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A SIMPLE CONSTITUTIVE MODEL FOR UNDRAINED SHEAR CHARACTERISTICS OF SANDY SOILS

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A vast amount of past experimental examinations reported that the internal peak angle of sand was jointly affected by the density and effective stress level. Several relationships were proposed between these elements. The dependence of dilatancy characteristics on the internal state of a granular material was examined and revealed. A simple constitutive model framework was established on a basis of several well-proven and experienced relationships for granular materials to simulate their undrained shear behavior. A basic hardening law connecting the varying tendency of the stress ratio with shear strain was employed. This model is capable of predicting the undrained monotonic stress-strain relationship of granular materials at different densities and various confining pressures. A series of parametric studies are conducted to investigate the susceptibility of the simulation results to the selected parameters. The simulation results also confirm the influential influences of dilatancy and deformability on the shear characteristics of granular materials at the critical state.

Keywords: Undrained shear behavior, Granular material, Stress-dilatancy, Constitutive relation, Critical state.

1 INTRODUCTION

The shear strength and deformation characteristics of granular soils subjected to shearing are of great interest in geomechanics and geoengineering practice (Bolton 1986). The sands during shearing tend to contract at loose state and show dilative tendency at dense state. Thus, the density state is regarded as an influential factor governing the shear response of sands. The mean stress level is also an important parameter affecting the shear response of sand.

In undrained conditions, flow failure of loose sand resulting from the steady state deformation following the unstable behavior is liable to occur. The steady state (Castro and Poulos 1977) is a state of deformation of soil without effective stress increment or decrement and with migration of pore water. The steady state is regarded as anonymous to the critical state in many experimental investigations (Been *et al*. 1991, Yamamuro and Lade 1998). The latter term is employed in the following section. Shear strength at the critical state was believed to be the undrained strength of granular soils with a flow failure. Flow deformation would continue infinitely once the outer force exceeds the undrained shear at the critical state. It was realized that the determination of shear strength at the critical state was more important than the decision of the stress triggering liquefaction during associated evaluations.

Some well-proven equations acquired from the drained triaxial tests are combined to formulate a simple constitutive model with the capacity of simulating the shear response of grain soils in this investigation. This study intends to provide a better understanding of the shear properties of granular materials with particular emphasis on the existence of critical state. The establishment of this simple model is also expected to be employed to evaluate the liquefaction phenomena of sand in laboratorial test.

2 CONSTITUTIVE MODEL

Some well-known equations associated with the strength and deformation characteristics of granular materials were summarized based on a large amount of experimental data in past investigations. Relevant relationships for this simple constitutive model are reviewed as below.

2.1 Peak Shear Strength Estimation

Past investigations revealed that the peak friction angle ϕ_p of granular materials was jointly governed by the levels of initial density and confining pressure (Bolton 1986, Gutierrez 2003, Lashkari 2009). A rise in density and a reduction in confining pressure enhance the peak friction angle ϕ_p of granular soils. The peak friction angle ϕ_p is composed of the frictional angle at the critical state ϕ_{cr} and the portion due to dilation behavior. The friction angle ϕ is correlated with the stress ratio η by the equation $\sin\phi=3\eta/(6+\eta)$ at any state during triaxial shearing. Analogous to this relationship, the peak stress ratio of granular materials η_p could be expressed using Eq. (1).

$$\eta_{\mathrm{p}} = \eta_{cr} + CD_r \ln(\frac{p_{cr}}{p}) \tag{1}$$

where η_p is the peak stress ratio, η_{cr} is the stress ratio at the critical state, D_r is the relative density and C is a material parameter related to the dilatancy characteristics of soils. p_{cr} refers to the mean stress when the peak stress ratio reduces to the stress ratio at the critical state. The peak stress ratio η_p only emerges for the soil at the dense state. η_p approaches to η_{cr} at the critical state as the mean stress p is increased to be identical to p_{cr}. It is indicated that the peak stress ratio η_p is independent on the density and confining pressures once the mean stress p exceeds p_{cr}. Besides, p_{cr} is also greatly affected by the level of density.

2.2 Stress-dilatancy

Stress-dilation relationship is also an important mechanical characteristic to describe the shear response of soils. This relationship captures the evolution of strain increment in accompany with the stress ratio of granular materials in drained shearing test. Well prediction of the stress-dilatancy characteristic plays a major role in the usefulness of a constitutive model. Some classical expressions were proposed to characterize the stress-dilation behavior. The empirical equation in Cam-clay model (Roscoe *et al*. 1963) is written in Eq. (2).

$$D = M - \eta \tag{2}$$

where $D=d\varepsilon_{vd}/d\varepsilon_s$ is the rate of dilation, $d\varepsilon_{vd}$ is the volumetric strain increment due to dilation and $d\varepsilon_s$ is the shear strain increment. M represents the stress ratio at the critical state and is regarded as synonymous to η_{cr} in Eq. (1). It is noted that the summary of rate of dilation D and stress ratio η is a constant in Eq. (2). Li and Dafalias (2000) proposed a state-dependent stress-dilatancy form using the state parameter ψ as the state variable in Eq. (3).

$$D = d_o\left(e^{m\psi} - \frac{\eta}{M}\right) = \frac{d_o}{M}\left(Me^{m\psi} - \eta\right) \tag{3}$$

where m and d_o are two positive parameters. It is noticed that the Eq. (2) is the specific case of Eq. (3) when m is equal to zero and d_o is identical to M. At the critical state, the $\psi = 0$ and $\eta = M$ lead to a zero dilatancy. Thus, Eq. (3) is positively included in this simple model.

