCTION

The Natural Strain suggested in this paper is effective strain representation, which can systematically describe strains ranging from infinitesimal deformations to large deformations. This finite strain is associated with a specific identical line element in the body, and it is obtained by integrating an infinitesimal strain rate on an identical line element over the whole process of the deformation path. Consequently, the shearing component of this strain is defined as a pure angle strain. Hence, this strain is useful because the rigid body rotation can be clearly removed from the shearing strain component. Moreover, since the additive law of strain on an identical line element can be satisfied, the strain rate in the Natural Strain theory can be clearly decomposed into elastic and plastic components; further, the elastic component can also be decomposed into deviatoric strain and volumetric strain component in the same manner as conventional infinitesimal deformation theory.

In our previous research, the shape of the yield surface obtained after applying the pre-deformation were examined by using the Natural Strain theory. The shape of the yield surface formed after applying the pre-deformations of uni-axial tension, under which the line elements on the principal axes of deformation are fixed in a body, were investigated as the most fundamental case (Kato *et al.* 2012). In subsequent work, the shape of yield surfaces formed under the pre-deformations of simple shear, under which the line elements of the principal axes are rotated and replaced, were investigated (Kato and Kazama 2014).

As the next step of this research, the shape of the yield surface, obtained after applying the pre-deformation of combined loading of tension and shear with proportional loading state, is examined in this paper. Using specimens where pre-deformation is already applied, the yield stress in arbitrary directions in the stress space was investigated by conducting the proportional loading tests again by altering the proportion of tension and torsion. In general, nonlinear phenomena occur in the stress and strain relations for the effect of hysteresis. Hence, the stress and strain relation in elastic regions, represented by a straight line, turns into a smoothly-curved line just before yield. Therefore, in this research, the method of utilizing the slope of a tangent in the principal deviatoric stress-deviatoric strain curve in the neighborhood of the yield surface is adopted for determining yield stress. Since yield stress is already revealed from the pre-deformation, the slope of a tangent when the materials arrived at yield in the reloading process can be determined in advance. Hence, as a method for estimating yield stress, the stress satisfying the specified tangent modulus is assumed to be the yield stress in this paper, and the shape of the yield surface estimated by this method is compared with the shape of the proof stress, generally known as a method for estimating yield stress. Moreover, the strain-hardening modulus h can be derived from the tangent modulus in each deviatoric stress-deviatoric strain curve, and the development of anisotropy can be revealed by investigating the distribution of h.

2 METHOD FOR ESTIMATING THE SHAPE OF THE YIELD SURFACE

The method for estimating yield stress based on the slope of the deviatoric stress-deviatoric strain curve, obtained by conducting proportional loading tests for tension and shear, is described in this section. As an example of the proportional loading tests, Figure 1 shows the schematic diagram in the direction of the pre-deformation, i.e., 65°, and in the opposite direction of it, i.e., 155°. In this figure, the value of yield stress in the same direction as the pre-deformation is different from the value in the opposite direction for the Bauschinger effect. Moreover, the linear relation between the stress and the strain in the elastic region turns gradually into a curved line as the stress approaches the yield point. Hence, it is not easy to determine the yield limit accurately.

For these reasons, the proof stress, defined by the stress generating a specified residual strain after removal of the load, is commonly used instead of the yield limit. However, since the deviatoric stress and deviatoric strain curve in the opposite direction of the pre-deformation becomes a shallow curve compared with the curve on the same side of the pre-deformation, it can be predicted that the yield stress based on proof stress will be smaller on the opposite side of the pre-deformation, as shown in Figure 1 (see point C'). Hence, it seems that the method by proof stress is not always sufficient for representing an actual yield phenomenon. As indicated in Figure 1, since the yield stress is already measured from the pre-deformation (see point B), the slope of the tangent at yield, namely, the tangent modulus, which arrives at the yield stress during the process of reloading, can be determined in advance (see point A'). Thus, it will be assumed in this paper that the stress, when the tangent modulus in the deviatoric stress - deviatoric strain curve coincides with the value of the specified tangent modulus, will be regarded as the yield stress. Namely, a point D in Figure 1, which has the same tangent modulus as the specified tangent modulus, is assumed to be a yield stress.

