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A NEW METHOD TO DETERMINE TENSILE-STRAIN SOFTENING CURVE OF QUASI-BRITTLE MATERIALS

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Based on weight integration to obtain a closed solution of cohesive crack problem, a new method is proposed to determine the tensile-strain softening curve (TSC) for quasi-brittle materials. The key technique is to determine the weight function by superposition of the solution with different fictitious crack lengths to satisfy a given crack opening displacement within cohesive crack surfaces. As an example, a central crack problem under uniform tension with given crack opening displacement in the fracture process zone (FPZ) was analyzed, the corresponding TSC was determined, and then the solution for stress and displacement field was obtained.

Keywords: Quasi-brittle materials, Cohesive crack, Stress function, Crack opening displacement (COD), Tensile strain softening curve (TSC), Weight function.

1 INTRODUCTION

The cohesive crack model (CCM), also called the fictitious crack model (Hillerborg et al. 1976), is generally accepted as a means to explain the fracture characteristics of brittle or quasi-brittle materials. According to the model, there is a fracture process zone (FPZ) at the crack tip, where the cohesive stress is interrelated with the crack-opening displacement (COD), and will drop from f_t as the crack opening increases. A real crack will propagate if the cohesive stress drops to zero corresponding with the critical crack opening. The correlation curve of σ to w is named the tensile-strain softening curve (TSC), and the length of FPZ b is the distance from the real crack tip to the cohesive crack tip.

Much effort has been spent on the TSC of concrete. Although there is an ideal method to tension the specimen directly, it is difficult to do accurately (Petersson 1981). An inverse-analysis method based on the load-displacement or the load crack mouth opening displacement (CMOD) is used in practice (Zhao et al 2010, Su et al 2012) by three-point bending, wedge splitting, or other specimen tests.

The theoretical solution to express the cohesive crack was proposed by Duan and Nakagawa (1988) in the weight-integration method, which refers to both the finite-stress concentrating distribution and the smoothed-crack opening shape within the FPZ. Under this method, the solution is closely related with the weight function. In this paper, a new technique is proposed to obtain the weight function by superposition of the solution with different cohesive crack length, in order to satisfy the given COD within FPZ, and to obtain the TSC.

2 STRESS FUNCTION BASED ON THE "DUAN AND NAKAGAWA MODEL"

Considering an elastic-plane problem, the complex-stress function is expressed as

$$\nabla^2\nabla^2 F(Z,a) = 0$$

$$F(Z,a) = \bar{Z}F_1(Z,a) + F_2(Z,a) \tag{1}$$

$$Z = x + iy\,,\quad \bar{Z} = x - iy \tag{2}$$

in which, $F(Z,a)$ is the stress function gotten with stress singularity by linear elastic fracture mechanics, a is the notched length. Then the components of stress and displacement can be stated in the following form:

$$\sigma_x = 2F_1' - \bar{Z}F_1'' - F_2'' \tag{3}$$

$$\sigma_y = 2F_1' + \bar{Z}F_1'' + F_2'' \tag{4}$$

$$\tau_{xy} = -i(\bar{Z}F_1'' + F_2'') \tag{5}$$

$$2G(\mu + i\upsilon) = \kappa F_1 - (\bar{Z}F_1'' + F_2'') \tag{6}$$

in which, G is shear modulus. For the plane stress state, $\kappa = 3-\nu/4-\nu$, and for the plane strain state, $\kappa = 3-4\nu$, ν is the Poisson's ratio.

To obtain the aimed analytical solutions, it is necessary to eliminate the stress singularity at the crack tip. A weighted stress function is constructed as:

$$Q\left(Z,a,b\right) = \int_a^{a+b} \rho\left(t\right) F\left(Z,t\right) dt \tag{7}$$

in which, $\rho(t)$ is the assumed weight function defined in the interval (a, $a+b$), b is the cohesion crack length. The stress function $Q(Z,a,b)$ is still bi-harmonic, which has a finite stress concentration at the crack tip, but varies with $\rho(t)$. With this, we can determine weight function $\rho(t)$ based on the given COD as per the following section.

3 DETERMINING THE WEIGHT FUNCTION BASED ON THE GIVEN COD

To determine the weight function based on the given COD, the main procedures are shown as follows:

(1) Assume the weight function form as $\rho(t) = 1/b$ initially.

(2) Divide the length of b into n parts as Figure 1.

