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INFLUENCE OF MOMENT GRADIENT ON ROTATION CAPACITY OF STEEL FLEXURAL MEMBERS

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Current seismic design procedures implicitly permit the inelastic structural deformations under strong ground motions for economic reasons. The local ductility is used as a parameter for evaluating the available inelastic performances of structures. Current code provisions take into account the ductility at the level of section, but it is necessary to study member ductility. One of the main factors influencing the value of ductility is moment gradient. This research presents a new method to determine ductility for constant-moment loading of flexural members based on theoretical and numerical studies. A numerical study was carried out on welded I-beams under a monotonic static loading, using nonlinear buckling analysis considering local and overall instabilities. The results illustrate that this new method is more accurate than others, can be used to predict ductility for constant-moment loading, and may be employed for different gradients of moments.

Keywords: Ductility, Buckling, Constant moment, Finite element, Nonlinear analysis.

1 INTRODUCTION

In classical plastic theory it is assumed that in the correspondence of a plastic hinge of a bent member, the plastic moment remains constant as far the rotation increases. This assumption neglects two important aspects: 1) the additional moment capacity due to the strain-hardening effects, and 2) the possibility of moment reduction due to the local buckling of flanges and/or web. Neglecting the first aspect is on the safe side, but the omission of the second aspect can produce an uncontrolled plastic redistribution followed by an important increasing of lateral displacement. To avoid that, a method for eliminating the achievement of the strong degradation is required. Building codes use the concept of cross-section behavioral classes to provide a sufficient plastic rotation capacity for a well-controlled redistribution. This is a qualitative method, in which the actual behavior cannot be determined. A more accurate method to determine the rotation capacity is based on the actual behavioral M-θ curve. On this base, two different approaches can be followed (Figure 1). In the first approach, rotation capacity is defined by considering the stable part of M-θ curve (Eq. 1):

$$R_{\max} = \frac{\theta_{p,\max}}{\theta_p} = \frac{\theta_{\max} - \theta_p}{\theta_p} = \frac{\theta_{\max}}{\theta_p} - 1 \tag{1}$$

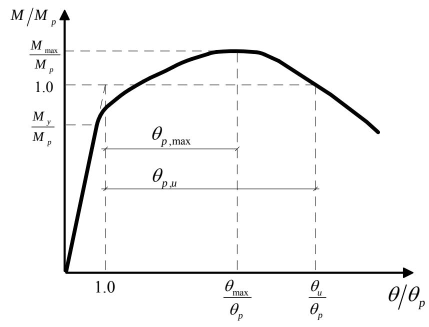

Figure 1. Moment-rotation curve.

$$R_u = \frac{\theta_{p,u}}{\theta_p} = \frac{\theta_u - \theta_p}{\theta_p} = \frac{\theta_u}{\theta_p} - 1 \tag{2}$$

where θ_p is the rotation corresponding to the full plastic moment M_p, and θ_{Max} is the plastic rotation corresponding to the maximum value of the moment M_{Max}. This approach has been followed by Mazzolani et al. (1993) and Kato (1990).

In the second approach, rotation capacity is defined by considering also the unstable branch of the M-θ curve up to the value θ_u, which corresponds to the plastic moment M_p in the lowering curve (Eq. 2). Under these conditions, the second approach seems to be more reliable. Nonetheless, for different moment gradients, Eq. (2) is not applicable due to the fact that, for example, in case of a constant-moment loading, the moment rotation curve may not reach line $M/M_p = 1$. Therefore, the reduced-moment method is recommended to determine ductility of flexural members under uniform moment loading.

Thus far, different moment reduction values have been employed by researchers—for example under $0.90M_p$ and $0.95M_p$ (Gioncu et al. 2002). Unfortunately there is no standard definition of the ductility for constant-moment loading in flexural members; therefore the current work is focused on this issue, presenting a new method to determine ductility for such cases.

2 THEORETICAL STUDY

2.1 Equivalent Plastic Modulus

According to the results of numerical and experimental studies conducted on simply-supported beams subjected to uniform loading, it was observed that there is merely one plastic hinge located at the center of the beam span in most cases (see Figure 2b). The number of plastic hinges may take with N = r + 1, where N is number of plastic hinges in the structure and r is degree of indeterminacy. Consequently, for different loading conditions of a simply-supported beam, the number of plastic hinges is similar, and with this assumption it can consider the same rotation value for 3- and 4-point bending when focused on the collapse mechanism. The beam-end rotation can be determined by the integration of the curvature diagram between support and mid-span (see Figure 2).

$$\theta_{3P} = \theta_{4P-Eq} \tag{3}$$

$$\int_0^{L/2} \chi_{3P} = \int_0^{L/2} \chi_{4P-Eq} \tag{4}$$

$$\frac{M_p.L}{4EI} = \frac{M_{pe}.L}{3EI} \tag{5}$$

For 3-point bending (moment gradient), the plastic moment is $M_p = Z_p f_y$, and for 4-point bending (constant moment), an equivalent value for the plastic moment and plastic modulus is considered in this research as:

$$M_{pe} = Z_e f_y \tag{6}$$

$$Z_{eq} = f_{eq} Z_p \tag{7}$$

where M_{pe} is equivalent plastic moment, Z_{eq} is the equivalent plastic modulus, and f_{eq} is equivalent factor. Increasing the number of point loads may change the moment gradient in flexural members, but the same procedure can be employed to determine the equivalent factor for these cases. The results of the illustrated equivalent factor for 4-point bending are equal to 0.75 (see Figure 2). For other moment gradients, the equivalent factor is very close to this value.

