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SHAPE OF YIELD SURFACE ESTIMATED BY USING NATURAL STRAIN UNDER PRE-DEFORMATION OF SHEAR AFTER UNIAXIAL TENSION

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In the present paper, the anisotropy of yield surface generated under large deformation is investigated on the basis of the Natural Strain theory, which can be treated systematically from the infinitesimal deformation to the large deformation. After applying the pre-deformation of uniaxial tension, simple shear and combined load of them to the test pieces, proportional loading tests are carried out again with changing the ratio of tension and torsion. Then, the shape of yield surface is investigated by estimating the yield stress in an arbitrary direction in the stress space. As the method for determining the yield stress, the slope of the tangent at yielding in the deviatoric stress- deviatoric strain curve is used. In order to reveal the influence of deformation history on the development of anisotropy in yield surface, the pre-deformation that the deformation history is different from previous study, namely, the pre-deformation of shear after uniaxial tension, is examined in this study. Then, the relation between the development of anisotropy in the yield surface and the deformation history of pre-deformation is revealed by comparing with the results of previous study.

Keywords: Deformation history, Yield stress, Anisotropy, Finite deformation, Elasto-plastic analysis, Ductile materials, Proof stress, Slope of tangent.

1 INTORODUCTTION

In general, as for the yield surface under the multi-axial loading conditions, the combined hardening model that the isotropic hardening and kinematic hardening coexists is used. On the other hand, in the case of the theory for representing the anisotropy of yield surface, the models by ellipse shape and convex shape are commonly used. However, in any case, almost these models are defined on the basis of infinitesimal deformation theory, and as for the yield surface under large deformation, it is not always fully elucidated.

The Natural Strain suggested in this study is associated with a certain identical line element in a body, and it is obtained by integrating an infinitesimal strain rate on an identical line element over the whole process of the deformation path. Consequently, since the shearing strain component of Natural Strain is represented by a pure angular strain, the rigid body rotation can be clearly removed. Moreover, since the additive law of strain on an identical line element can be satisfied, this strain rate can be clearly decomposed into the elastic component and the plastic component. Furthermore, the elastic component can also be decomposed into deviatoric part and volumetric part in the same manner as conventional infinitesimal deformation theory. Therefore, the Natural Strain is the effective strain representation which can be systematically treated from infinitesimal deformation to the large deformation.

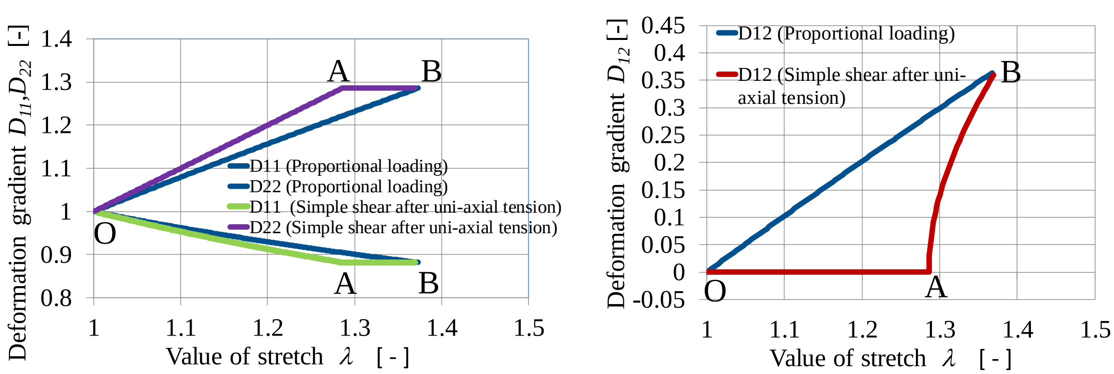
In a series of our previous study (Kato and Kazama 2013), after applying the large pre-deformation of uniaxial tension or simple shear to the test pieces, proportional loading tests have been carried out again with changing the ratio of tension and shear. Then, the shape of yield surface has been estimated by investigating the slope of the tangent in those deviatoric stress and deviatoric strain curves. Consequently, it was revealed that the shape of yield surface in a stress space becomes convex at the pre-deformation side and flat at the opposite side of the pre-deformation. Thus, the existence of anisotropy in the yield surface has been confirmed. However, in these studies, the pre-deformation was limited to the uniaxial state of tension or shear. Hence, in subsequent work (Kato 2014), the pre-deformation of combined loading of tension and shear with the proportional loading state has been investigated.

