Sustainable Solutions in Structural Engineering and Construction
Edited by Chantawarangul, K., Suanpaga, W., Yazdani, S., Vimonsatit, V., and Singh, A.

ISBN: 978-0-9960437-0-0

MODAL PARAMETER ESTIMATION OF LTI SYSTEM USING HILBERT-HUANG TRANSFORMATION OF MEASURED WIRELESS SENSOR DATA

MEDA VINAY TEJA, SWARUP MAHATO, and ARUNASIS CHAKRABORTY

Dept of Civil Engineering, Indian Institute of Technology, Guwahati, India

An iterative Hilbert-Huang transformation (HHT) based algorithm is developed to extract the modal parameters of a linear time invariant (LTI) system excited by recorded non-stationary ground motion. The acceleration responses are measured using wireless sensors, which are filtered to avoid mode mixing prior to evaluate the instantaneous amplitude and phase using HHT. The band width is adjusted in successive iterations to achieve convergence in modal parameter estimation. The numerical study presented in this work discusses the efficiency of the identification strategy in the light of noise contaminated earthquake responses.

Keywords: HHT, EMD, IMF, System identification, Time-frequency analysis.

1 INTRODUCTION

Estimation of dynamic properties (natural frequencies, mode shapes and damping ratios) of structures are the primary step in health monitoring and prognosis of structural elements like beams/columns (Yang and Nagarajaiah 2014), rotors in mechanical systems (Li *et. al.* 2013), bridges (Hester and Gonzalez 2012). In the last two decades extensive research has been carried out to develop various system identification strategies that estimate modal properties. Parameters estimation from the structural vibration responses are done in three different paradigms – (a) Frequency domain analysis, (b) Time domain analysis and (c) Time-Frequency domain analysis. Frequency domain analysis involves mostly Fourier transform based signal processing. However, the major drawback of any Fourier based analysis of non-stationary signals lie with its inherent ability to average out frequency information with time. Compare to Fourier based algorithms, time domain techniques are more robust but faces difficult to implement for real life structures. In this context, time-frequency based techniques have gained popularity in the recent past due to their inherent ability to extract localized features of any signal. Feldman (1997) proposed system identification technique based on the Hilbert transform (HT) which is applicable to mono-component signals. Other time-frequency based methods such as short time Fourier transform (Narasimhan and Nagarajaiah 2005, Nagarajaiah and Varadarajan 2005) and wavelet transform (Chakraborty *et. al.* 2006, Basu *et. al.* 2008) have received considerable attention in the recent past. Among other time-frequency based approach, HHT based non-stationary signal processing as proposed by Huang *et. al.* (1998) is efficient to extract the modal features of a dynamical system. This method is applicable to non-stationary and non-linear signals (Carbajo *et. al.* 2013, Yang *et. al.* 2003). However, HHT based technique to extract modal parameters are mostly limited to impulse or ambient response only, as the method suffers serious setback to identify dynamic parameters from earthquake response due to mode mixing. With this in view, present study aims to address the use of HHT to identify LTI system parameter from the earthquake response. For this purpose, the acceleration response of a steel frame tested in laboratory using recorded El-Centroaccelerogram. The numerical results demonstrate the efficiency of the proposed iterative HHT based identification scheme in presence of noise contamination.

2 HHT BASED PARAMETER ESTIMATION

The dynamic response of $\boldsymbol{n}$ degrees of freedom (dof) system subjected to an excitation $\mathbf{P}(\boldsymbol{t})$ is obtained by solving the differential equation:

$$\boldsymbol{M}\ddot{\boldsymbol{Y}}(t) + \boldsymbol{C}\dot{\boldsymbol{Y}}(t) + \boldsymbol{K}\boldsymbol{Y}(t) = \boldsymbol{P}(t) \tag{1}$$

where, $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ are $n \times n$ mass, damping and stiffness matrices respectively and $\boldsymbol{Y}(t)$ is the $n \times 1$ displacement vector. The over dotting in Eq. (1) represents differentiation with respect to time. The acceleration response ($\ddot{\boldsymbol{Y}}(t)$) can be represented as the summation of the modal accelerations and may be expressed as:

$$\ddot{\boldsymbol{Y}}(t) = \sum_{j=1}^{n} \phi_j \ddot{y}_j(t) \tag{2}$$

The term $\phi_j$in the above equation representsj^{th} mode shape of the system. Assuming the forcing function in Eq. (1) to be impulse (i.e., $\mathbf{P}(t) = P_0$), one can evaluate the acceleration in j^{th}mode as:

$$\ddot{y}_j(t) = \frac{\phi_j P_0 \omega_{dj}}{m_j} e^{-\eta_j \omega_{nj} t} \sin(\omega_{dj} t + \psi_j) \tag{3}$$