In this study, the experimental equation of shallow curve is formulated as:

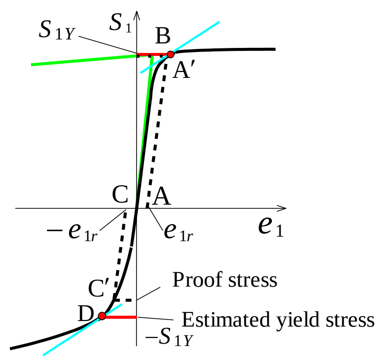


Figure 1. Method for determination of yield stress.

$$S_1 = a\left(1 - exp\left(b e_1\right)\right) + c e_1 + d \tag{1}$$

where *a*, *b*, *c*, *d* are coefficients determined by the non-linear least-squares method. Moreover, in order to derive the slope of the tangent in the principal deviatoric stress-deviatoric strain curve, the following equation is obtained by differentiating Eq. (1) with respect to the principal deviatoric strain e_1, i.e.,

$$\frac{dS_1}{de_1} = -a\,b\,exp\left(b e_1\right) + c \tag{2}$$

Then, the strain hardening modulus *h*, which is obtained under the proportional loading tests in each direction, is derived as:

$$h = \frac{2G}{\frac{dS_1}{de_1}} - 1 \tag{3}$$

As previously mentioned, since the slope of tangent at yield is already specified, the value of the strain hardening modulus *h* at yield can also be specified. Therefore, the shape of the yield surface can be estimated from the distributions of the strain-hardening modulus *h* (see Figure 2).

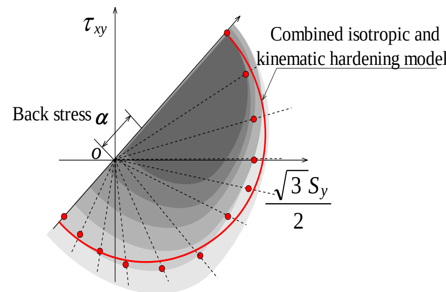


Figure 2. Distributions of strain hardening modulus *h* and boundary of yield locus.

3 EXPERIMENTAL METHOD

In this research, the experiments were composed of two stages. Namely, the experiments applying the pre-deformation of proportional loading for tension and torsion to the specimens were conducted first. Next, the experiments of the proportional loading for tension and torsion were conducted in arbitrary directions in the stress space. Hence, the multi-axial testing machine, which can be applied as tension and torsion at the same time, was used in these experiments. The displacement meters equipped to the circular jig were used for the measurements of longitudinal displacements, and the rotary encoders were used for the measurements of torsional angles. In these experiments, a cylindrical specimen made of pure copper (i.e., the so-called tough pitch copper), with gauge length 30 mm and purity 99.99%, was selected from the ductile materials. Next, regarding the experimental conditions for pre-deformation, the experiments of proportional loading for tension and torsion (the direction of principal axis of stress being 65°) were conducted with the condition that the final deformation of each pre-deformation is the same value (i.e., all principal stretch is λ =1.39 [-]).

On the other hand, after attaching tri-axial strain gauges to the specimens that were already pre-deformed, experiments of proportional loading were conducted with the principal axis of stress fixed in arbitrary angles, i.e., 65°, 70°, 80°, 90°, 95°, 105°, 110°, 115°, 120°, 130°, 135°, 145°, 150°, and 155°. The principal strain in the proportional loading tests was estimated from the extensional strain components of the tri-axial strain gauge in consideration of the Natural Strain theory, and the principal deviatoric stress-deviatoric strain curves in each tests were drawn. Next, the value of yield stress was determined in accordance with the method explained in the previous section, and the shape of the yield surface in the stress space was investigated. Then these estimated results were compared with the results based on conventional proof stress.