2.3 Distortion Hardening Law

The distortion relationship linking the development of stress ratio η with shear strain $d\varepsilon_s$ was proposed and well-proven in the hyperbolic stress-strain model (Duncan and Chang 1970). This simple hardening law assumes that granular soils are constantly approaching to the peak shear stress ratio η_p with increasing shear strain ε_s in Eq. (4).

$$\eta = \frac{\eta_p \varepsilon_s}{A + \varepsilon_s} \tag{4}$$

where A is a parameter associated with the initial shear modulus of soils.

2.4 Model Construction

Particular expressions reviewed above are combined and formulated to create a new simple constitutive model. The specific procedure to implement the calculation using the proposed simple model is explained in this section. The total volumetric strain increment $d\varepsilon_v$ is supposed to be composed of the portion due to consolidation $d\varepsilon_{vc}$ and the portion induced by dilation $d\varepsilon_{vd}$. The volumetric strain increment due to consolidation $d\varepsilon_{vc}$ in Eq. (5) could be calculated from the relationship between the mean stress p and the void ratio e during isotropic consolidation loading.

$$d\varepsilon_{vc} = \frac{\lambda}{1+e_o}\frac{dp}{p} \tag{5}$$

where λ is the slope of isotropic consolidation line, e_o represents the initial void ratio of sand.

The total volumetric strain increment $d\varepsilon_v$ could be acquired using Eqs. (3) and (5)

$$d\varepsilon_v = d\varepsilon_{vc} + d\varepsilon_{vd} = \frac{\lambda}{1+e_o}\frac{dp}{p} + \frac{d_o}{M}\left(Me^{m\psi} - \eta\right)d\varepsilon_s \tag{6}$$

For undrained condition, the total volumetric strain increment $d\varepsilon_v$ should be zero for a saturated sample when the pore fluid and soil particle compressibility are neglected. Thus, the expression in Eq. (6) is rearranged to display the mean stress increment dp,

$$dp = -\frac{p(1+e_o)}{\lambda} d_o\left(e^{m\psi} - \frac{\eta}{M}\right)d\varepsilon_s \tag{7}$$

Putting the Eqs. (4) and (1) into the Eq. (7), we further obtain the exact form of mean stress increment dp,

$$dp = -\frac{p(1+e_o)}{\lambda} d_o\left[e^{m\psi} - \frac{\varepsilon_s}{\varepsilon_s + A}(1 + \frac{CD_r}{M}\ln\frac{p_{cr}}{p})\right]d\varepsilon_s \tag{8}$$

Eq. (8) descripts the variation in mean stress dp with the rise in shear strain increment $d\varepsilon_s$during undrained shearing in triaxial shearing test. The calculation implementation of dp is integrated numerically. Firstly, dp could be decided using Eq. (12) for a given increment $d\varepsilon_s$. Secondly, mean stress ($p \leftarrow p$+dp) and shear strain ($\varepsilon_s \leftarrow \varepsilon_s + d\varepsilon_s$) could be updated using superposition method. Then, peak stress ratio η_p could be re-determined using the new value of p and Eq. (1). The stress ratio η could also be updated using Eq. (8) to decide the new deviatoric stress q. Subsequently, the iteration calculation would continue with the same given increment $d\varepsilon_s$. Whole calculation would be completed once the desire level of shear strain is attained.

3 SIMULATION RESULTS

The simulation capacity of this simple constitutive model is demonstrated in this section. Figures 1-3 show the measured and predicted stress-strain response of Toyoura sand at loose, medium-dense and dense states at various confining pressures. In each figure, subtitle (a) describes the effective stress paths of sands and (b) shows the corresponding stress-strain curves of sands. The solid symbols are the measured results and the solid lines represent the predicted values. Verdugo and Ishihara (1996) conducted an extensive set of monotonic drained and undrained triaxial tests on Toyoura sand at a wide range of stresses and densities to determine the ultimate condition of sand. The initial void ratios of Toyoura sand are prepared as 0.907 (loose), 0.833 (medium-dense), 0.735 (dense). A set of high quality data was reported. The maximum and minimum void ratios of the Toyoura sand are 0.977 and 0.597, respectively.

