Sustainable Solutions in Structural Engineering and Construction
Edited by Chantawarangul, K., Suanpaga, W., Yazdani, S., Vimonsatit, V., and Singh, A.

ISBN: 978-0-9960437-0-0

MODELING OF CONCRETE BEHAVIOR UNDER BIAXIAL FATIGUE LOADING WITH VARIOUS MEAN STRESSES

ASHKAN SABOORI[1], SIAMAK YAZDANI[1], ANDREW REBERG[2], MIJIA YANG[1], DENVER TOLLIVER[3], and SARA MAMANI[1]

[1]*Civil and Environmental Engineering Dept, North Dakota State University, Fargo, USA*
[2]*Black and Veatch Corporation, Bloomington, USA*
[3]*Upper Great Plains Transportation Institute, North Dakota State University, Fargo, USA*

In this paper, a general approach is proposed for modeling the behavior of concrete under biaxial fatigue loading with various mean stresses (stress range). Using damage mechanics and principles of thermodynamics, a generalized bounding surface approach is proposed. It is mentioned that limit surface, determined by model, is a condition in which the number of fatigue loading is set to one. In other words, limit surface represents the strength of material under monotonic loading with various load paths. By increasing the number of loading cycles, the limit surface is allowed to contract and to form surfaces representing residual strength curves. The evolution of the subsequent surfaces depends on load magnitude, load range (mean stress), and load path. Within the formulation, a softening function is postulated which captures the reduction in the strength of concrete under fatigue loading. A comparison of model predictions against experimental data shows a good correlation.

Keywords: Anisotropic, Bounding surface, Concrete, Damage mechanics, Fatigue.

1 INTRODUCTION

Concrete fatigue has been studied more closely in the past few decades. This can be attributed to the increasing use of concrete as a construction material in various types of structures, such as dams, bridges, airport/highway pavements, and pressure vessels where cyclic loading plays an important role in their design life. Many experiments have been done on concrete under uniaxial cyclic loading, although there are a relatively small number of experiments done under a biaxial state of loading (Nelson et al. 1988, Su and Hsu 1988, Yin and Hsu 1995). It has been shown that concrete under fatigue loading loses strength rapidly as the number of cyclic loadings increases. This deterioration of concrete strength and mechanical behavior can be attributed to the nucleation and propagation of microcracks during the fatigue process. It has been argued by Su and Hsu (1988) that the fatigue strength of concrete under biaxial compression is greater than under uniaxial compression for any given number of load cycles. The increase in strength of concrete under biaxial fatigue loading, as opposed to uniaxial loading, is because of the effects of existing confinement pressure in biaxial load path. This confinement restricts the nucleation and propagation of microcracks.

Given that fatigue loading has a significant effect on the design life of concrete structures, often leading to sudden ruptures, there is a great need for a reliable model to predict the life of concrete under a fatigue environment. This paper presents a damage-mechanics approach to model fatigue loading for concrete. Using the bounding-surface approach proposed by Wen (2012), a limit surface, representing the monotonic strength of concrete, will be developed. This surface will be allowed to contract as the number of cycles of loading increases. The resulting residual strength surface represents the strength of concrete for various biaxial load paths. A softening function is proposed to capture the effects of cyclic loading, stress range, and loading path on loss of strength.

2 GENERAL FORMULATION

The general formulation shown in the following is based on the damage-mechanics approach, and follows the framework of the internal variable theory of thermodynamics. Based on studies done by Ortiz (1985) and Yazdani (1993) for small deformations, the Gibbs Free Energy is defined as follows:

$$G(\boldsymbol{\sigma}, k) = \frac{1}{2}\boldsymbol{\sigma}:\boldsymbol{C}(k):\boldsymbol{\sigma} - A^i(k) \tag{1}$$

where **C** is the compliance tensor, **σ** is the stress tensor, k is a scalar damage parameter, and A^i (k) is a scalar function associated with the surface energy of microcracks. The symbol **":"** represents a tensor contraction operation. A constitutive relation for brittle materials is used as:

$$\boldsymbol{\varepsilon} = \boldsymbol{C}(k):\boldsymbol{\sigma} \tag{2}$$

where **ε** represents strain tensor. The compliance tensor, **C**, is assumed to take an additive decomposition form as:

$$\boldsymbol{C}(k) = \boldsymbol{C}^0 + \boldsymbol{C}^c(k) \tag{3}$$

where $\mathbf{C^0}$ and $\mathbf{C^c}$ are the initial undamaged compliance tensor of the material, and the added flexibility tensor associated with the accumulation of damage, respectively. Due to the nonlinearity behavior between stress and strain for brittle materials produced by damage, the rate form of the flexibility tensor is considered as:

$$\dot{\boldsymbol{C}}(k) = \dot{\boldsymbol{C}}^c(k) = \dot{k}\boldsymbol{R} \tag{4}$$

In Eq. (4), response tensor, **R**, determines the direction at which damage occurs. For isothermal and small deformation, the internal dissipation inequality can be represented by Gibbs Free Energy as:

$$\frac{\partial G(\boldsymbol{\sigma}, k)}{\partial k}\dot{k} \geq 0 \tag{5}$$

It is also assumed that the damage is an irreversible phenomenon, i.e., concrete cannot be healed after damage occurs. By combining Eq. (1) through (5) in the absence of any viscosity considered, the general form of the damage surface is given by:

$$\Psi(\boldsymbol{\sigma},k) = \frac{1}{2}\boldsymbol{\sigma}:\boldsymbol{R}:\boldsymbol{\sigma} - \frac{1}{2}t^2(\boldsymbol{\sigma},k) = 0 \tag{6}$$

where t(**σ**,k) is called damage function. Ψ(**σ**,k)<0 expresses the elastic condition for the material enclosed by damage surface Ψ(**σ**,k)=0. The condition Ψ(**σ**,k)>0 is not acceptable for rate-independent process.

Guided by experimental data, the following form for the damage function is postulated by (Ortiz, 1985):

$$t(\boldsymbol{\sigma},k) = f_c e \frac{\ln(1+E_0 k)}{(1+E_0 k)} \tag{7}$$

where f_c is the compressive strength of concrete, E_0 is the initial stiffness, and e represents the natural number. In order to form the strength surface, the response tensor should be defined. In this paper only the compression mode of damage is considered. The damage mode is identified by response tensor **R**, given as:

$$\boldsymbol{R} = \frac{\boldsymbol{\sigma}^- \otimes \boldsymbol{\sigma}^-}{\boldsymbol{\sigma}^- : \boldsymbol{\sigma}^-} + \alpha H(-\lambda)(\boldsymbol{I} - \boldsymbol{i} \otimes \boldsymbol{i}) \tag{8}$$

where "⊗" is the tensor product operator, $\boldsymbol{\sigma}^-$ represents the negative cone of the stress tensor, $H(-\lambda)$ is defined as the Heaviside function of the maximum eigenvalue of $\boldsymbol{\sigma}^-$, and **I** and **i** are the fourth and second order identity tensors, respectively. α is a material parameter that can be obtained by a biaxial monotonic loading test.

3 BOUNDING SURFACE APPROACH

The bounding surface approach was first proposed by Wen et al. (2012) to predict the behavior of woven fabric composites under fatigue loading. In the bounding-surface approach, limit surface is a mathematical surface in the biaxial strength space, which represents the strength of the material under monotonic loading. This surface is shown schematically in Figure 1. In the case of fatigue loading, as cyclic loading is applied, the limit surface contracts and forms residual strength curves. This reduction in strength is caused by damage and microcracks generated during the fatigue process. As the number of load cycles increases, the strength continues to decrease further and the residual surface shrinks. The reduction in strength happens to a point at which the residual strength is equal to the magnitude of loading. At this point, a failure surface is formed, and the material cannot withstand any additional cycles as they result in failure of the material.

