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# MATHEMATICAL MODEL AND SIMULATION OF VEHICLE PLATOON OVERTAKING OTHER VEHICLE

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A vehicle platoon is the method of grouping vehicles into platoon and driving synchronously. The vehicles in the platoon can travel safely in very small vehicle distance with the help of the mechanical and electric systems and thus, the traffic flow can be increased without any additional road construction. In this study, the interaction behavior between the vehicle platoon of three vehicles and the other vehicle is discussed. When the platoon of three vehicles overtakes the other vehicle, the vehicles in the platoon change the lane, overtake the preceding slow-travelling vehicle and changes the lane again. The vehicle velocity is controlled according to the vehicle following model (Bierley 1963, Chandler *et. al.* 1958, Gazis *et. al.* 1961, Helly 1959). The control models of vehicle velocity and behavior are defined in the mathematical model and discussed in the experiments of the robot vehicle. The results show that the model can make the platoon overtakes the other vehicle safely. In the near future, the model should be improved in order to enhance the safety and the efficiency of the vehicle platoon.

*Keywords*: Vehicle velocity, Vehicle following model, LEGO MINDSTORM.

## 1 INTRODUCTION

In vehicle platoon, vehicles travel as a coordinated array with small vehicle head distance by the help of the electric and mechanical control system. It is considered as the important technique for increasing the traffic volume safely. The most important issue of the vehicle-platoon control system is to keep safe and adequate head distance between vehicles even when some vehicles in the platoon show down suddenly. Simply, it is most important to keep the vehicle platoon stable. For this purpose, authors studied the velocity control model of the vehicles in the platoon (Kita *et. al.* 2014, 2016). When, however, the vehicle platoon travels along the highway and inter-city road network, the control of vehicle platoon becomes more complicated. One of such situations is the overtaking of the platoon. When the platoon of a few vehicles overtakes the other vehicle, each vehicle in the platoon departs from the group control of the platoon and then, overtaken the vehicle.

The aim of this study is to present the control model of the vehicle velocity in the platoon. The model is formulated according to the vehicle following model in case that the platoon of three vehicles overtakes the other vehicle. The validity of the control model is discussed in the computer simulation and the experiment of LEGO MINDSTORM (LEGO).

The remaining part of this paper is organized as follows. In section 2, the model is formulated by means of vehicle following model. In section 3, the model parameters are determined in the computer simulation. In section 4, the model with designed parameters is applied for the experiments of LEGO MINDSTORM. In section 5, the conclusion is summarized again.

## 2 MATHEMATICAL MODEL OF VELOCITY CONTROL

### 2.1 Traffic Situation

One vehicle platoon and two single vehicles travel in the same direction (Figure 1). The platoon is composed of three vehicles which can travel individually. One of two single vehicles travels in from of the platoon in the same direction as the platoon at the slower speed than the platoon. The other travels along the other lane. Three vehicles in the platoon are named as the lead, the first following and the second following vehicles, respectively.

The behavior of the vehicles in the platoon is summarized as follows.

1) The lead vehicle of the platoon determines whether the platoon can overtake its preceding travelling vehicle.
2) When it is decided that the platoon cannot overtake, the lead vehicle slow down to the velocity of the frontal vehicle.
3) When it is decided that the platoon can overtake, each vehicle of the platoon changes the lane, overtake the frontal vehicle and change the lane again.

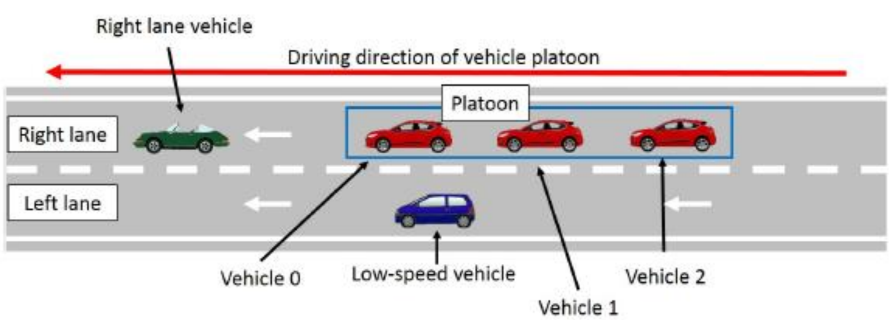


Figure 1. Traffic situation composing of one vehicle platoon and two single travelling vehicles.

