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ON EFFICIENCY OF EKF FOR PARAMETER ESTIMATION OF LTI SYSTEM FROM NON-STATIONARY ACCELERATION RESPONSES

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Parameters of the linear time invariant (LTI) dynamic system using extended Kalman filter (EKF) are identified in this work. The efficiency of EKF for parameter estimation of LTI system is studied. For this purpose, a three-story steel frame is used in the laboratory, and the recorded ground motion is applied to measure the acceleration response at different floor levels. Using these responses, the EKF-based predictor-corrector algorithm is used to identify the modal parameters. It has been observed that the EKF-based identification scheme can converge to different system matrices (*i.e.,* mass and stiffness) in different experiments for the same structure. However, their eigen values (*i.e.*, natural frequency and mode) remain the same.

Keywords: Extended Kalman filter, Non-stationary excitation, System identification, State estimation.

1 INTRODUCTION

The Kalman filter (KF) (Kalman 1960), essentially a method of sequential least-squares estimation, has been successfully applied in many field and many applications in structural dynamics, some of which are discussed here. Hoshiya and Saito (1984) introduce this method to evaluate structure conditions. The original KF method is applicable to systems with linear sate vector so, more specifically, they use an extended Kalman filter (EKF) proposed by Kalman and Bucy (1961) for non-linearly-related state vectors. In their study, they used a three-degrees of freedom system (3DOF) to identify the stiffness and damping, and also used an equivalent nonlinear model to identify the hysteresis parameter of the system. In this study, they proposed using a forgetting factor for some parameters to get better convergence of parameters. Though they did not give any formulation to evaluate this forgetting factor (except with different values for it), they present good results. In 1994, Xia *et al.* give a proposal to formulate the calculation for this fading factor or adaptive factor. In this formulation, residual covariance is minimized for some targeted parameter. Thus, its memory of last value is faded. The type of this factor is exponential, i.e., the value is reduced as more samples are contributed to the filter.

In 1994, Lin and Zhang successfully applied the method of global iteration proposed by Hoshiya and Saito for the Bouc-Weng nonlinear model, where the input is simulated earthquake data. They show that with a high value of weight, the obtained result is more realistic. Instead of applying the factor in a covariance matrix, Sato and Sato (1997) suggest using a neural network method, where the observed and estimated vectors will be factorized after the first iteration to obtain good convergence. To prove this, they used a 2DOF nonlinear system and updated the state vector. In their work they also presented their results for a building of 8DOF with nonlinearity. In this case, estimated displacement and force-displacement hysteresis were compared with recorded displacement and force-displacement hysteresis, obtaining a physically-affordable result. This method was also used in parametric estimation and damage identification for different models by Corigliano and Mariani (2004), Yang *et. al.* (2006), Gao and Lu (2006), and Ghanem and Ferro (2006).

According to the method proposed by Ghosh, Roy and Manohar (2007), there is no need to evaluate Jacobian matrix in each iteration. They gave two methods: locally-transversal linearization (LTL) and multi-step transversal linearization (MTrL). In the LTL method, the nonlinear states or vector field was replaced by a time-invariant conditionally-linearized vector field for a time step, and then KF estimation was applied. In their second proposal, the filter worked over multiple time-steps, finding the system transition matrix of the conditionally linearized vector field through Magnus' expansion. Though they did not check for ultimate convergence, in their study they showed this method is less sensitive to a computational time step. In their study only the time invariant parameter is considered, so there is no insight in case things like sudden parameter property changes happened.

Tipireddya, Nasrellah, and Manohar (2009) showed their results on parameter identification under a moving load and an incremental static load. For moving-load analysis, a point load over a beam was taken. In the case of incremental static load, a truss was considered. As a result, the reaction due to the moving load was estimated. In addition to the methods by Ghosh *et al.* above, Saha and Roy (2009) proposed another new approach to avoid calculations of Jacobian matrix, which is derivative free local linearization (DLL). To establish this method, one SDOF oscillator and 3DOF shear frame building with constant parameter was taken. Their study showed that more efficiency is achievable with a higher order DDL.