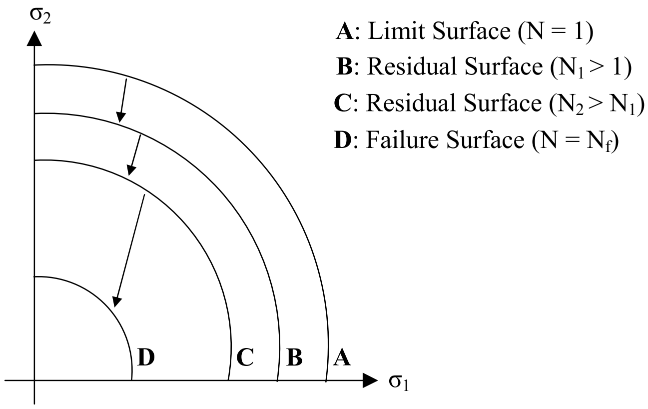

Figure 1. Schematic representation of bounding surfaces in biaxial stress space.

In order to capture the described behavior of concrete under cyclic loading, an evolutionary equation is needed to predict the failure surface. To accomplish this task, a new damage function is proposed as follows:

$$t(\sigma, k, N, r) = F(N,r).t(\sigma, k) \tag{9}$$

where F(N,r) is regarded as the softening function. The number of cycles of loading is given by N, and r is the stress ratio (ratio of minimum fluctuating stress to maximum fluctuating stress). By considering a fatigue uniaxial compression, and plugging Eq. (9) into Eq. (6), the following relation is obtained for softening function:

$$F(N,r) = \frac{\sigma}{f_c} \tag{10}$$

where σ is the residual strength of the concrete after specific number of cyclic loading. Eq. (10) is a representation of so-called S-N curves. Based on studies done by Aas-Jakobsen and Lenschow (1973), Hsu (1981), and Qiao and Yang (2006), different parameters affect the fatigue life of composite materials such as concrete. These factors include amplitude of loading, σ_{max}, stress ratio, r, and load path. While the fatigue life of concrete is adversely affected by the amplitude of loading, Aas-Jakobsen and Lenschow (*ibid*) reported that increasing the stress ratio results in a greater fatigue life at a given stress. Moreover, considering the data provided by Yin and Hsu (1995), it is apparent the rate of reduction in strength is not the same for different load paths. With these findings, the softening function is proposed as:

$$F(N,r) = N^{\left[A(1-r)\left(\frac{tr^2(\sigma)}{\sigma:\sigma}\right)^B\right]} \tag{11}$$

where N is the number of cyclic loadings, and A and B are material parameters that can be obtained by uniaxial and biaxial fatigue loading tests, respectively, r is the stress ratio, and σ represents the load path. By incorporating this softening function into Eq. (6), residual strength surfaces could be obtained under various load paths.

4 NUMERICAL EXAMPLES

In this section, results predicted by the model are compared with experimental data obtained from literature. Three material parameters α, A, and B are calculated based on the experimental data presented. Figure 2 shows the prediction results of residual strength surfaces in biaxial stress space against experimental data work of Nelson et al. (1988). The damage surfaces show a good correlation for monotonic loading when N = 1, as well as for fatigue loading when N = 10, 100, and 1000 with experimental data.

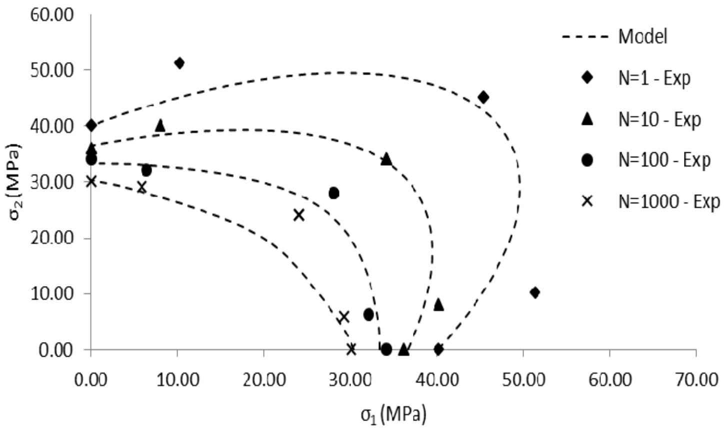

Figure 2. Residual strength surfaces for various number of cyclic loading.