### 2.2 Velocity Control Model

The vehicle platoon is composed of three vehicles. Their velocity is controlled by the vehicle following model, which controls the vehicle velocity according to the mathematical model with the information from the frontal vehicles. Chandler model is adopted in this study. The vehicle acceleration rate is defined as the linear function of the velocity distance of the vehicle and its frontal vehicle as follows.

$$\ddot{x}_1(t+\Delta t) = \alpha\left(\dot{x}_{n-1}(t) - \dot{x}_n(t)\right) \quad (n = 1,2) \tag{1}$$

The variables $x_n(t)$ and $x_{n-1}(t)$ are the positions of the vehicle $n$ and its nearest preceding vehicle $(n-1)$, respectively. The terms $\dot{x}_{n-1}(t) - \dot{x}_n(t)$ denotes the vehicle velocity difference between the vehicles $(n-1)$ and $n$. The parameters $\alpha$ is the sensitivity parameter.

### 2.3 Overtaking Condition

Each vehicle in the platoon overtakes the frontal vehicle by turns. The overtaking situation is shown in Figure 2. The vehicle P will overtake the vehicle L in order to avoid the collision with the vehicle R.

Assume that the positions of the vehicles L, P and R are referred as $x_L(t)$, $x_P(t)$ and $x_R(t)$ at the time $t$, respectively. The following condition is held for them.

$$x_L(t) > x_P(t) > x_R(t) \tag{2}$$

The velocities of the vehicles L, P and R are as follows.

$$v_L(t) > 0, v_P(t) > 0, v_R(t) > 0 \tag{3}$$

The distance between the vehicle P and the vehicle R is given as follows.

$$x_P(t) - x_R(t) - \frac{l_P + l_R}{2} \tag{4}$$

Assume that the passing time of the vehicle P over the vehicle L is $\Delta t$. If the vehicle R doesn't come into collision with the vehicle P, the following condition has to be satisfied.

$$\left(x_P(t) - x_R(t) - \frac{l_P + l_R}{2}\right) + v_P(t)\Delta t \geq v_R(t)\Delta t$$

$$x_P(t) - x_R(t) \geq \{v_R(t) - v_P(t)\}\Delta t + \frac{l_P + l_R}{2} \tag{5}$$

When the vehicle P starts to overtake the vehicle L, the distance between the vehicle L and the vehicle P is $x_L(t) - x_P(t)$. The following relation is held.

$$v_P(t)\Delta t = v_L(t)\Delta t + x_L(t) - x_P(t) + \frac{l_L + l_P}{2} \tag{6}$$

The following equation is obtained from equations (5) and (6).

$$x_P(t) - x_R(t) \geq \frac{v_R(t) - v_P(t)}{v_P(t) - v_L(t)}\left\{x_L(t) - x_P(t) + \frac{l_L + l_P}{2}\right\} + \frac{l_P + l_R}{2} \tag{7}$$

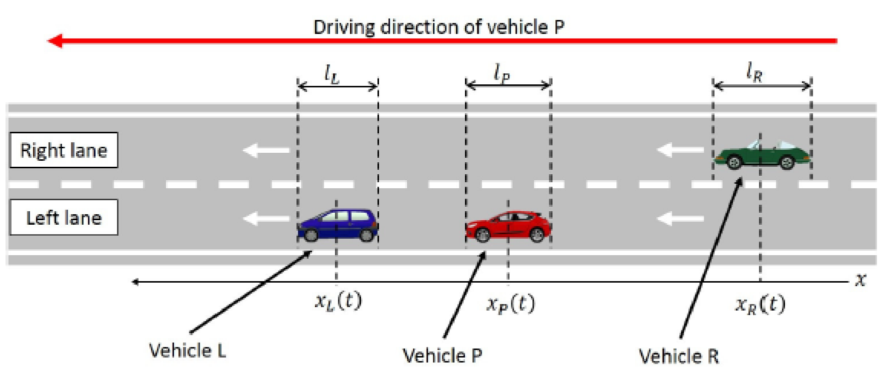


Figure 2. The vehicle p overtakes the vehicle L for avoiding the collision with the vehicle R.