Where, m_j, ω_{nj}, ω_{dj} and η_j are the modal mass, natural frequency, damped natural frequency and damping ratio respectively.

2.1 Hilbert–Huang Transformation

A brief overview of HHT is presented here for completeness. The Hilbert transform (HT) of a mono component signal ($f(t)$) with finite energy is given by (Bendat 2010):

$$\tilde{f}(t) = \int_{-\infty}^{+\infty} \frac{f(t)}{\pi(t-\tau)} d\tau \tag{4}$$

Using the above expression for $\tilde{f}(t)$, the analytic signal is constructed as :

$$S(t) = f(t) + i\tilde{f}(t) = A(t)e^{i\alpha(t)} \tag{5}$$

Where, $A(t)$ and $\alpha(t)$ are the instantaneous amplitude and phase of the signal $f(t)$. Huang *et. al.* (1998) furthers this method for multi-component non-stationary signals, where the non-stationary signal $\bar{f}(t)$ is decomposed into its empirical modes as:

$$\bar{f}(t) = \sum_{k=1}^{m} f_k(t) + r_c \tag{6}$$

Where, $f_k(t)$ and $r_c$are the k^{th} intrinsic mode function (IMF) and the residue left after IMF extraction. Once the IMFs are obtained, HT can be used on each $f_k(t)$to obtain the instantaneous amplitude and phase as discussed above.

2.2 Estimation of Modal Properties

The HHT based time-frequency analysis discussed in the previous section is further used for modal parameter estimation. For this purpose, HHT of Eq. (3) is considered to evaluate the instantaneous amplitude and phase forj^{th} mode from its analytic signal, which takes the following form:

$$A_j(t) = \frac{\phi_j P_0 \omega_{dj}}{m_j} e^{-\eta_j \omega_{nj} t} \tag{7.a}$$

$$\alpha_j(t) = \omega_{dj} t + \psi_j \tag{7.b}$$

Taking logarithms of both sides of the Eq. (7.a), one can show that:

$$\ln\left(A_j(t)\right) = -\eta_j \omega_{nj} t + \ln\left(\frac{\phi_j P_0 \omega_{dj}}{m_j}\right) \tag{8}$$

Eq. (8) represents a straight line whose slope represents the product of η_j and ω_{nj}. Furthermore, the differentiation of both sides of Eq. (7.b) provides the damped natural frequency ω_{dj}, which is given by:

$$\frac{d\left(\alpha_j(t)\right)}{dt} = \omega_{dj} \tag{9}$$

Eq. (8) and Eq. (9) together provide the modal parameters of the system. The force due to earthquake ground motion has arbitrary magnitude that can be modeled as a train of impulse at different time instant $t_i; i = 1,2,3, \ldots n$ which is given by:

$$P(t) = \sum_{i=1}^{n} P_i \delta\,(t - t_i) \tag{10}$$

Using Eq. (10), one can estimate the total response due to earthquake load by method of superposition. Finally, Eq. (7) and Eq. (8) are adopted to evaluate modal parameters. It may thus be noticed that a direct implementation of HHT on the total response to identify the parameters often suffer inaccuracy due to mode mixing (Montejo *et al.* 2012). To avoid this problem, an iterative band pass filtering is proposed prior to applying HHT. The width of the band pass filters are controlled from coarser to finer in each iteration. This procedure of band pass filtering is stopped as soon the difference in estimated modal parameters falls below the prefixed threshold. Finally, the mode shape is obtained from the IMFs at different dof, given by:

$$\phi_j^l = \frac{IMF_j^l}{IMF_j^1} \tag{11}$$

where, $l = 1,2,\ldots,n$. It is seen that without loss of generality the value of mode shape at the first dof is considered to be unity while others are expressed as the ratio to it.