However, the detailed studies concerning the effect of the deformation history of pre-deformation have not been conducted before. So, as the next step of this research, another types of pre-deformation, namely, the deformation path is different but the final deformation is the same as the pre-deformation of proportional loading of tension and shear, have been chosen as the subject of this research. Especially, in a recent study (Kato 2016), the deformation history of tension after simple shear has been examined.

In the present study, in order to elucidate the effect of deformation history on the development of anisotropy of yield surface more in detail, the pre-deformation that is reversed the order of deformation path, namely, the pre-deformation of shear after uniaxial tension, is examined.

2 THE DEFORMATION HISTORY OF PRE-DEFORMATION

In the present paper, the pre-deformation, which the deformation path is different but the final deformation state is the same as the deformation path of tension after simple shear that has been investigated in the previous study (Kato 2016), is chosen as the subject. Namely, the shear after uniaxial tension is examined as the deformation history of pre-deformation. Figure 1 (a), (b) shows the differences in the deformation history between the pre-deformation of the present paper（O→A→B）and the pre-deformation of proportional loading（O→B）by comparing the deformation gradients (the normal components D_{11} , D_{22} and the shearing component D_{12}). As for the deformation path in this study, in the first stage, the uniaxial tension is applied during O→A, and after that, in the second stage, the simple shear is applied during A→B.



(a) Deformation gradient D_{11}, D_{22} (b) Deformation gradient D_{12}

Figure 1. Deformation path during pre-deformation (shear after uniaxial tension).

3 METHOD FOR ESTIMATION OF YIELD STRESS

Figure 2 shows the schematic diagrams of the principal deviatoric stress- deviatoric strain curve generated after applying the large pre-deformation. Since the yield stress has been already found from the final stress condition of the pre-deformation (see point B), the slope of tangent at yielding can also be specified in advance (see strait blue line at point A').

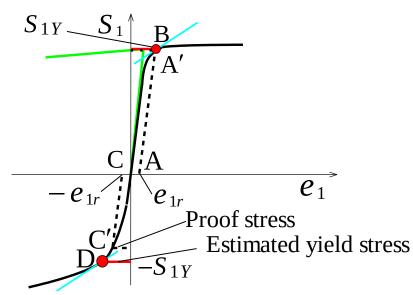


Figure 2. Principal deviatoric stress-deviatoric strain curve.

First, the experimental equation for the principal deviatoric stress, which becomes shallow curve around yielding point, is formulated as

$$S_1 = a\left(1 - exp\left(b e_1\right)\right) + c e_1 + d \tag{1}$$

where, *a*, *b*, *c*, *d* are coefficients determined by using the *Levenberg-Marquardt Method*, which is one of the non-linear least-squares method. And, in this figure, the experimental equation is described by the red solid curve. Moreover, in order to derive the slope of the tangent, the following equation is obtained by differentiating Eq.(1), i.e.,

$$\frac{dS_1}{de_1} = -a b\, exp\left(b e_1\right) + c \tag{2}$$

Hence, the slope of the tangent at yielding can be specified by using Eq.(2). As one example, considering the case of the opposite side of the pre-deformation, a point D, which has the same value as the slope in pre-deformation side, is assumed to be a yield stress. On the other hand, the yield stress by proof stress, which is defined by using residual strain, is estimated smaller as shown in point C'.