Zhou, Wu, and Yang (2008) used a weighted EKF method over experimental results of a 3DOF system, where some system property changes with time. For their study, they built a three-story shear frame model and added an external device to change the stiffness property. They used white noise and ElCentro and Kobe earthquake records as input. At a certain time during their experiment, they triggered the external device to change the stiffness of the model. In their result, they identified the change in stiffness property.

The main objective of this paper is to apply EKF for a non-stationary earthquake, and show that the evaluated element properties are dependent on the responses, i.e., the convergence values change from response to response. In this study it is also shown that the combined effects of these different elementary properties are nullified in overall structural behavior. With the different values of elements' stiffness, the natural frequencies converge at almost same the value irrespective of response sources.

2 OVERVIEW OF EXTENDED KALMAN FILTER

From a general scenario, let us consider a multi-degree of freedom dynamic system whose equation of motion is expressed as:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{M}\{\boldsymbol{\iota}\}\ddot{\boldsymbol{u}}_{\mathbf{g}} \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are mass, damping and stiffness matrices respectively. In Eq. (1) $\ddot{\boldsymbol{u}}$, $\dot{\boldsymbol{u}}$ and $\boldsymbol{u}$ are acceleration, velocity and displacement respectively, where $\boldsymbol{u} = [u_1, u_2, u_3, \ldots, u_n]^{\mathrm{T}}$ and n is degree of freedom. The vector $\{\boldsymbol{\iota}\}$ and $\ddot{\boldsymbol{u}}_{\mathbf{g}}$ represent the influence vector and ground excitation respectively. The governing equation of motion in Eq. (1) can be represented in state-space in the following form:

$$\frac{d\boldsymbol{X}(t)}{d\boldsymbol{u}} = \mathbf{g}(\boldsymbol{X}, \boldsymbol{f}, t) + \boldsymbol{w}(t) \tag{2}$$

where $\boldsymbol{f}$ is external force in dynamic system (here it is $\boldsymbol{M}\{\boldsymbol{\iota}\}\ddot{\boldsymbol{u}}_{\mathbf{g}}$). It may be noted that the dynamic state in Eq. (2) is given by:

$$\boldsymbol{X}(t) = \{\boldsymbol{u}^{\mathrm{T}}, \dot{\boldsymbol{u}}^{\mathrm{T}}\}^{\mathrm{T}} \tag{3}$$

The respective observation equation is given by:

$$\boldsymbol{Y}_{i+1} = \mathbf{h}(\boldsymbol{X}_{i+1}, \boldsymbol{f}_{i+1}, t_{i+1}) + \boldsymbol{\eta}_{i+1}(t) \tag{4}$$

where $\boldsymbol{Y}_{i+1}$ is observation vector at t_{i+1}. The covariance matrix of measurement noise (i.e. $\boldsymbol{\eta}_{i+1}$) is expressed as $\mathrm{E}\left[\boldsymbol{\eta}_k \boldsymbol{\eta}_j^{\mathrm{T}}\right] = \mathbf{R}_i \delta_{kj}$ and it is Gaussian independent and identical in nature. Here, $\mathrm{E}[.]$ represents expectation operator and δ_{kj} is the Kronecker delta. To apply estimation theory in EKF, the non-linear term ($\mathbf{g}(\boldsymbol{X}, \boldsymbol{f}, t)$, $\mathbf{h}(\boldsymbol{X}, \boldsymbol{f}, t)$) in Eq. (2) and (4) are linearized by Taylor's expansion around the estimated state ($\widehat{\boldsymbol{X}}$) of each iteration. In this case, Taylor's expansion up to second order is considered, and $\mathbf{G}_{(i|i)}$ and $\mathbf{H}_{(i+1|i)}$ is Jacobian matrix of state and observation equation respectively. In each iteration, the estimation state value $\widehat{\boldsymbol{X}}_{(i+1|i+1)}$ is evaluated by minimizing the sum square error of observation $\boldsymbol{Y}_{i+1}$ and predicted state $\widehat{\boldsymbol{X}}_{(i+1|i)}$. In an iterative way, one may write this relation as given by:

$$\widehat{\boldsymbol{X}}_{(i+1|i+1)} = \widehat{\boldsymbol{X}}_{(i+1|i)} + \overline{\mathbf{K}}_{i+1}\left[\boldsymbol{Y}_{i+1} - \mathbf{h}\left(\widehat{\boldsymbol{X}}_{(i+1|i)}, \boldsymbol{f}_{i+1}, t_{i+1}\right)\right] \tag{5}$$