4 EXPERIMENTAL RESULTS

Figure 3 shows the locus of stress points in each proportional loading tests, which are conducted after applying the pre-deformation to the specimens. Here, ① is the result of 65° (i.e., the direction of the pre-deformation), ② is 70°, ③ is 80°, ④ is 90° (i.e., the case of uni-axial tension), ⑤ is 95°, ⑥ is 105°, ⑦ is 110°, ⑧ is 115°, ⑨ is 120°, ⑩ is 125°, ⑪ is 130°, ⑫ is 135° (i.e., the case of the simple shear), ⑬ is 140°, ⑭ is 145°, and ⑮ is 155° (i.e., the opposite direction of the pre-deformation). As the typical example among these loading paths, the principal deviatoric stress-deviatoric strain curves in case of ①, ④, ⑧, ⑫ and ⑮ are described in Figure 4 (a), (b), (c), (d), and (e), respectively.

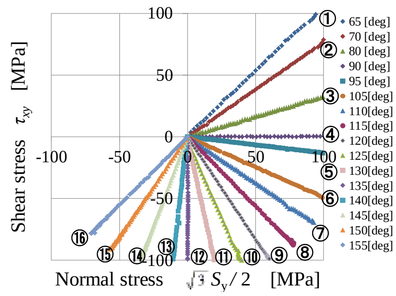


Figure 3. Stress points in each proportional loading tests.

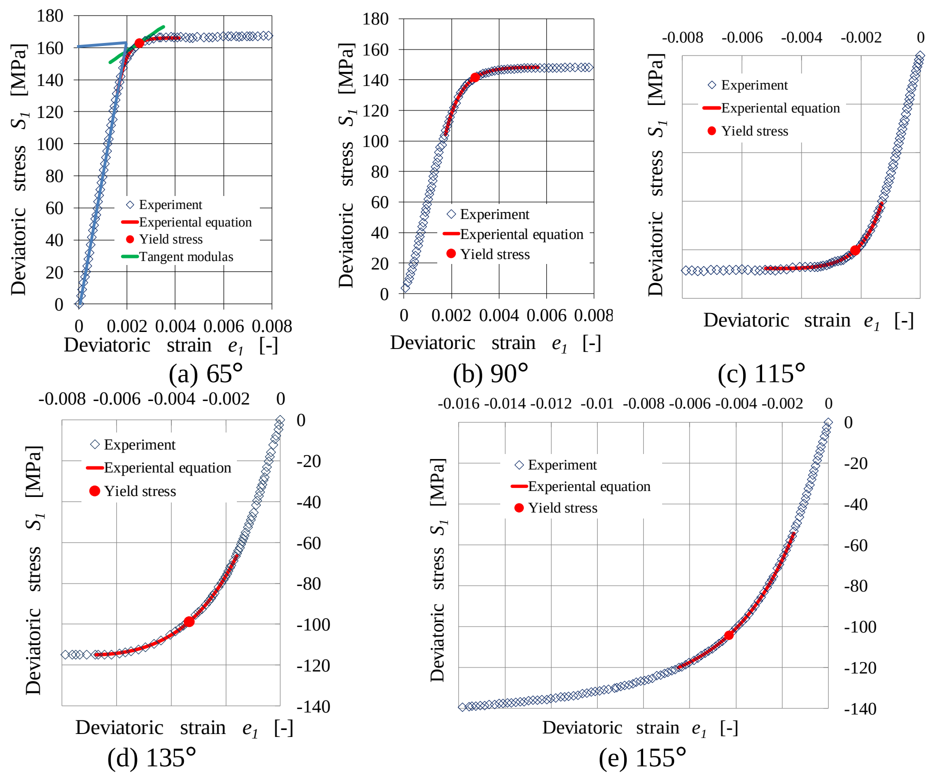
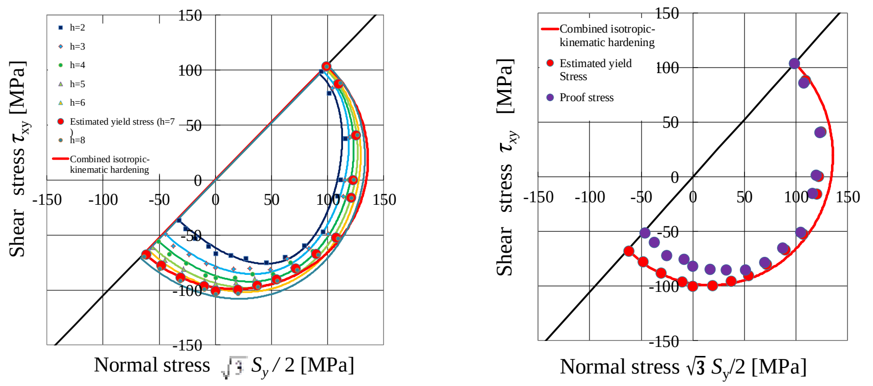


