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FRACTURE-PROCESS ZONE ANALYSIS OF REINFORCED BARS VERTICAL TO MATRIX CRACKS

SHUJIN DUAN, YANLONG ZHANG, RUIMEI AN, and QUANMIN GUO

Research Institute of Structural Engineering, Shijiazhuang Tiedao University, Shijiazhuang, China

This study focuses on a crack mechanics model—an infinite center-cracked concrete panel with a steel bar across a crack loaded by uniform tensile stress, under which the slip between reinforcement and concrete is neglected and the force produced by the reinforcement is regarded as a centralized force to close the upper and lower surface of the crack. The critical sizes of the fracture process zone (FPZ) are obtained by the use of a power-exponent tensile-strain softening model under the maximum tensile stress criterion and the maximum tensile strain criterion. The results show that the critical sizes of fracture process zone at both crack tips decrease with the increasing steel bar area, but the distance between the reinforcement and the crack tip or the decreasing Poisson ratio increase with the increasing of the tensile-strain softening index.

Keywords: Reinforced concrete, Fracture mechanics, Fracture process zone, Secondary elastic crack tip stress.

1 INTRODUCTION

The fracture behavior of concrete is mainly influenced by the fracture process zone (FPZ). The FPZ size ahead of the concrete crack can be measured through various experiments – e.g., the laser speckle method or acoustic emission and photo elastic coating – but satisfactory results cannot be obtained because the complexity of concrete cracks. During the past few decades, many models, e.g., the fictitious crack model (Hillerborg 1980), the crack-band model (Bazant *et al.* 2007), and the Duan-Nakagawa model (Duan *et al.* 1988), have been proposed to study FPZ, increasing the understanding of the concrete fracture process. The critical sizes of FPZ for the concrete were derived from local solutions, based on the Westergaard stress function, with the secondary elastic crack tip stress (Duan *et al.* 2013).

It has been a topic of extensive research to study reinforced concrete members with low reinforcement ratio by fracture mechanics. Zhao *et al*. (1994) derived an integral equation for reinforced concrete plates with cracks, solved the stress intensity factors of the plate by numerical methods based on linear elastic-fracture mechanics, and discussed the main effects, such as the ratio of steel and the length of crack and bond-slip rigidity, on the anti-cracking behavior of the plate.

This study is focused on a crack-mechanics model—an infinite center-cracked concrete panel with a steel bar across the crack loaded by an uniform tensile stress σ at infinity, which is assumed to behave elastically everywhere except inside the FPZ. In this paper, the slip between reinforcement and concrete is neglected and the force produced by the reinforcement is regarded as a centralized force to make the upper and lower surface of the crack closed. The problem shown in Figure 1 (a) can be modeled as Figure 1 (b).

Based on the Westergaard stress function with the secondary elastic crack tip stress, the critical sizes of FPZ are obtained by the use of a power exponent tensile strain softening model under two criteria: the maximum tensile stress criterion and the maximum tensile strain criterion.

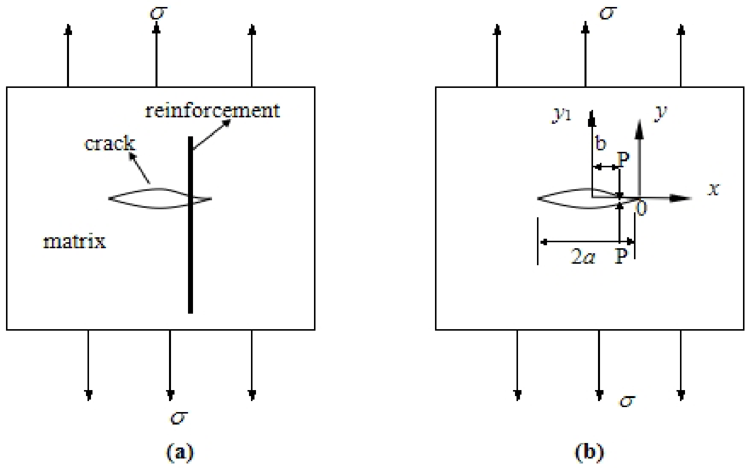

Figure 1. The problem and its mechanical model (a)⇒(b).