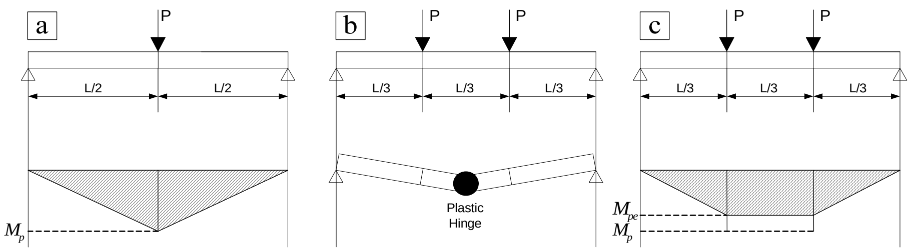

Figure 2. Moment gradient VS constant-moment loading and forming plastic hinge.

2.2 Correction Coefficient

The above procedure may adjust the moment-rotation curve to meet the theoretical value of plastic moment. However, the theoretical study has shown that basically there is a difference in the amount of ductility obtained from 3-point and 4-point bending. It is necessary to revise the results of the last step after considering the equivalent factor. This approach needs to calculate the theoretical value of rotation capacity for these two loading patterns, and derive a correction coefficient to equalize the value to 3-point bending rotation capacity. This has been used as a reference load pattern by previous researchers. For 3-point bending (see Figure 3), the length of the fully plastic zone is given by Gioncu et al. (2002) as:

$$\frac{L_p}{L} = 1 - (\frac{M_p}{M_{ph}});\ M_{ph} = m_h M_p;\ \chi_{ph} = s_h . \chi_p;\ s_h = s + (m_h - 1)e_h\ ;\ e_h = \frac{E_h}{E} \tag{8}$$

where M_{ph} is the moment in the hardening strain range, E_h is the strain hardening modulus, and s is the length of the yielding plateau. The beam-end rotation and rotation in the strain-hardening range can be determined by the integration of the curvature diagram (Figure 3):

$$\theta_p = \int_0^{L/2} \chi . d_x \tag{9}$$

$$\theta_u = \int_0^{\frac{L}{2}-\frac{L_p}{2}} \chi . d_x + \int_{\frac{L}{2}-\frac{L_p}{2}}^{\frac{L}{2}} \chi . d_x \tag{10}$$

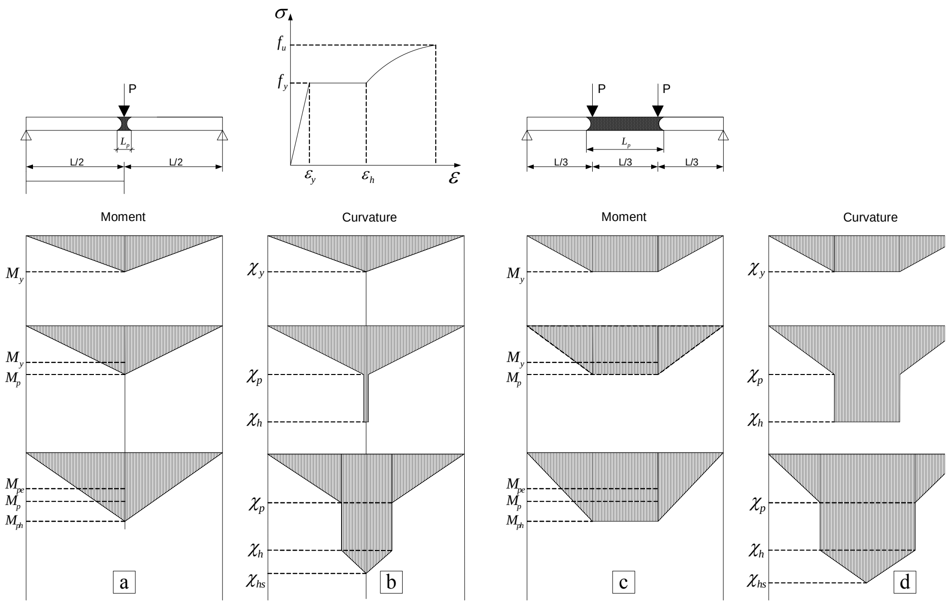

Figure 3. Moment and curvature diagram for constant moment and moment gradient in hardening-strain portion.