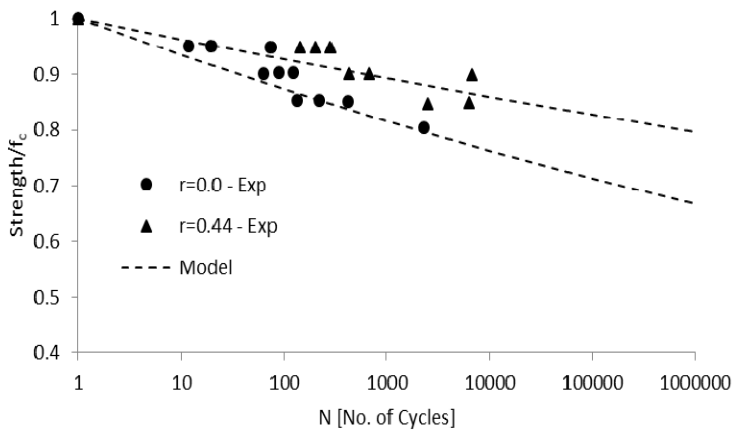

Figure 3. Effect of R on S-N curve of concrete under uniaxial loading path.

Figure 3 illustrates the comparison between the experimental data provided by Awad (1971), and S-N curves obtained by the model for two uniaxial fatigue loadings with different stress ratios. As shown, the model captures the effect of stress range on fatigue life of concrete.

5 CONCLUSION

An anisotropic model was utilized to predict the strength behavior of concrete under biaxial compressive fatigue loading. Since nucleation of cracks is the main type of damage in concrete under loading, a class of damage mechanics theory was used to establish the limit surface. Under cyclic fatigue loading, limit surface collapses and forms new surfaces identified as residual strength surface. A softening function was subsequently proposed based on the factors which affect the fatigue life of concrete, e.g., maximum stress, stress ratio, and load path. By incorporating a softening function into the model, the concept of bounding surface is achieved. Comparing the experimental data with model showed a good compatibility.

Acknowledgments

The authors gratefully acknowledge financial support from the United States Department of Transportation and the Department of Civil and Environmental Engineering of North Dakota State University through the Mountain Plains Consortium.

References

Aas-jakobsen, K., and Lenschow, R., Behavior of Reinforced Columns Subjected to Fatigue Loading. *ACI Journal Proceedings*, 1973.

Awad, M. E., *Strength and Deformation Characteristics of Plain Concrete Subjected to High Repeated and Sustained Loads*, University of Illinois Engineering Experiment Station, College of Engineering, University of Illinois at Urbana-Champaign, 1971.

Hsu, T. T., Fatigue of Plain Concrete, *ACI Journal Proceedings*, 1981.

Nelson, E. L., Carrasquillo, R. L., and Fowler, D. W., Behavior and Failure of High-Strength Concrete Subjected to Biaxial-Cyclic Compression Loading. *ACI Materials Journal*, v. 85, n. 4, 1988.

Ortiz, M. A., Constitutive Theory for the Inelastic Behavior of Concrete, *Mechanics of materials*, v. 4, n. 1, 67-93, 1985.

Qiao, P., and Yang, M., Fatigue Life Prediction of Pultruded E-Glass/Polyurethane Composites. *Journal of Composite Materials*, v. 40, n. 9, 815-837, 2006.

Su, E. C., and Hsu, T. T., Biaxial Compression Fatigue and the Discontinuity of Concrete, *ACI Materials Journal*, v. 85, n. 3, 1988.

Wen, C., Bounding Surface Approach to the Modeling of Anisotropic Fatigue Damage in Woven Fabric Composites, *Open Journal of Composite Materials*, v. 2, 2012.

Yazdani, S., On a Class of Continuum Damage Mechanics Theories, *International Journal of Damage Mechanics*, v. 2, n. 2, 162-176, 1993.

Yin, W., and Hsu, T. T., Fatigue Behavior of Steel Fiber Reinforced Concrete in Uniaxial and Biaxial Compression, *ACI Materials Journal*, v. 92, n. 1, 1995.