2 BASIC EQUATIONS

It was assumed that the steel bar area is so small that the steel bar yields before the matrix failure. It was also assumed that the steel bar is the ideal elastic-plastic material with yield stress f_y in this paper. So the Westergaard stress function of the model ahead of reinforced concrete FPZ in Figure 1(b) can be written in the form:

$$\phi = \mathrm{Re}\,\bar{\bar{Z}} + y\,\mathrm{Im}\,\bar{Z} + \frac{\sigma\left(x^2 - y^2\right)}{4} \tag{1}$$

where Z is a stress function:

$$Z(z) = \frac{K_{I1}}{\sqrt{2\pi z}}\left[1 + \frac{3}{4}\left(\frac{z}{a}\right) - \frac{5}{32}\left(\frac{z}{a}\right)^2 + \frac{21}{384}\left(\frac{z}{a}\right)^3 - \cdots\right] - \frac{K_{I2}}{\sqrt{2\pi z}}\left[1 - \left(\frac{1}{4} + \frac{a}{a-b}\right)\frac{z}{a} + \cdots\right] \tag{2}$$

where $z=x+iy$ is the complex variable measured from the right crack tip for this analysis; K_{I1} and K_{I2} are the usual stress intensity factor, $K_{I1}=\sigma(\pi a)^{(1/2)}$, $K_{I2}=P((a+b)/((a-b)\,\pi a))^{(1/2)}$; a is the total half-crack length; P is the centralized force that the steel bar applies to concrete; $P=f_y A$, A is the steel bar area; b ranged from -*a* to *a* as a coordinate value of the steel bar center in the coordinate x-y_1 system.

Taking the first and the secondary elastic crack tip stress, the Westergaard stress function is:

$$Z(z)=\frac{K_{I1}}{\sqrt{2\pi z}}\left[1+\frac{3}{4}\left(\frac{z}{a}\right)\right]-\frac{K_{I2}}{\sqrt{2\pi z}}\left[1-\left(\frac{1}{4}+\frac{a}{a-b}\right)\frac{z}{a}\right] \tag{3}$$

After derivation of equation (3), then:

$$Z'_{|z|\to 0}=-\frac{C_1}{2r\sqrt{2\pi r}}\cos\frac{3\theta}{2}+\frac{C_2}{2r\sqrt{2\pi r}}\cos\frac{\theta}{2}+i\left[\frac{C_1}{2r\sqrt{2\pi r}}\sin\frac{3\theta}{2}+\frac{C_2}{2r\sqrt{2\pi r}}\sin\left(-\frac{\theta}{2}\right)\right] \tag{4}$$

where $C_1=K_{I1}-K_{I2}$, $C_2=\frac{3K_{I1}}{4a}+\left(\frac{1}{4}+\frac{a}{a-b}\right)\frac{K_{I2}}{a}$.

For the opening mode (Mode I) crack, the stresses are determined by the Westergaard function, Z(z), as follows:

$$\begin{cases}\sigma_x=\mathrm{Re}\,Z-y\,\mathrm{Im}\,Z'-\sigma\\ \sigma_y=\mathrm{Re}\,Z+y\,\mathrm{Im}\,Z'\\ \tau_{xy}=-y\,\mathrm{Re}\,Z'\end{cases} \tag{5}$$

In terms of polar coordinates (r, θ), substituting equation (3) and equation (4) into equation (5) , the stress components ahead of the crack tip can be obtained as:

$$\begin{cases}\sigma_x=\frac{C_1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+C_2\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(1+\sin^2\frac{\theta}{2}\right)-\sigma\\ \sigma_y=\frac{C_1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)+C_2\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(1-\sin^2\frac{\theta}{2}\right)\\ \tau_{xy}=\frac{C_1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}-C_2\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\cos^2\frac{\theta}{2}\end{cases} \tag{6}$$