3 NUMERICAL RESULTS AND DISCUSSION

A three-storied steel frame as shown in Figure 1 is used in this study for experimental validation. The model has equal mass of 15.20 kg at each floor level and stiffness of 41.987, 76.842 and 74.812 kN/mat 1st, 2nd and 3rd floor respectively. The recorded accelerogram (El-Centro) is applied using the uni-axial shake table and the accelerations are recorded at each floor using wireless sensors as shown in Figure 2. Although this figure shows only the forced response, the recording continued to capture the free response of the model. The proposed iterative scheme is then adopted on these forced responses to identify the modal parameters. For this purpose, coarser band pass filters with 3 divisions in a frequency range of 0 to 25Hz are initially adopted. Using these filtered signals, the HHT based algorithm as discussed in Eq. (7) to Eq. (11) are used to identify the natural frequencies and damping ratios.

The frequency range is further sub-divided into smaller divisions and the identification process continued until the convergence in parameter estimation is achieved. Figure 3 shows the IMFs corresponding to three modes obtained at the end of the iterations. Figures 4 and 5 show the instantaneous amplitude and phase of the three measurements as described in Eq. (8) and Eq. (9). It may be noticed that the instantaneous amplitude varies initially due to the transients present in the measurements. Using these figures, modal parameters are identified in Table 1.

From this table, one can notice that the errors in estimating frequencies are well within 5% (the modal damping ratios have more error when evaluated from the seismic response than the free responses). Finally, Figure 6 shows the mode shapes as described in Eq. (11). A close match is noticed between the theoretical and estimated results, which clearly indicate the accuracy of the proposed algorithm.

Table 1. Original and estimated values of modal parameter.

Mode	Original	Estimated			
		Free vibration response		Earthquake response	
	ω_n (Hz)	ω_n (Hz)	η (%)	ω_n (Hz)	η (%)
1st	4.1565	4.1972	0.27	4.1911	0.15
2nd	12.8093	12.8527	0.16	12.8075	0.22
3rd	19.8511	19.9532	0.21	19.9199	0.00

Figure 1. Laboratory test set-up.

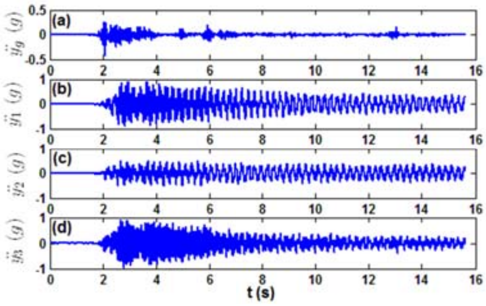

Figure 2. Excitation and responses (a) El-Centro motion; Response of (b) 3rd, (c) 2ndand (d) 1stfloor.

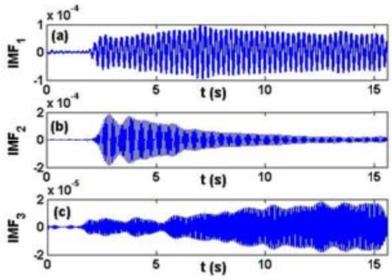

Figure 3. IMFs obtained from top floor response (a) 1st(b) 2nd and (c) 3rdmode.

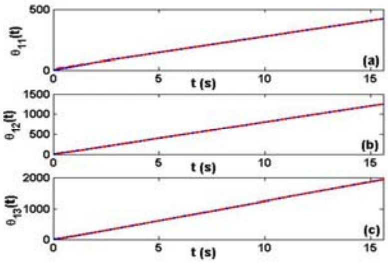

Figure 4. Instantaneous phase angle (a) 1st (b) 2nd and (c) 3rdmode.

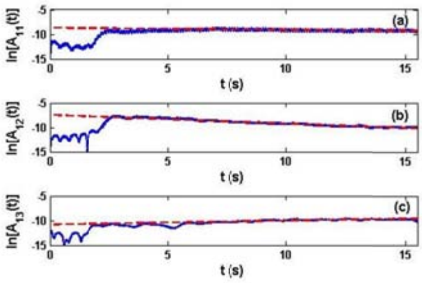

Figure 5. Instantaneous Amplitude (a) 1st (b) 2nd and (c) 3rd mode.