Table 1. Initial position and velocity of vehicles.

| Vehicle | Initial velocity (cm/s) | Initial position (cm) |
|---|---|---|
| Vehicle 0 | 11 | 0 |
| Vehicle 1 | 11 | -33 |
| Vehicle 2 | 11 | -66 |
| Low-speed vehicle | 7.5 | 48 |
| Right lane vehicle | 16.5 | -33 |

## 3 SIMULATION AND EXPERIMENTAL RESULTS

Along the road, one vehicle platoon and two single vehicles travel in the same direction. The vehicle platoon is composed of three vehicles, which are named as Vehicle 0, Vehicle 1 and Vehicle 2. One single vehicle, "the low-speed vehicle", travels along the same road at the slower speed than the vehicle platoon. Another single vehicle, "the right lane vehicle", travels along the other lane. Their initial position and the initial velocity are listed on Table 1. The vehicle 0, the vehicle 1 and the vehicle 2 of the platoon, the low-speed vehicle and the right lane vehicle locate at 0 time-step at 0 cm, -33 cm, -66 cm, 48 cm and -33 cm, respectively. The low-speed vehicle and the right lane vehicle travel at the constant velocity of 7.5 cm/s and 16.5 cm/s, respectively. The platoon starts at the velocity of 11 cm/s.

The numerical simulation results are shown in Figure 3. The platoon follows the preceding travelling vehicle from 24 to 67 time-steps and at 68 time-step, starts to overtake the preceding travelling vehicle. The vehicle 2, which is the last vehicle of the platoon, overtakes the preceding vehicle at the 163 time-step.

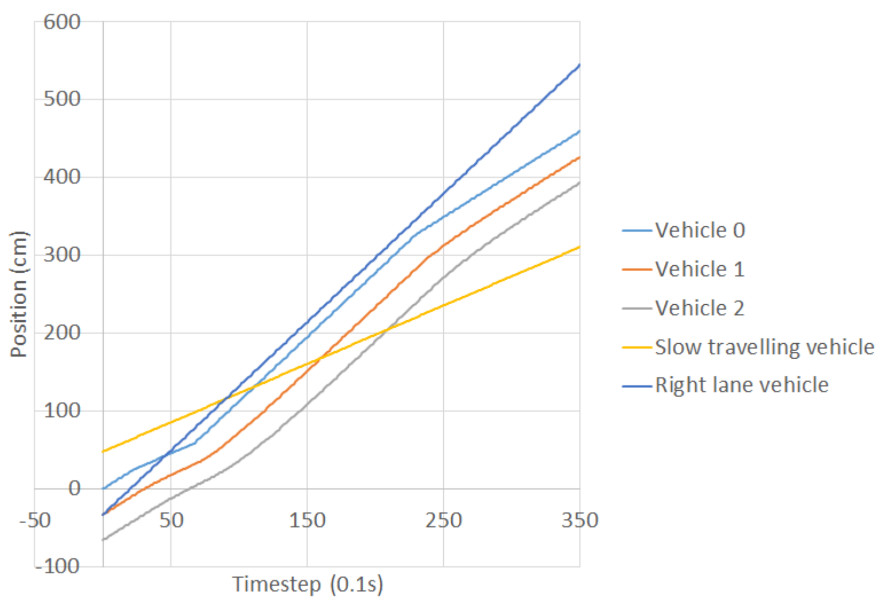


Figure 3. Trajectory of vehicle positions in simulation.

The velocity control model is used for the vehicles in the experimenal result of LEGO MINDSTORMS EV3. The experiment is illustrated in Figure 4. The trajectroy of the vehicle positions is shown in Figure 5. The low-speed and the right lane vehicles travel at the constant speed, just like as the siumulation in Figure 3. The trajectories of the vehicles in the platoon are also similar to them in the siumulation in Figure 3. The differences between the vehicles in the platoon in Figures 3 and 5 are caused by the delay time of the velocity control model and the noise included in the input data such as the sensor information, the lane changing of vehicles, and so on.

