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# UTILIZATION OF ARMA MODELS TO MEASURE DAMAGE POTENTIAL IN SEISMIC RECORDS

M. K. EL-CHOUM

*Dept of Civil, Environmental and Ocean Engineering, Stevens Institute of Technology, Hoboken, USA*

ARMA parameters are developed to assess the damage potential in seismic records. The parameters are chosen to fit acceleration time series of particular earthquake records using the maximum-likelihood method. For each event, a set of random accelerograms is generated and used to establish statistically-valid structural response spectra. Since the number of earthquake accelerograms in any seismic region is limited, non-stationary stochastic models are used to characterize earthquake ground motion. From a sample of earthquakes, the mean and variance of response spectral ordinates are obtained for damage predictors. Structural design spectra for earthquakes are based on smoothed linear response spectra obtained from different events scaled by their peak values. Such an approach does not incorporate other characteristics of the excitation represented by measured data. Samples of acceleration records are generated for each event. In this study, individual records for an earthquake are treated as one realization of an underlying non-stationary process that actually characterizes the earthquake. This research provides a reliable description of the information contained within acceleration records, and can also provide a reasonable estimate of the average nonlinear demand spectra.

*Keywords:* Autoregressive Moving Average (ARMA), Time Series, Earthquake, Response Spectra, Modeling, Ductility, Hysteretic Energy and Stochastic Process.

## 1 INTRODUCTION

Earthquake data in the form of time series are of great interest in structural investigation and design. These time series are highly irregular, indicating time-varying frequency and amplitude characteristics. Measured earthquake records, which are only one sample of records of a non-stationary stochastic process, can be thought of as one random realization of a stochastic process. Therefore, earthquakes must be regarded as a time series of a non-stationary character. Most of the existing modeling of a ground-acceleration time series as a stochastic process has been based on one of two approaches: *a frequency domain* and a *time domain* (Shinozuka 1987). Over past decades, authors have worked with ARMA (autoregressive moving average) models. The interaction between theory and practice has attempted to develop practical techniques to model earthquake data using ARMA models, which are sufficiently flexible to describe ground motions.

In current design practice, measured earthquake time series are characterized by their peak characteristics, such as peak acceleration or velocity. A response spectrum is the principal method of representing ground motion for current structural design procedures. However, the conventional design for earthquakes has serious limitations. The approach that normalizes seismic accelerations is flawed for the following reasons:

(1) The damage potential for linear and nonlinear systems is dependent on other earthquake characteristics, such as frequency content and duration.

(2) An earthquake time series should be regarded as a sample realization from a population that could have been generated by the process.

This study investigates the use of simple non-stationary ARMA models that can be considered to be characteristic of specific historical earthquakes. Measured records are used to estimate the parameters of the underlying population using maximum-likelihood techniques (Turkstra et al. 1987). Simulated records from this population are then used to obtain average response spectra for each historical event, along with an estimate of the variance of response. To assess damage potential, response spectra for damped, single degree-of-freedom (SDOF) systems were used. Spectra for samples of simulated acceleration records for bi-linear and stiffness softening systems were applied. Damage predictors include ductility demand and normalized hysteretic-energy demand.

## 2 SEISMIC-PROCESS MODELS

The time-domain approach concerned with using autoregressive moving average (ARMA) models to characterize earthquake records data is relatively new in this field. A survey paper by Kozin (1988) is a good guide to this approach applied to earthquake data. The techniques of ARMA models are well known (Box and Jenkins 1976, Kozin 1978). Early applications of ARMA models by Chang et al. (1979), Kozin (1976), and Kozin and Nakajima (1980), attempted to reproduce the details of historical records. Another extensive study of ARMA models was completed by Ellis and Cakmak (1987). An ARMA model at any time step $\boldsymbol{k}$ may be represented as follows:

$$A_{\mathbf{k}} - \varphi_{\mathbf{1}} \bullet A_{\mathbf{k-1}} - \ . \ . \ . \ - \varphi_{\mathbf{p}} \bullet A_{\mathbf{k-p}} = W_{\mathbf{k}} - \theta_{\mathbf{1}} \bullet W_{\mathbf{k-1}} - \ . \ . \ . \ \theta_{\mathbf{q}} \bullet W_{\mathbf{k-q}} \tag{1}$$

where φ, θ are constant coefficients.

The left side of Eq. (1) is known as the autoregressive (AR) part of order p. The time series $\{A_k\}$ is the sequence of measured data. The right side of Eq. (1) is known as the moving average (MA) part of order $\boldsymbol{q}$. The sequence $\{W_k\}$ is a set of independent, identically-distributed Gaussian random variables.

In the present study, a digitized data is first divided by its modulating function to obtain a stationary time series. The method of Pradlwarter (1987) was used to estimate the modulating function, and when obtained is used to fit a parametric envelope function, of the form (Eq. 2):

$$S(t) = \begin{cases} \alpha \bullet [t/Tp] \bullet [t/Tp] & t \leq Tp \\ \alpha \bullet exp[-\beta(t - Tp] & t > Tp \end{cases} \tag{2}$$

The constant **α**, **β** and the peak time $\mathbf{T}_p$ can be found through an iterative least-square procedure. The stationary time series obtained is then used to estimate the ARMA model parameters. Alternative models of the ARMA process were compared with the use of the Akaike information criteria (AIC) (1974).