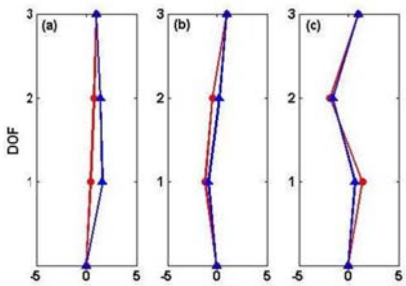

Figure 6. Identified (--o--) and theoretical (--Δ--) mode shapes. (a) 1st (b) 2nd and (c) 3rd mode.

4 CONCLUSIONS

In this paper, an iterative HHT based identification scheme is presented to evaluate the dynamic parameters of a LTI system from earthquake response. The methodology works well to identify the natural frequencies and mode shapes from the non-stationary response and hence can be adopted immediately after any seismic event as the application of known excitations (e.g. impulse, sinusoids) are not recommended without assessing the condition of the structure. Once the modal parameters are identified, they can be used to infer structural health, which is essential for maintenance and future modification if any.

Acknowledgements

Authors acknowledge the financial support received from BRNS, Govt. of India (Grant No. 2012/36/03/-BRNS dated 25th July 2012) for this work.

References

Basu, B., Nagarajaiah, S., and Chakraborty, A., Online Identification of Linear Time-varying Stiffness of Structural Systems by Wavelet Analysis, *Structural Health Monitoring*, 7(1), 21-36, 2008.

Bendat, J., and Piersol, A., *Randam Data: Analysis and Measurements Procedures*, 4th Edition, Jhon Willey & Sons, Inc., 2010.

Carbajo, E.S., Carbajo, R.S., Goldrick, C.M., and Basu, B., ASDAH: An automated structural change detection algorithm based on the Hilbert–Huang transform, *Mech. Syst. Signal Process*, 2013.

Chakraborty, A., Basu, B., and Mitra, M., Identification of modal parameters of a mdof system by modified L–P wavelet packets, *Journal of Sound and Vibration*, 295, 827–837, 2006.

Feldman, M., Non-Linear Free Vibration Identification Via The Hilbert Transform, *Journal of Sound and Vibration*, 208(3), 475-489, 1997.

Hester, D., and Gonzalez, A., A wavelet-based damage detection algorithm based on bridge acceleration response to a vehicle, *Mech. Syst. and Signal Processing* 28, 145–166, 2012.

Huang, N., Shen, Z., Long, S., Wu, C., Shih, H., Zheng, Q., Yen, N.-C., Tung, C., and Liu, H., The Empirical Mode Decomposition and the Hilbert Spectrum for Nonlinear and Non-Stationary Time Series Analysis, *Math., Physical and Engrg. Sciences*, 454(1971), 903-995, 1998.

Li, B., Zhang, C., and He, Z., HHT-based crack identification method for start-up rotor, *Frontier Mechanical Engineering*. 7(3), 300–304, 2012.

Montejo, L.A., Velazquez, L.R., Ramirez, R.I., and Jang, S., Wavelet and HHT based identification of different levels of inelastic action of RC structures, *15th WCEE*, 2012

Nagarajaiah, S., and Varadarajan, N., Short time Fourier transform algorithm for wind response control of buildings with variable stiffness TMD, *Engrg. Structures*, 27, 431–441, 2005.

Narasimhan, S., and Nagarajaiah, S., A STFT semiactive controller for base isolated buildings with variable stiffness isolation systems, *Engrg. Structures*, 27, 514–523, 2005.

Yang, J., Lei, Y., Pan, S., and Huang, N., System identification of linear structures based on Hilbert–Huang spectral analysis. Part 1: normal modes, *Earthquake Engrg. Structural Dynamics*, 32, 1443–1467, 2003.

Yang, Y., and Nagarajaiah, S., Structural damage identification via a combination of blind feature extraction and sparse representation classification, *Mechanical Systems and Signal Processing*, 45(1), 1 - 23, 2014.