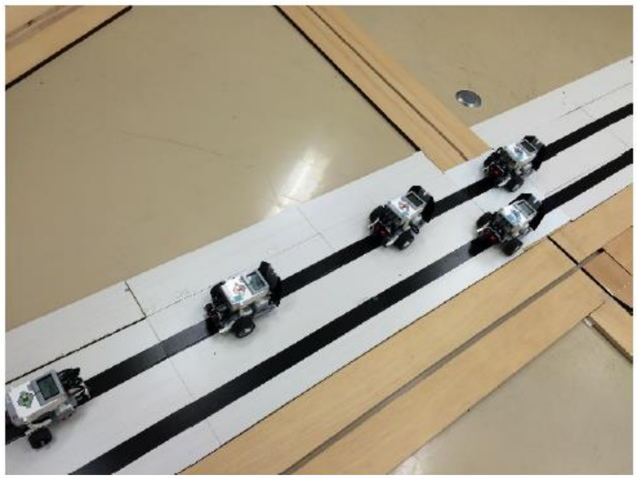

Figure 4. Experimental status.

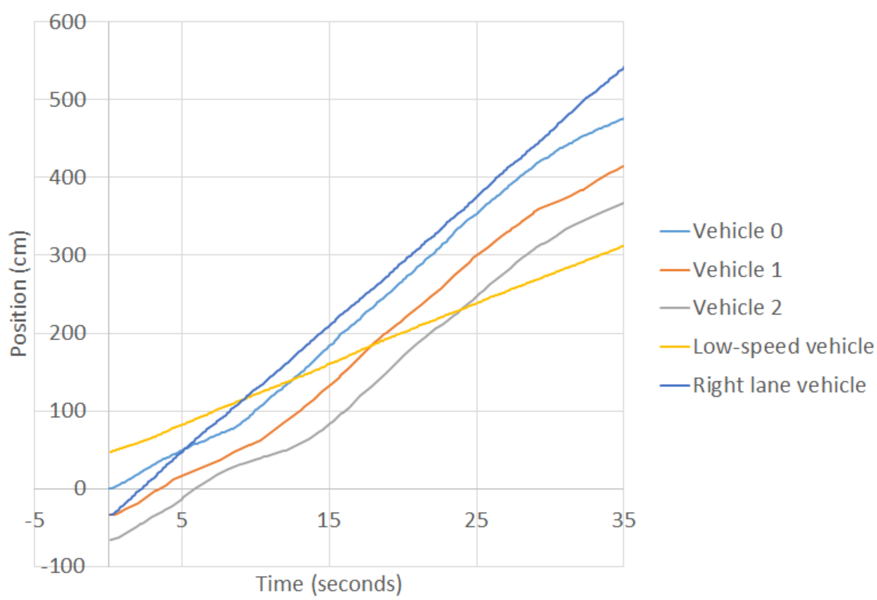


Figure 5. Trajectory of vehicle positions in experiment of LEGO MINDSTORM.

## 4 CONCLUSIONS

This paper describes the velocity control model of the vehicles in the platoon, when the platoon avoids the vehicle travelling on the other lane and overtakes the preceding traveling vehicle. The model is defined as the Chandler model which controls the vehicle velocity according to the velocity difference between the vehicle and its preceding vehicles. The overtaking condition which the vehicles in the platoon can overtake the preceding travelling vehicle is defined from the positions and velocities of the vehicles and their distances.

The model was applied for the velocity control of the vehicles in the computer simulation and the experiments of LEGO MINDSTORMS EX3. The experimental result was compared with the computer simulation result. The vehicle behavior in the experimental result is agreed relatively well with the vehicle behavior in the computer simulation result although the slight difference between the computer simulation and the experimental results is observed. It is assumed that the difference is caused by the delay time of the velocity control model and the noise included in the input data such as the sensor information, the lane changing of vehicles, and so on.

**Acknowledgments**

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