The stresses at the crack tip are:

$$\sigma_{1,2}=\frac{A}{\sqrt{2\pi r}}\cos\frac{\theta}{2}+B\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}-\frac{\sigma}{2}\pm\sqrt{\frac{\sigma^2}{4}-C_3+C_4} \tag{7}$$

where $C_3=\sigma\left(C_2\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\sin^2\frac{\sigma}{2}-\frac{C_1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\sigma}{2}\sin\frac{3\sigma}{2}\right)$

$$C_4 = \left(\frac{C_1}{2\sqrt{2\pi r}} \sin\theta \right)^2 + \left(C_2 \sqrt{\frac{r}{8\pi}} \sin\theta \right)^2 - \frac{C_1 C_2}{4\pi} \sin^2\theta \cos\theta$$

3 THE ANALYTICAL EXPRESSION OF CRITICAL SIZE OF FRACTURE PROCESS ZONE

In the paper, the analytical expression of critical size of FPZ is obtained based on three assumptions: 1) The FPZ ahead the crack tip is a band distribution along the crack direction; 2) Within the FPZ, the stress decays from σ_u to zero at the tip of the traction-free crack; and 3) the rate of decay is consistent with the stress-strain relationship of the concrete, assumed to be a power exponent expressed as:

$$\sigma = \sigma_u \left(\frac{\varepsilon}{\varepsilon_u} \right)^{-n}, 0 \le n < 1 \tag{8}$$

where σ_u is the ultimate tensile strength of concrete; n is the concrete tensile-strain softening index ahead the crack tip; and ε_u is the ultimate tensile strain of concrete.

Along the direction of crack, the σ_y ahead the crack tip for the mode I crack can be written as (Schmidt *et al.*1980):

$$\sigma_y = \sigma_u \left(\frac{r_p}{r} \right)^{\frac{n}{n-1}}, 0 \le n < 1 \tag{9}$$

where r_p is the size of FPZ.

It is assumed to behave elastically everywhere except inside the FPZ. Similar to the Dugdale model, the stress singularities assumed in linear elastic fracture mechanics can vanish from the crack tip, and the FPZ can be formed (i.e., the process of decreasing traction with increasing opening).

$$\int_0^{r_p} \sigma_u \left(\frac{r_p}{r} \right)^{\frac{n}{n-1}} dr = \int_0^{r_0} \sigma_y dr, 0 \le n < 1 \tag{10}$$

Along the direction of cracks, the analytical expression of the critical size of FPZ can be obtained by substituting equation (6) with equation (10):

$$r_p = \frac{3C_1 + C_2 r}{3(1-n)\sigma_u} \sqrt{\frac{2r}{\pi}} \tag{11}$$

Substituting the critical values of C_1 and C_2 into equation (11), the analytical expression of the critical size of FPZ under the maximum tensile stress criterion and the maximum tensile strain criterion can be obtained:

$$r_{pc} = \frac{6C_{1c}C_{2c}C_5 + C_5^3\pi}{6(1-n)C_{2c}^2\sigma_u} \tag{12}$$

where $C_5 = \sigma_u - \sqrt{\sigma_u{}^2 - \frac{2C_{1c}C_{2c}}{\pi}}$ (the maximum tensile-stress criterion); $C_5 = \frac{\sigma_u - \sigma\nu}{1-\nu} - \sqrt{\left(\frac{\sigma_u - \sigma\nu}{1-\nu}\right)^2 - \frac{2C_{1c}C_{2c}}{\pi}}$ (the maximum tensile-strain criterion, i.e., plane stress); $C_5 = \frac{\sigma_u - \sigma\nu - \sigma\nu^2}{1-\nu-2\nu^2} - \sqrt{\left(\frac{\sigma_u - \sigma\nu - \sigma\nu^2}{1-\nu-2\nu^2}\right)^2 - \frac{2C_{1c}C_{2c}}{\pi}}$ (the maximum tensile-strain criterion, i.e., plane strain); K_{I1c} is a quantity related to the fracture toughness of concrete; K_{I2c} is a quantity related to the fracture toughness of the steel bar; and v is the Poisson ratio.

The critical sizes of FPZ for different steel bar areas are plotted in Figure 2 when b/a=0.2, ν= 0.2 and n=0.5 under both criteria. It shows that the critical size of FPZ decreases when the reinforcement area increases.

