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NUMERICAL DYNAMIC ANALYSIS OF ORTHOTROPIC PLATES UNDER LOCALIZED BLAST LOADING

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This paper analyzes the dynamic response of fixed supported orthotropic plates under localized blast loading using the method of modal superposition. The analysis procedure is used to quantify the linear transient response of such plates to the localized blast load at different positions. Many studies are currently available, in which the blast load is considered to be spatially uniform across the plate, with a temporal distribution described by a Dirac delta function. The novel aspect considered here is the case for which the blast load is modeled as a linear triangular function, and the orthotropic plate is fixed along its edges. A Mathematica program is used to solve the first and the second auxiliary Levy-type problem to determine the values of the natural frequencies of the system. The results presented here are collected from the results of analyses performed on localized blast-loaded orthotropic plates, for a variety of parameters important with regard to the dynamic response. Conclusions are drawn concerning the influence of the various parameters on the nature of the orthotropic-plate response.

Keywords: Modal superposition, Dirac delta function, Linear triangular function, The first and the second auxiliary Levy-type problem.

1 INTRODUCTION

The study of blast effects on structures has been the subject of considerable research effort. A bomb explosion within or immediately nearby a building can cause catastrophic damage on the building's external and internal structural members, collapsing walls, blowing out large expanses of windows, and shutting down critical life-safety systems. Loss of life and injuries to occupants can result from many causes, including direct blast effects, structural collapse, debris impact, fire, and smoke. The indirect effects can also prevent timely evacuation, thereby contributing to additional casualties.

Strategies for blast protection have become an important consideration for structural designers as global terrorist attacks continue. Conventional structures normally are not designed to resist blast loads. Because the magnitudes of design loads are significantly lower than those produced by most explosions, conventional structures are susceptible to damage from explosions. Further, often conventional structures are susceptible to damage from explosions. With this in mind, engineers increasingly are seeking solutions for potential blast situations, to protect building occupants and the structures.

Pandey et al. (2006) investigated the behavior of reinforced concrete panels reinforced with top and bottom steel meshes subjected to blast load. Blast wave characteristics, including incident and reflected pressures and impulses, as well as panel central deflection, were measured. The post-blast damage and mode of failure of each panel was observed, and those panels that were not completely damaged by the blast were subsequently statically tested to find their residual strength. The results of this study indicate that the retrofit may not be suitable in every situation, and that quantifying its strengthening effects will need more actual blast testing rather than merely theoretical modelling.

Khadid et al. (2007) studied the fully-fixed stiffened plates under the effect of blast loads to determine the dynamic response of the plates with different stiffener configurations, and to consider the effect of time duration.

Alisjahbana and Wangsadinata (2013) studied the dynamic behavior of orthotropic plates subjected to the outside blast. The outside blast is at a constant position and the dynamic deflections were calculated for different values of time duration and number of stiffeners. The results of their work show that the dynamic response of the orthotropic plates is dependent on the number of stiffeners and the time duration of the blast loading.

To further understand the dynamic response of the plate subjected to blast loading, this research studies the effect of the position of the blast loading and the maximum dynamic deflection of the orthotropic plate. These numerical results may serve as design guidelines for structures under the blast loading. It should be noted that experimental studies are costly and dangerous, whereby their result is not always ensured and always show some degree of uncertainty.

2 SOLUTION OF THE GOVERNING EQUATION

The deflection of the orthotropic plate is governed by the following differential equation:

$$\begin{aligned} & D_x \frac{\partial^4 w(x,y,t)}{\partial x^4} + 2B \frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w(x,y,t)}{\partial y^4} + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} \\ & + \gamma h \frac{\partial w(x,y,t)}{\partial t} = p(x,y,t) \end{aligned} \tag{1}$$

where $w(x,y,t)$ is the vertical deflection of the orthotropic plate at point (x,y) and time t; D_x is the bending rigidity in the x-direction, B is the torsional rigidity, D_y is the bending rigidity in the y-direction, ρ is the mass density of the plate, h is the thickness of the plate, and γ is the damping ratio of the plate. The dynamic concentrated load $p(x,y,t)$ is written as:

$$p(x,y,t) = P(t)\delta[x - x(t)]\delta[y - y(t)] = P(t)\delta[x - x_0]\delta[y - y_0] \tag{2}$$

where δ[.] = Dirac delta function; $P(t)$ = concentrated load; $x(t)$ = the position of the concentrated load in the; $y(t)$ = the position of the concentrated load in the y-direction; x_0 = the initial position of the blast loading in the x-direction; and y_0 = the initial position of the blast loading in the *y*-direction. The localized blast loading $P(t)$ which is a linear triangular function can be expressed as:

$$P(t) = P_0\left(1 - \frac{t}{t_d}\right) \tag{3}$$

where P_0 is the maximum amplitude of the blast loading, t_d is the duration of the blast loading. In modal form, the vertical deflection of the orthotropic plate is expressed as:

$$w(x, y, t) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} W_{mn}(x, y)T_{mn}(t) \tag{4}$$

where $W_{mn}(x,y) = X_m(x,y)Y_n(x,y)$; $X_m(x,y)$ and $Y_n(x,y)$ = *m*th and *n*th normal modes of free vibration of the orthotropic plate in the *x*-direction and *y*-direction, respectively. This can be determined from the first and the second auxiliary Levy-type problem (Alisjahbana and Wangsadinata 2013).

2.1 First Auxiliary Levy-type Problem

The solution of the free vibration problem of Eq. (1) can be expressed as:

$$W_{mn}(x, y) = X_m(x)\sin\frac{\pi q y}{b} \tag{5}$$

satisfying the boundary conditions:

$$X_m(x) = \frac{dX_m(x)}{dx} = 0 \text{ at } x = 0 \text{ and } x = a \tag{6}$$

Substituting Eq. (5) into the homogeneous form of Eq. (1) results in an ordinary differential equation for $X_m(x)$:

$$X_m(x) = \cosh(F_1 x) + \left(\frac{bp(c_1 - C_1)}{-\beta s_1 + bpS_1}\right)\sinh(F_2 x) - \cos(F_2 x) - \left(\frac{-\beta c_1 + \beta C_1}{\beta s_1 - bpS_1}\right)\sin(F_2 x) \tag{7}$$

where:

$$\beta = \sqrt{\frac{2Bq^2a^2}{D_x} + p^2b^2}\ ;\ F_1 = \frac{\beta\pi}{ab}\ ;\ F_2 = \frac{p\pi}{ab}\ ;\ C_1 = \cosh\left(\frac{\beta\pi}{b}\right);\ c_1 = \cos(p\pi)$$

$$S_1 = \sinh\left(\frac{\beta\pi}{b}\right);\ s_1 = \sin(p\pi)$$

2.2 Second Auxiliary Levy-type problem

The solution of the free vibration problem of Eq. (1) can be expressed as:

$$W_{mn}(x, y) = \sin\frac{\pi p x}{a} Y_n(y) \tag{8}$$

satisfying the boundary conditions:

$$Y_n(y) = \frac{dY_n(y)}{dy} = 0 \text{ at } y = 0 \text{ and } y = b \tag{9}$$

Substituting Eq. (8) into the homogeneous form of Eq. (1) results in an ordinary differential equation for $Y_n(y)$:

$$Y_n(y) = \cosh(F_3 y) + \left(\frac{bq(c_2 - C_2)}{-\theta s_2 + bqS_2}\right)\sinh(F_3 y) - \cos(F_4 y) - \left(\frac{-\theta c_2 + \theta C_2}{\theta s_2 - bqS_2}\right)\sin(F_4 y) \tag{10}$$

where:

$$\theta = \sqrt{\frac{2Bq^2b^2}{D_y} + q^2a^2}\ ;\ F_3 = \frac{\theta\pi}{ab}\ ;\ F_4 = \frac{q\pi}{ab}\ ;\ C_2 = \cosh\left(\frac{\theta\pi}{a}\right);\ c_2 = \cos(q\pi)$$

$$S_2 = \sinh\left(\frac{\theta\pi}{a}\right);\ s_2 = \sin(q\pi)$$

The unknown quantities p and q are calculated from the transcendental equations as follows (Pevzner et al. 2000):