Moreover, the strain hardening modulus *h* is derived as follows.

$$h = \frac{2G}{\dfrac{dS_1}{de_1}} - 1 \tag{3}$$

As previously mentioned, since the slope of tangent at yielding is already specified, the strain hardening modulus at yielding can also be specified. Hence, the shape of the yield surface can be estimated based on the distributions of *h*.

4 EXPERIMENTAL METHOD

In the experiments, in order to apply a large pre-deformation to the specimens, the cylindrical specimens made from high purity tough pitch copper, i.e., outer diameter 22 [mm], inner diameter

16 [mm] , the gauge length 30 [mm] and purity 99.99%, are adopted. And, the multi-axial loading test machine, which can be applied tension and torsion at the same time, was used. As for the measurements of longitudinal displacements, the displacement meters equipped to the circular jig are used, and the rotary encoders are used for the measurements of twisting angles (see Fig.3).

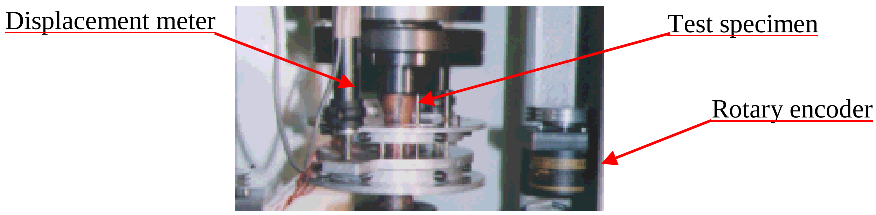


Figure 3. Test machine.

Next, the experiments are composed of two stages. Namely, the experiments applying the pre-deformation of shear after uniaxial tension to the specimens are firstly conducted. In the second place, the proportional loading tests of tension and torsion are conducted.

(i) Experimental condition of pre-deformation of shear after uniaxial tension

The uniaxial tension, i.e., the principal stretch m =1. 28, is firstly applied, then, the torsion, i.e., shear deformation D_{12} =0.37, is applied. However, the final deformation is the same value in all experiments, i.e., the principal stretch λ =1.38 [-].

(ii) Experimental condition of proportional loading tests for tension and shear

Attaching the tri-axial strain gages to the specimens, the experiments are conducted with the principal axis of stress being fixed in arbitrary angle, i.e., 0, 10, 25, 35, 45, 50, 55, 65, 70, 80, 90, 100, 110, 120, 135, 145, 155,170 [deg.]. Here, the principal strain is measured from the extensional strain components of the tri-axial strain gauge in consideration of the Natural Strain theory. Then, according to the method explained in the previous chapter, the value of yield stress is determined. Moreover, the shape of the yield surface in a stress space is estimated from the distribution of the yield stress.

5 EXPERIMENTAL RESULTS

Figure 4 shows the locus of stress points in each proportional loading tests, which are conducted after applying the pre-deformation to the specimens. Here, ① is the result of 0 [deg.], i.e., the case of uni-axial compression, ② is 10[deg.], ③ is 25[deg.], ④is 35[deg.], ⑤ is 45[deg.], i.e., the case of the simple shear and it is the final direction of the pre-deformation, ⑥ is 50[deg.], ⑦ is 55[deg.], ⑧ is 65[deg.], ⑨ is 70[deg.], ⑩ is 80[deg.],⑪ is 90[deg.], i.e., the case of uni-axial tension, ⑫ is 100[deg.], ⑬ is 110[deg.], ⑭ is 120[deg.], ⑮ is 135[deg.], i.e., the simple shear in the reversed direction, ⑯ is 145[deg.], ⑰ is 155[deg.] and ⑱ is 170[deg.].

As the example, the principal deviatoric stress- deviatoric strain curves in case of ⑤, ⑪ and ⑮ are described in Figure 5 (a), (b) and (c), respectively. Here, the experimental equation, i.e., Eq. (1), is described by the solid red curve in each figure. As is obvious from these figures, the stress and strain relation is a sharp curve at shear side of ⑤. However, it becomes gradually shallow curve as approaching the reverse shear side, which is the opposite direction to the final state of pre-deformation.