The critical sizes of FPZ for different steel bar positions are plotted in Figure 3 when A=0.0004m^2, ν= 0.2 and n=0.5 under both criteria. It shows that the critical size of FPZ decreases with the distance between the reinforcement and crack tip increases.

The critical sizes of FPZ for different Poisson ratios are plotted in Figure 4 when A=0.0004m^2, b/a=0.2 and n=0.5 under the maximum tensile strain criterion. It shows that the critical size of FPZ decreases with Poisson ratio increasing.

The critical sizes of FPZ for different concrete tensile strain softening indexes are plotted in Figure 5 when A=0.0004m^2, b/a=0.2 and ν=0.2 under both criteria. It shows that the critical size of FPZ increases with the tensile strain softening index increases.

4 CONCLUSIONS

It is reasonable to assess the critical sizes of FPZ ahead of the reinforced concrete crack tip under the maximum tensile-stress criterion and the maximum tensile-strain criterion; (2) The critical sizes of FPZ ahead of reinforced concrete crack tip decrease with the increasing reinforcement area under both failure criteria; (3) The critical sizes of FPZ decrease with the increasing distance between the reinforcement and the crack tip under the both failure criterion; (4) The critical sizes of FPZ ahead of reinforced concrete crack tip increase with the decreasing Poisson ratio under the maximum tensile strain criterion, and approach that of the maximum stress criterion when ν= 0; (5) The critical sizes of FPZ ahead of reinforced concrete crack tip increase with an increasing concrete tensile-strain softening index.

Acknowledgments

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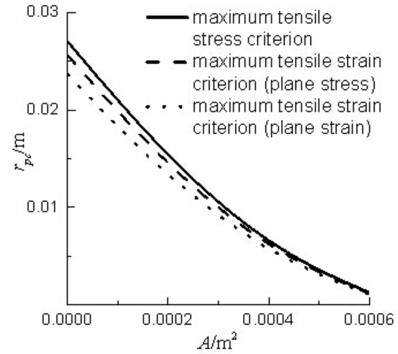

Figure 2. The fluence of A to r_{pc}.

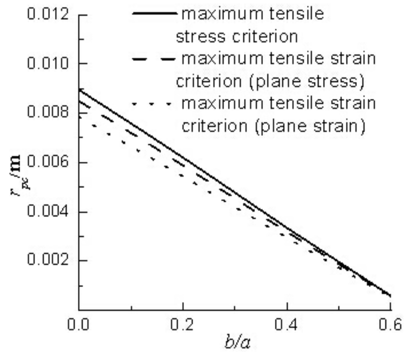

Figure 3. The fluence of b/a to r_{pc}.

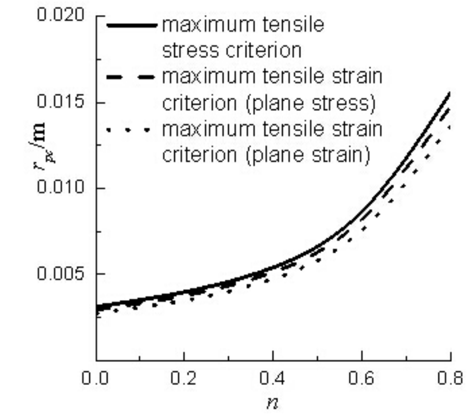

Figure 4. The fluence of v to r_{pc}.

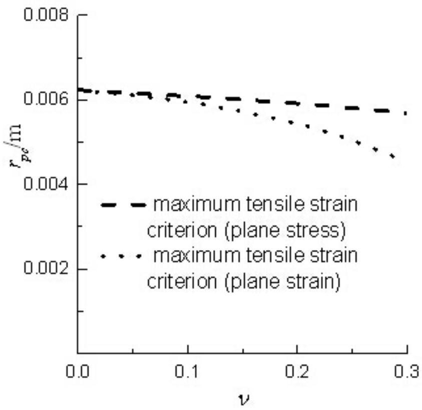

Figure 5. The fluence of n to r_{pc}.

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