The data considered in this study consists of four major earthquakes. El Centro, California, NS 1940, (M = 6.7) digitized at 0.02 seconds with 2688 data points; Parkfield, California, NE 1966 (M = 5.6) digitized at 0.02 seconds with 2183 data points; Mexico N90W 1985 (M = 8.1) digitized at 0.02 seconds with 9004 data points; and Nahanni in Canada 1985 (M = 6.9) digitized at 0.005 seconds with 4085 data points.

## 3 RESPONSE ANALYSIS

A fundamental element in practical design for earthquakes is the response spectra for SDOF systems (Clough and Penzien 1993, Newmark and Rosenblueth 1973). To establish these spectra, acceleration records from particular earthquakes are used as input to linear and nonlinear models; response measures, such as maximum displacement, are calculated. For nonlinear structures, the ratio of maximum displacement to yield displacement—or ductility factor—is used as a design parameter. For any single record, the irregular response spectra obtained are normally smoothed into tripartite-linear approximation (Newmark and Blume 1973, Newmark and Hall 1969).

In summary, peak seismic displacement, velocity and acceleration were used as a basis for response spectra and damage prediction. These methods of analysis did not incorporate other characteristics of the earthquake record such as duration and frequency content. The set of accelerations used to construct response spectra is composed of different realizations of time series; each belongs to a characteristic stochastic process.

As a basis for comparison of alternative earthquake process models, the response of a SDOF system with viscous damping was adopted. Bi-linear systems (Velestos and Newmark 1960, Riddell and Newmark 1979, Iwan and Gates 1979, Nau and Hall 1984, Zahrah and Hall 1984, Lin and Mahin 1985) and stiffness softening systems were used (Riddell and Newmark 1979, Mahin and Bertero 1980). Response spectra were obtained by numerical integration of the general equation assuming linear acceleration in each time step (Clough and Penzien 1993) (Eq. 3):

$$M \bullet \ddot{u}(t) + C \bullet \mathring{u}(t) + R(u,t) = -M \bullet \ddot{u}_{\boldsymbol{g}}(t) \qquad \textit{or}$$
$$\ddot{u}(t) + 2 \bullet \omega \bullet \xi \bullet \mathring{u}(t) + R(u,t)/M = -\ddot{u}_{\boldsymbol{g}}(t) \tag{3}$$

where u(**t**) is the relative displacement with respect to the ground, **M** is the mass, **C** is the damping coefficient, $\ddot{\mathbf{u}}_{\boldsymbol{g}}$ is the ground acceleration, R(u,t) is the restoring force, $\omega$ is the initial frequency, and $\boldsymbol{\xi}$ is the fraction of the critical damping of the structure.

## 4 RESULTS

Nonlinear response analysis of the four historical events indicates that the ARMA(2, 1) process using samples of twenty simulated earthquakes provides a reliable description of the information contained within acceleration records. It can also provide a reasonable estimate of the average nonlinear demand spectra (El-Choum and Brahimi 1997). The ARMA(2,1) concludes that the order (2,1) is sufficient to estimate the damage effect of an earthquake with relatively few parameters (*ibid*). It produced graphical representation of the four major earthquakes showing displacement ductility, and normalized hysteretic energy demand obtained from the (1) original earthquake, (2) mean spectra from samples of twenty, and (3) one standard deviation confidence intervals, with 5% damping ratio and yield strength ratio of 0.1.

Most response spectra lies within the ± one standard deviation bound. For this set of data, it can be concluded that the measured record drawn from the model process may be accepted within the confidence intervals. Graphical representation of the data indicates that the model is reliable and acceptable. Graphs show the displacement ductility and normalized hysteretic energy with 5% damping ratio ($\xi$), and yield strength ratios (Y) of 0.05, 0.1 and 0.2, using elasto-plastic and stiffness degrading systems for El-Centro and Nahanni earthquakes.

## 5 SUMMARY AND CONCLUSIONS

In this study a method was presented for using non-stationary ARMA models to characterize specific historical earthquakes, and develop structural earthquake-response spectra for damage prediction.

As a basic concept in this analysis, a measured earthquake acceleration time series was regarded as one sample realization from a population of such a series associated with the event. The underlying process was assumed to be an ARMA process. The parameters of the ARMA process were estimated using maximum-likelihood approach, and the order of the model was found using Akaike Information Criteria (AIC). From these models, a set of acceleration records was generated for each event and used to estimate damage and response spectra.

Based on the four earthquakes analyzed, this study suggests the following conclusions:

- A time-domain approach using ARMA models provides a simple approach with a limited number of parameters. For wide-scale use in practice, a limited number of model parameters are essential.
- The assumption that an earthquake event is one sample realization from a population of such time series corresponding to an underlying event, allows a good description of average response spectra and damage potential. The use of average spectra for an event is statistically more valid than the use of spectra from a single record.
- Analysis of displacement ductility and normalized hysteretic energy for elasto-plastic and stiffness degrading systems indicated many similarities. Nonlinear analysis of the four events indicates that ARMA(2,1) process, using samples of twenty simulated earthquakes, can provide an acceptable description of earthquake properties.
- The hypothesis that an earthquake is one sample realization from a population of such time series, corresponding to the underlying event characterized by ARMA(2,1) and one peak envelope function, is a valid representation of the damage effects of the underlying earthquake.

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