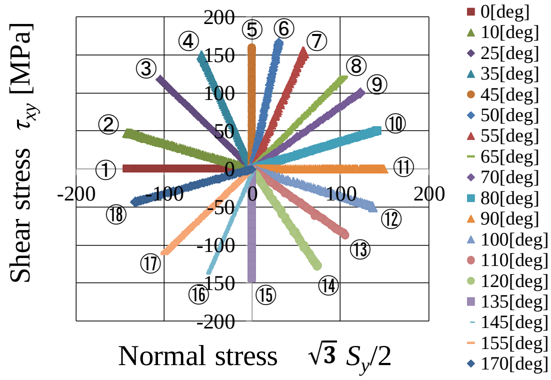


Figure 4. Stress points in each proportional loading tests.

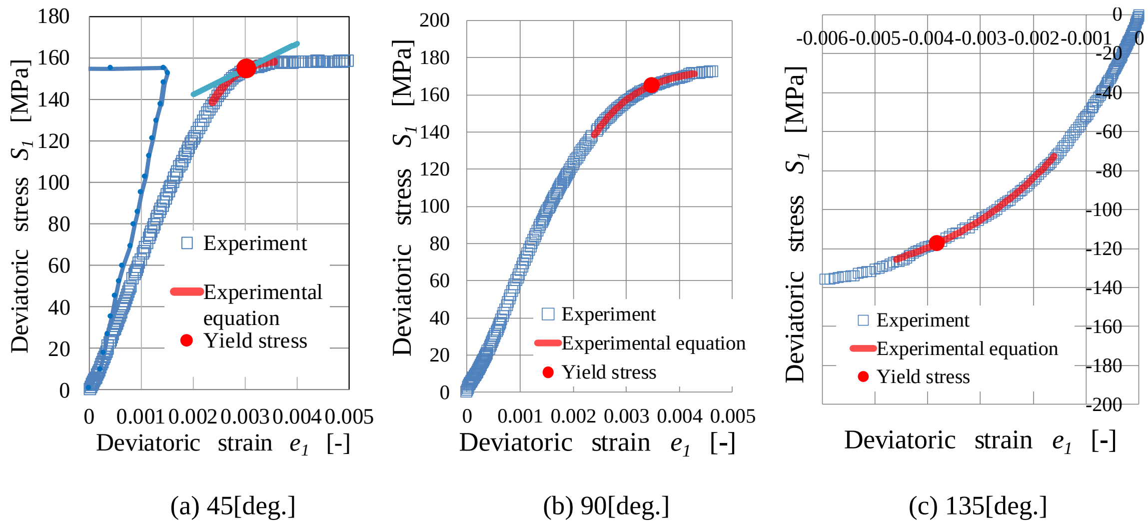


Figure 5. Principal deviatoric stress – deviatoric strain diagram.