$$\frac{\pi}{a^2b^2}\left(2bp\beta - 2bp\beta\cos(p\pi)\cosh\left(\frac{\pi\beta}{b}\right) - \left(b^2p^2 - \beta^2\right)\sin(p\pi)\sinh\left(\frac{\pi\beta}{b}\right)\right) = 0 \tag{11}$$

$$\frac{\pi}{a^2b^2}\left(2aq\theta - 2aq\theta\cos(p\pi)\cosh\left(\frac{\pi\theta}{a}\right) - \left(a^2q^2 - \theta^2\right)\sin(q\pi)\sinh\left(\frac{\pi\theta}{a}\right)\right) = 0 \tag{12}$$

The normal modes are determined as the product of Eq. (7) and Eq. (10).

3 NUMERICAL CALCULATIONS

Based on the theory above, numerical calculations are carried out to illustrate the dynamic deflection response of an orthotropic plate to the blast loading at different positions and values of t_d. The orthotropic plate is subjected to a blast loading at different positions. In the following, a four-edge fixed supported orthotropic rectangular plate is studied as an example.

Let us illustrate with a concrete numerical example. Assume a concrete rectangular orthotropic plate, given E = 210 GPa, ρ = 2400 kg/m^3, γ = 5%, υ = 0.2, a = 5.5 m, b = 4.5 m, h = 0.16 m, 0.18 m and 0.2 m, P_0 = 1.3 MPa (Khadid 2008). In order to study the effect of the position of the blast loading, four positions have been used in this study as follows: $x_0 = 0.125a$; $0.25a$; $0.375a$; and $0.5a$. Modified Bolotin Method (MBM) has been applied to obtain the natural frequencies of the fixed orthotropic plate.

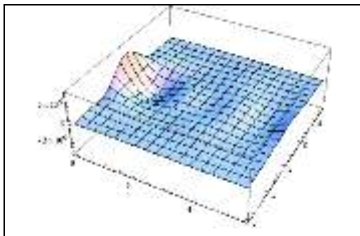
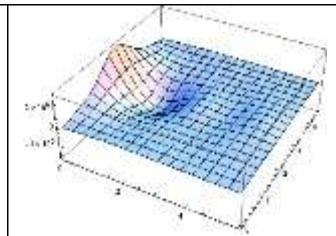
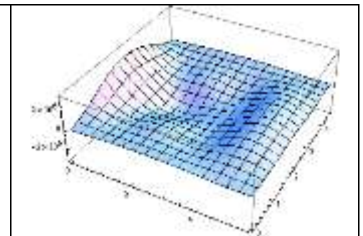
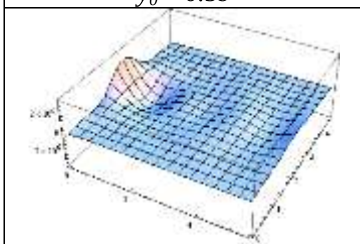
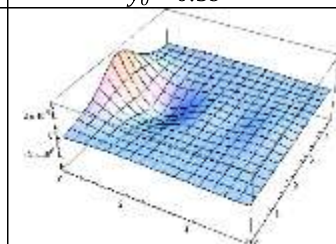
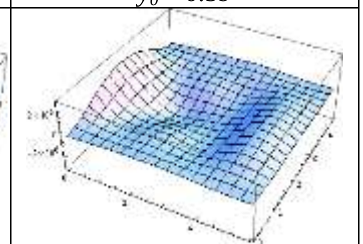
		
h = 16 cm, $x_0 = 0.125a$, $y_0 = 0.5b$	h = 16 cm, $x_0 = 0.25a$, $y_0 = 0.5b$	h = 16 cm, $x_0 = 0.375a$, $y_0 = 0.5b$
		
h = 20 cm, $x_0 = 0.125a$, $y_0 = 0.5b$	h = 20 cm, $x_0 = 0.25a$, $y_0 = 0.5b$	h = 20 cm, $x_0 = 0.375a$, $y_0 = 0.5b$

Figure 1. Three-dimensional moment-x distribution of orthotropic plate to blast loading (γ = 5%, t_d = 2ms).