It may be noticed that above equation is similar to actual Kalman filter as in Eq. (5). Here, $\widehat{\boldsymbol{X}}_{(i+1|i)}$ is the estimated state at $(i+1)^{\text{th}}$ time instant which is given by:

$$\widehat{\boldsymbol{X}}_{(i+1|i)} = \widehat{\boldsymbol{X}}_{(i|i)} + \int_{t_i}^{t_{i+1}} \mathbf{g}(.)\mathrm{d}t \tag{6}$$

The Kalman gain in Eq. (19) is evaluated by:

$$\overline{\mathbf{K}}_{i+1} = \textstyle\sum_{(i+1|i)} \mathbf{H}_{(i+1|i)}^{\mathrm{T}} \left[\mathbf{H}_{(i+1|i)} \textstyle\sum_{(i+1|i)} \mathbf{H}_{(i+1|i)}^{\mathrm{T}} + \mathbf{R}_{i+1}\right]^{-1} \tag{7}$$

In the above equation $\sum_{(i+1|i)}$ is the error covariance matrix of $\widehat{\boldsymbol{X}}_{(i+1|i)}$ and it is given by:

$$\sum_{(i+1|i)} == \boldsymbol{\Phi}_{i+1,i} \sum_{(i|i)} \boldsymbol{\Phi}_{i+1,i} + \widetilde{\boldsymbol{W}}_i \tag{8}$$

Here, $\boldsymbol{\Phi}_{i+1,i}$ is the state transition matrix of the linearized system, evaluated as:

$$\boldsymbol{\Phi}_{i+1,i} \approx \mathbf{I} + \Delta t.\mathbf{G}_{(i|i)} \tag{9}$$

where $\mathbf{I}$ represents the unit matrix. The error covariance matrix is given by:

$$\sum_{(i|i)} = \left[\mathbf{I} - \overline{\mathbf{K}}_i \mathbf{H}_{(i|i-1)}\right] \sum_{(i|i-1)} \left[\mathbf{I} - \overline{\mathbf{K}}_i \mathbf{H}_{(i|i-1)}\right]^{\mathrm{T}} + \overline{\mathbf{K}}_i \mathbf{R}_i \overline{\mathbf{K}}_i^{\mathrm{T}} \tag{10}$$

Finally to identify the parameter, the state vector is modified with unknown parameters as follows:

$$\boldsymbol{X}(t) = \{\boldsymbol{u}^{\mathrm{T}}, \dot{\boldsymbol{u}}^{\mathrm{T}}, \boldsymbol{\theta}^{\mathrm{T}}\}^{\mathrm{T}} \tag{11}$$

in which $\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3, \dots, \theta_n]^{\mathrm{T}}$ is the n unknown system parameters. This can include any system parameter, *e.g.*, stiffness, damping etc. With the augmented state vector, the EKF algorithm mention in Eq. (2) and (10) are used to identify $\boldsymbol{\theta}$.

3 NUMERICAL RESULTS AND DISCUSSION

To execute the presented algorithm, a three-floor steel shear frame laboratory model was assembled. This model was tested on the shake table and applied with a past earthquake accelerogram at its base. The laboratory model had a floor plate of $600 \times 300 \times 10$ mm and a column of $600 \times 30 \times 10$ mm (Figure 1). The base was rigidly fixed to the shake table to smoothly transfer the vibrating motion to the model. For this system, system parameters were evaluated by finite element modeling (see Table 1). For this experiment, El Centro 1940 earthquake motion was applied to the model. For simplicity in application, the recorded ground motion was given a shorter time period (Figure 2a). The response due to this excitation was recorded by a force-based accelerogram at each floor level. On this recorded acceleration response EKF algorithm was applied, estimating the acceleration. Both recorded and estimated acceleration for the top floor are shown in Figure 2b. From this figure it is clearly visible that the state is perfectly identified by this algorithm.