Figure 4. Principal deviatoric stress – deviatoric strain diagram.

Here, the experimental equation, i.e., Eq. (1), is described by the solid curve in each figure. For 65°, the value of yield stress was already decided from the pre-deformation, and indicated by the circular plot (●). The tangent modulus on this plot, i.e., the straight line in Figure 4 (a), can be derived by using Eq. (2). On the other hand, the estimated yield stress are indicated by the circular plots (●) in Figure 4 (b), (c), (d), and (e), respectively. As is obvious from these figures, the stress and strain relation becomes a shallow curve for ①, ④, ⑧, ⑫, and ⑮. Therefore, in the case of proof stress, as the loading direction approaches the opposite side of pre-deformation, the yield stress was estimated to be smaller.

Figure 5 (a) shows the results of the estimated yield stress of all directions in the stress space. The circular plots (●) indicate the estimated yield stress, and the distribution of *h* near the yield surface is described in this figure. As the loading direction approaches the side of pre-deformation, the distance of curves in each modulus *h* becomes narrow. However, it widens as the loading direction approaches the opposite side. The shapes of *h* become convex at the side of pre-deformation.

Finally, Figure 5 (b) shows the results of comparison between the estimated yield surface and conventional proof stress. The shapes of both yield surfaces almost coincide at the pre-deformation side. However, in the opposite side of pre-deformation, the yield surface based on the proof stress becomes smaller compared with the estimated yield surface.



(a)Distribution of *h* and yield surface (b) Comparison of yield surface

Figure 5. Shape of yield surface and distributions of strain hardening modulus *h*.

5 CONCLUSIONS

In this study, the yield stress was estimated by examining the slope of tangent for the principal deviatoric stress-deviatoric strain curve in the proportional loading tests. It was revealed that the shape of the yield surface becomes convex at the side of pre-deformation and becomes flat in the opposite direction of pre-deformation. Moreover, the shape of the estimated yield surface was compared with the shape of conventional proof stress. Consequently, it became clear that the shapes of both yield surfaces almost coincide at the side of pre-deformation, but on the opposite side, the yield surface by proof stress becomes smaller compared with the estimated yield surface.

References

Kato, Y., Kazama, A., and Otsuka, Y., Elasto-plastic analysis for finite deformation using natural strain (shape of yield surface obtained after pre-deformation of uni-axial tension), *Research, Development, and Practice in Structural Engineering and Construction*, Vimonsatit, V., Singh, A. and Yazdani, S. (ed.), 157-162, RPS Publishing, 2012.

Kato, Y., and Kazama, A., Elasto-plastic analysis using natural strain: Shape of yield surface generated after pre-deformation of large simple shear, *Advanced Materials Research* 875-877, 518-523, Feb, 2014.