Table 1. The maximum dynamic deflection of the orthotropic plate subjected to blast loading as a function of position (t_d = 2ms).

x_0	***h* = 16 cm** w_{max} **(m)**	***h* = 18 cm** w_{max} **(m)** **γ = 0%**	***h* = 20 cm** w_{max} **(m)**	***h* = 16 cm** w_{max} **(m)**	***h* = 18 cm** w_{max} **(m)** **γ = 5%**	***h* = 20 cm** w_{max} **(m)**
0.125 *a*	0.00137509	0.00111798	0.00089858	0.00073848	0.000601885	0.000510213
0.25 *a*	0.0030529	0.00234992	0.00199192	0.00147865	0.00154278	0.0013031
0.375 *a*	0.00405889	0.00319664	0.00257393	0.0034592	0.00271042	0.00210699
0.5 *a*	0.00500864	0.00397082	0.00306262	0.00428886	0.00332483	0.00258333

3.1 Effects of the Position of the Blast Loading

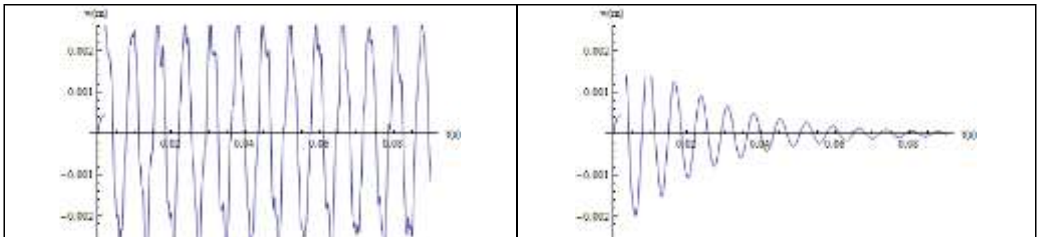
The maximum dynamic deflections of the orthotropic plate for different positions of the blast loading (x_0, y_0) are presented in Table 1. Four different positions of the blast loading and the three different plate thickness (h) are tabulated. It can be seen that the maximum dynamic deflection increased gradually as the blast loading moved to the middle span of the orthotropic plate. The maximum dynamic deflection of the system increased by 19.34% if the position of the load $x_0 = 0.375a$ moves to $x_0 = 0$; $5a$ for the value of h = 20 cm.

3.2 Effects of the Plate Thickness on the Dynamic Response of the Plate

The time history of the dynamic deflection at the center of the plate $w(a/2,b/2)$ was calculated and plotted for m = 1,2,3,…,5 and n = 1,2,3,…,5. The effect of the plate thickness is tabulated in Table 1. The maximum dynamic deflection decreased with the increase in the plate thickness. Increasing the plate thickness by 2 cm decreased the maximum dynamic deflection by 18.5% for position of the blast loading at $x_0 = 0.125a$.

3.3 Effects of the Damping Ratio

As seen in Fig. 2, the effect of the damping ratio is that the maximum dynamic deflection response decreased where damping became higher in all cases. It can be seen that the deflection magnitudes diminished with an increase in damping ratio in all cases. The dynamic deflection of the system at mid-span for the value of h = 20 cm and x_0 = 0.5a were reduced by 15.6%. By increasing the damping ratio, the internal forces and internal moment distribution also decreased significantly.



Dynamic deflection time history, γ=0%	Dynamic deflection time history, γ=5%

Figure 2. Dynamic deflection response of undamped orthotropic plate (left) and damped orthotropic plate (right) to blast loading. Parametric values: $x_0 = 0.5a$, $y_0 = 0.5b$, $t_d = 2ms$, h = 20 cm.

4 CONCLUSION

This paper offered an analytical solution for the dynamic response of the orthotropic plates with fully-fixed supported boundary conditions under the blast loading at different positions. The maximum dynamic deflection and the internal moment and shear force distribution of the plate were illustrated and analyzed. The results obtained are applicable to further research in this field. To supply a more accurate orthotropic plate for practical application, the idealization of the blast loading can be done by using the exponential function and the semi-rigid boundary conditions of the orthotropic plates. This should be considered in future studies.

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