As given in Eq. (14), in parallel to state updating, system parameters were also evaluated. In this identification technique, the main concentration was on three stiffnesses of the model. For identification of stiffnesses, initial value and error covariance were taken at 60,000 N-m/s and 10^6 respectively. Figure 3 shows the convergence of stiffness. In the end, the stiffness value converged at 51,770, 57,267, and 66,108 N-m/s. As the stiffness value was evaluated, the natural frequency was also calculated at each step with the identified stiffness. In Figure 4, estimated natural frequencies are shown with original values. From this figure, it is observed that after successful completion of the algorithm, the evaluated natural frequencies were 4.2547, 12.4092 and 18.0384 Hz. In case of identified stiffness, compared with the theoretical value, there were errors of 23.3%, -25.5% and -11.6% respectively for three stiffnesses. In contrast, the errors for the three natural frequencies were 2.4%, -3.1% and -9.1% respectively.

With the same input and initial conditions, this test was repeated three times and the results summarized in Table 1. From Table 1, it is clear that with same input and initial conditions, for a given system the convergence values of stiffness varies from response to response. On the other hand, the natural frequencies almost gave the same results for all responses. This happened because a system can be represented by different combinations of element stiffness matrices, and eigen values obtained from these combinations represent the same natural frequencies. In that case, the eigen vector will differ from case to case.

Figure 1. Laboratory model.

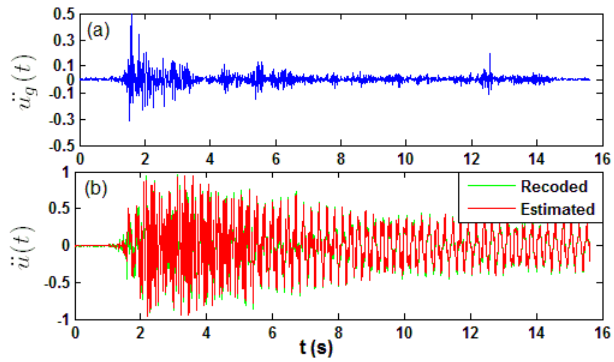

Figure 2. (a) Base excitation, (b) Recorded and estimation response of top floor.

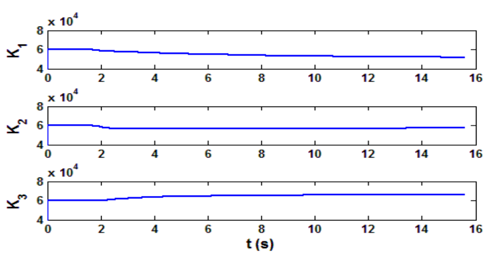

Figure 3. Estimated stiffness values.

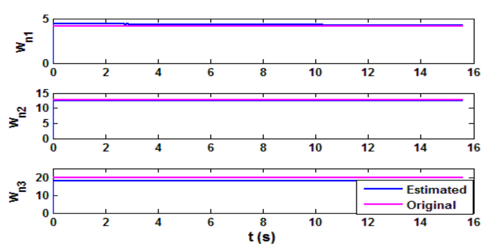

Figure 4. Updated natural frequencies.

Table 1. Identified system parameters.

Parameters	Original values	Estimated Parameter		
		Test 1	Test 2	Test 3
K_1 (N-m/s)	41987	51770	48908	50691
K_2 (N-m/s)	76842	57267	61324	56602
K_3 (N-m/s)	74812	66108	66480	67826
ω_{n1} (Hz)	4.1565	4.2547	4.2289	4.2248
ω_{n2} (Hz)	12.8093	12.4092	12.3916	12.4208
ω_{n3} (Hz)	19.8511	18.0384	18.3311	18.0848

4 CONCLUSION

In this study, the applicability of EKF is shown for non-stationary type signals in practical scenarios. It was observed that for state updating, this method was perfect irrespective of the source of response, but from the presented results it was clear that in the EKF algorithm, the identified stiffness values were not so conclusive. For the same structures, all other constraints being equal, these values can change from response to response. However, identified natural frequencies obtained from different response gave almost the same result. This happens because, for any given system, one can get different global system matrices with different values of element properties, and all of these system matrices will give the same natural frequencies.

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