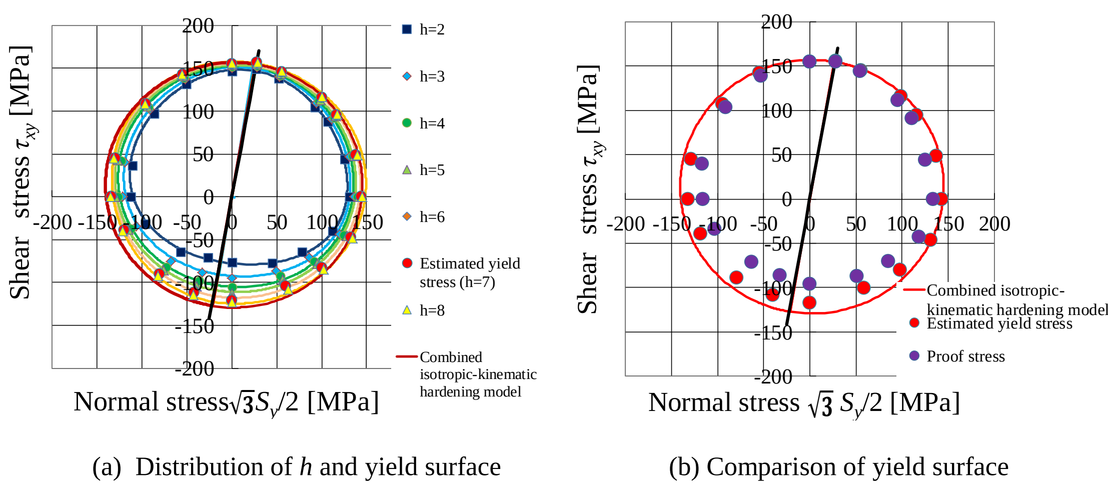
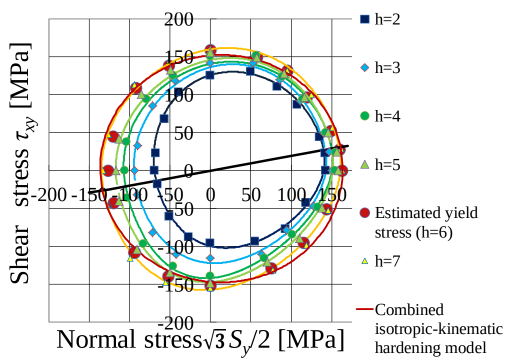
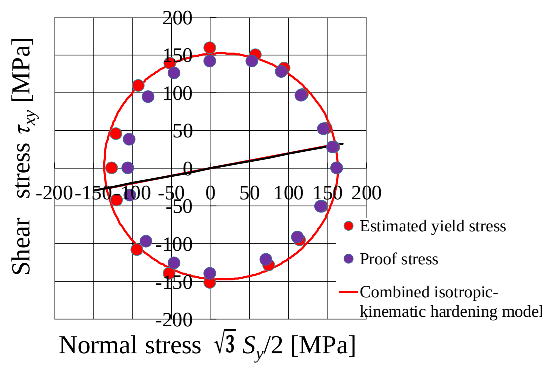


Figure 6. Shape of yield surface and distributions of *h* (shear after uniaxial tension).

Figure 6 (a) shows the results of the estimated yield stress in all directions in a stress space. In this figure, the circular plots (●) indicate the estimated yield stress. Moreover, the distributions of the strain hardening modulus *h* are also described in this figure. As the loading direction approaches to the simple shear side that is the second stage of the pre-deformation, the distance of curves in *h* becomes narrow. However, the distance becomes larger as the loading direction approaches the revers shear side, i.e., 135[deg.]. On the other hand, Figure 7 (a), (b) shows the results of yield stress generated under pre-deformation of uniaxial tension after simple shear in the previous study. It can be confirmed that the shapes of these curves become the convex at the tension side and become flat at compression side. Therefore, the anisotropy of the yield surface is closely related to the deformation applied at the second stage of pre-deformation.



(a) Distribution of *h* and yield surface



(b) Comparison of yield surface

Figure 7. Shape of yield surface and distributions of *h* (tension after simple shear).

6 CONCLUSIONS

In the present study, using the test piece applying the pre-deformation of shear after uniaxial tension, the yield stress was estimated by examining the slope of tangent on the principal deviatoric stress – deviatoric strain curve. As the result, since the shape of the yield surface becomes convex near the shear side and becomes flat at the reversed shear side, it was revealed that the anisotropy of the yield surface is formed closely related to the shear deformation applied at the second stage of pre-deformation. As for the distributions of strain hardening modulus *h*, it was revealed that the distance of these curves is small at the shear side, however, at the reversed shear, the distance becomes large. Moreover, the shape of estimated yield surface was compared with the shape by conventional proof stress. Consequently, both yield surfaces almost coincide at the shear deformation side, but at the opposite side, the yield surface by proof stress is small compared with the yield surface estimated in this study.

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