(3) First, point 0 is considered as the crack tip, and $\rho(t)$ is substituted by $1/b$ in

the Equation (7), where the displacement field will be worked out. The displacement of all the points in the x-axis is represented as:

$$V_0 = \{\upsilon_{0,1}, \upsilon_{0,2}, \cdots \upsilon_{0,i}, \upsilon_{0,n-1}, \upsilon_{0,n}\} \tag{8}$$

in which, $\upsilon_{0,i}$ is the displacement of point i when point 0 is considered as the crack tip. Now, considering that the measured *COD*/2 of point 1 is equal to $\upsilon_{0,1}$, then the load σ_0 will be worked out.

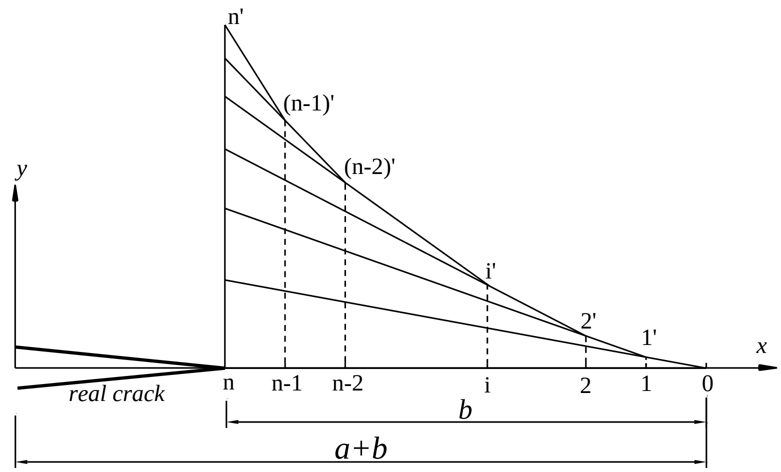

Figure 1. Sketch of calculating the weight solution.

(4) Assuming that point 1 is the crack tip, the weight function $\rho(t)$ is substituted by the $\dfrac{n}{b(n-1)}$ in the Eq. (7). The displacement field will be worked out secondly. The displacement of all the points in the x-axis is represented as:

$$V_1 = \{\upsilon_{1,2}, \cdots \upsilon_{1,i}, \upsilon_{1,n-1}, \upsilon_{1,n}\} \tag{9}$$

in which, $\upsilon_{1,i}$ is the displacement of point i when point 1 is considered as the crack tip. To give the given $COD/2$ to the sum of $\upsilon_{1,2}$ and $\upsilon_{0,2}$, then the load σ_1 will be worked out.

(5) Assuming that the crack tip changes from point 2 to point (n-2), the corresponding load is worked out. The load is respectively $\sigma_2, \sigma_3, \cdots \sigma_{n-2}$.

(6) Considering that point (n-1) is the crack tip, the weight function ρ (t) is substituted by the n/b in the Equation (7). The displacement in the x-axis is represented as:

$$V_{n-1} = \{\upsilon_{n-1,n}\} \tag{10}$$

in which, $\upsilon_{n-1,n}$ is the displacement of point *n* when point (*n*-1) is considered as the crack tip. To give the measured *COD*/2 to the sum of $\upsilon_{n-1,n}$, $\upsilon_{n-2,n}$, $\upsilon_{n-3,n}$ …, $\upsilon_{1,n}$ and $\upsilon_{0,n}$, then the load σ_{n-1} will be worked out.

(7) To superimpose the calculated loads that corresponds with the assumed crack tip from point 0 to point (n-1), the total load σ is obtained as:

$$\sigma = \sigma_0 + \sigma_1 + \cdots \sigma_i + \cdots \sigma_{n-1} \tag{11}$$

The final weight function will be determined by the superposition of each weighting as per the above steps. It will be normalized to its area as 1.0, with the calculated *v* satisfying the given *COD*.

From the above procedure, the weight function and then the stress function will be yielded out. Therefore, the stress and displacement fields for a crack problem can be demonstrated, which can reflect the material's fracture features, such as the TSC.

4 AN EXAMPLE

A Griffith crack problem under mode I is taken as an example, as shown in the Figure 2. The basic data are that *a*=30mm, *b*=10mm, *E*=3.5×10^4Gpa, f_t =3.0Mpa, and the measured *COD* is shown in Table 1.