The rotation capacity corresponding to plastic results is rendered as:

$$\mu_\theta = \left(\frac{\theta_u}{\theta_p}\right) - 1 \tag{11}$$

$$\mu_{\theta-3p} = \left(\frac{1}{m_h}\right)\left(m_h - 1\right)\left(s + e_h m_h - e_h\right) \tag{12}$$

where $\mu_{\theta-3p}$ is the theoretical value of rotation capacity for 3-point bending. With same procedure, a formula for 4-point bending can be derived as (Figure 3):

$$\frac{L_p}{L} = 1 - \left(\frac{2M_p}{3M_{ph}}\right) \tag{13}$$

$$\mu_{\theta-4p} = \left(\frac{1}{2m_h}\right)\left(1 + 3s.m_h - 2s + 3e_h m_h^2 - 5e_h m_h + 2e_h - 2m_h\right) \tag{14}$$

The ratio of 3-point to 4-point bending is used as the correction coefficient:

$$Coeff_{\left(3p/4p\right)} = \frac{\mu_{\theta-3p}}{\mu_{\theta-4p}} = \frac{2(m_h - 1)(s + e_h.m_h - e_h)}{\left(1 + 3s.m_h - 2s + 3e_h.m_h^2 - 5e_h.m_h + 2e_h - 2m_h\right)} \tag{15}$$

3 NUMERICAL STUDY

A nonlinear finite-element modeling technique was employed to conduct a parametric study on doubly-symmetrical steel I-shaped beams. The results were used to validate the above theoretical studies. Steel grade Q345 was used according to Chinese Standard GB/T1591 (2008). The considered beams consisted of compact webs with various flange slendernesses subjected to both 3- and 4-point bending.

The finite-element software package ANSYS 12.0 was employed in this research. In modeling studies where inelastic buckling was investigated, the strategy of seeding the finite element mesh with an initial displacement field was employed. In this technique, the finite element mesh was subjected to a linearized-eigenvalue buckling analysis from which an approximation to the first buckling mode of the flexural member was obtained. The displacement field associated with this lowest mode was then superimposed onto the finite element model as a seed imperfection for use in the incremental nonlinear analysis, the imperfection values being based on GB50205 (2001). The residual stress has a significant influence on rotational behavior of flexural members (Trahair et al. 2007). Consequently, for this numerical study, a residual stress pattern proposed by Ban et al. (2012) was employed.

4 RESULTS AND DISCUSSION

Figure 4a shows three moment-rotation curves which all resulted from numerical studies. 3P represents 3-point bending; 4P-original and 4P-new method illustrate the results of constant moment before and after applying an equivalent method. All three of these curves resulted from similar beams using identical material and geometry. As can be seen in this figure, the 4P-original is not able to meet $M/M_p = 1$.

As mentioned previously, prior works suggested that the reduced-moment method may be used for constant-moment loading, but these results show the accuracy of equivalent methods compared with the reduced-plastic-moment method. Table 1 and Figure 4b, present the ductility values for four different flange slenderness values (b represents free outstands of a compression flange). To compare the proposed method and reduced plastic moment method, different reduction factors (90% and 95%) were used. The average result of the proposed method illustrates that this method predicts closer ductility values when compared to other methods discussed.

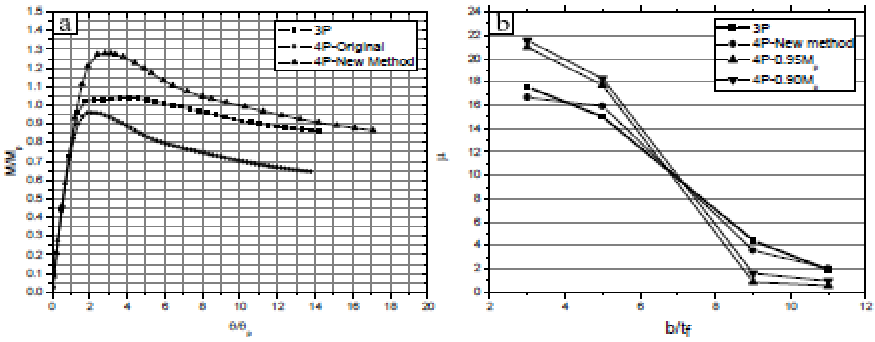

Figure 4. (a) Moment-rotation curves to compare proposed equivalent method and moment gradient; (b) Comparing the different methods to determine ductility with reference value.

Table 1. Results of ductility of different methods.

(b/t_f)	**3**	**5**	**9**	**11**
3P	17.58	15.02	4.42	1.92
4P-New method	16.67	15.92	3.57	2.02
$4P-0.95M_p$	20.96	17.75	0.85	0.54
$4P-0.90M_p$	21.55	18.30	1.61	0.98

5 CONCLUSION

This paper aimed to equalize the ductility value of constant-moment and moment-gradient patterns, as well as generalizing the results of previous works on these patterns. Its results illustrated that the new method to determine rotation capacity for uniform moment loading is more accurate than other methods, and results are closer to 3-point bending beams. Consequently this method can be used to predict ductility for constant-moment loading, and may be employed for different gradients of moments.

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