Table 1. The measured *COD*.

x/mm	30.0	31.0	32.0	32.5	34.0	35.0	36.0	38.0	40.0
COD/2/mm	0.1189	0.109	0.095	0.0884	0.0685	0.054	0.064	0.015	0

The weight function is assumed as $\rho(t) = 1/b$, $(a \le t \le c, c = a + b)$ and the stress function is obtained by the weight integral method as:

$$\Phi_3 = \frac{\sigma_0}{2b} x[\arcsin(\frac{c}{z}) - \arcsin(\frac{a}{z})] \tag{12}$$

The stress $\sigma_y\big|_{y=0}$ and displacement $\upsilon\big|_{y=0}$ along the *x*-axis is shown as:

$$\sigma_y \big|_{y=0} = \frac{\sigma}{b} |x| [\arctan(\frac{c}{\sqrt{x^2 - c^2}}) - \arctan(\frac{a}{\sqrt{x^2 - a^2}})], \qquad (|x| \geq c) \tag{13}$$

$$\upsilon \big|_{y=0} = \frac{\sigma}{bE} \{x^2 \ln x + c\sqrt{c^2 - x^2} - x^2 \ln[c + \sqrt{c^2 - x^2}]\}, \ (|x| \le c) \tag{14}$$

Based on the given *COD*, taking the cohesive crack lengths as 10.0mm, 8.0mm, 6.0mm, 5.0mm, 4.0mm, 2.5mm, 2.0mm, and 1.0mm respectively, the corresponding load is obtained and superimposed. Then the numerical weight function is worked out. After fitting and normalizing, the weight function is linearly expressed as:

$$\rho(t) = 0.0137t - 0.3795(a \le t \le a + b) \tag{15}$$

The numerical and fitting of weight function are shown in Figure 3:

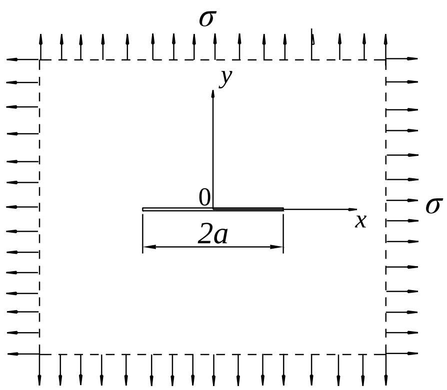

Figure 2. Griffith crack problem.

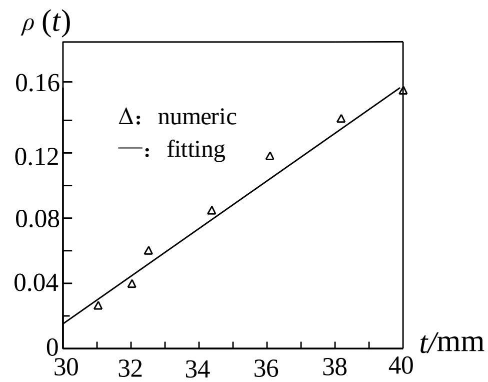

Figure 3. Weight function.

Based on Equations (7) and (15), the obtained cohesive stress distribution and COD distribution along the crack ligament are shown in Figure 4. The corresponding TSC is demonstrated in Figure 5.

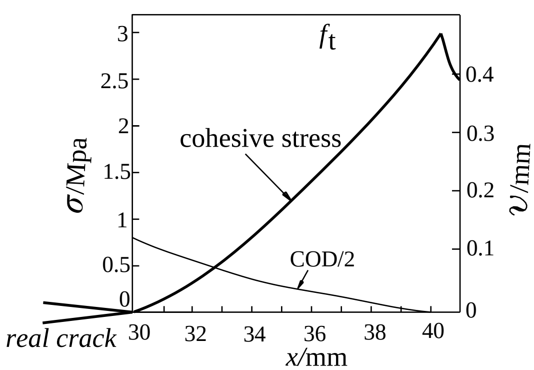

Figure 4. Cohesive stress and COD in FPZ.

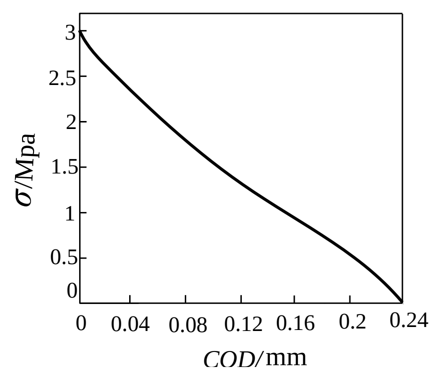

Figure 5. Obtained TSC.

5 CONCLUSIONS

A new method is proposed to determine the stress faction and the TSC for quasi-brittle materials based on the Duan and Nakagawa Model, which can satisfy the given COD within cohesive crack surfaces. It can be conveniently used to simulate the concrete fracture process if the COD within the FPZ is measured by the specimen fracture test.

Acknowledgments

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