The prediction values have a good agreement with the measured results of Toyoura sand subjected to undrained shearing in Figures 1-3. This constitutive model has capacity of describing the pure contractive behavior of loose samples at confining pressures of 1.0 and 2.0 MPa. This simple model is capable of predicting dilative behavior of medium-dense sand at confining pressures of 0.1 MPa and 1.0 MPa and contractive behavior of medium-dense sand at confining pressures of 2.0 MPa and 3.0 MPa in Figure 2. Both of predicted and measured results show that the deviatoric stresses of medium-dense sand at confining pressures of 0.1 MPa and 1.0 MPa monotonically increase with the shear strain progresses. However, medium-dense sand rapidly grains the peak shear strength and gradually loses the deviatoric stress with the rise in shear strain. For dense sand, simulated values are generally consistent with the observation results in Figure 3. The slight deviation of predicted result from the measured value at a confining pressure of 0.1 MPa in Figure 3 is probably resulted from the strong dilation estimation at low confining pressures by the selected stress-dilatancy relation in Eq. (7).

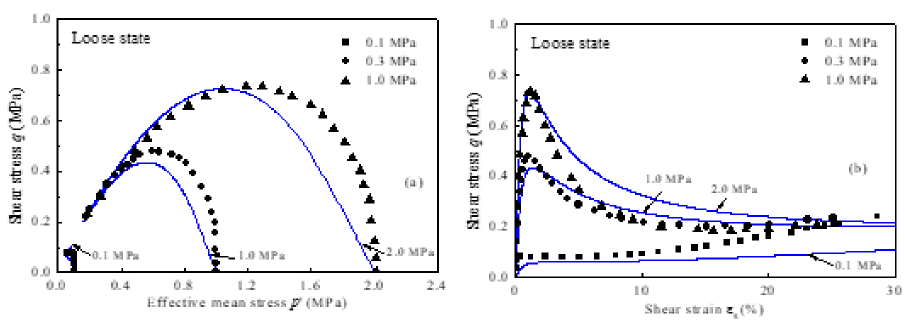

Figure 1. Predicted and measured stress-strain response of loose sand.

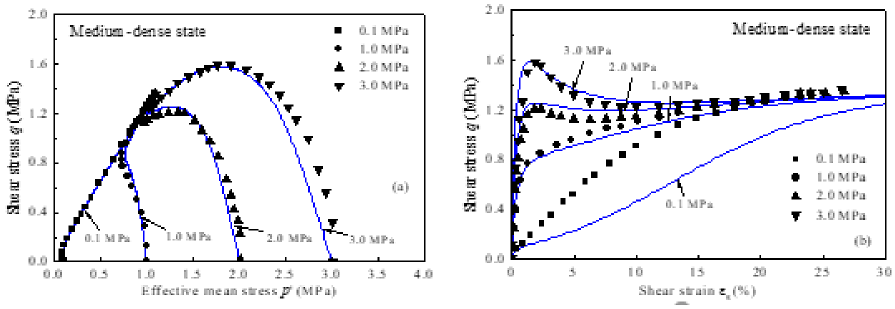

Figure 2. Predicted and measured stress-strain response of medium-dense sand.

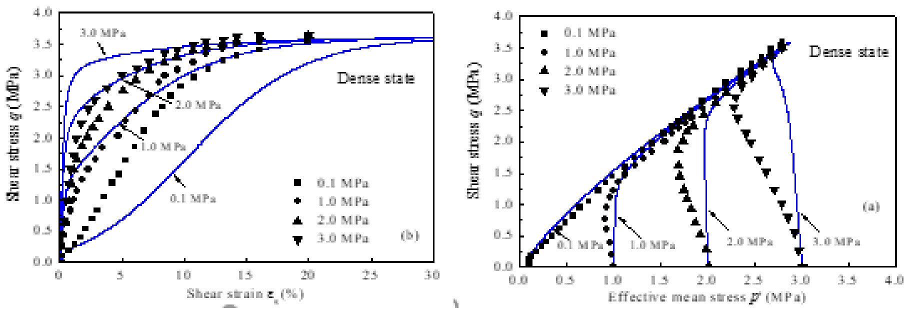

Figure 3. Predicted versus measured stress-strain response of dense sand.

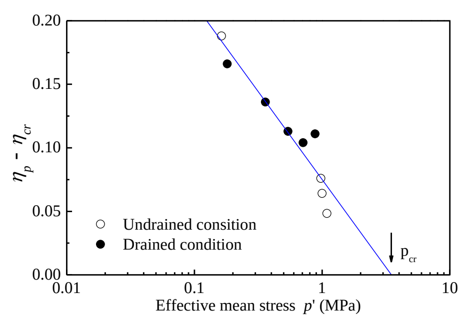

Figure 4. Determination of the mean stress at the critical state p_{cr}.

It is noted that the simulation results of sand specimens at different confining pressures and initial densities approach to the constant deviatoric stress *q*, which is the hint of the critical state as the axial strain progresses. It is indicated that the critical state could be attained at larger shear strains around 20% through the simulation using this simple constitutive model. Table 1 shows the set of simple model parameters for Toyoura sand at three densities. The determination method of the associated parameters is simply explained. The difference in the stress ratios at the peak state and the critical state are plotted against the effective mean stress to determine p_{cr} using Eq. (5) in Figure 4. It is seen that the rise in density increases the material parameter *C* and level of p_{cr}. With regard to the parameters related to the stress-dilatancy and the location of critical state line, they are independent on the density of sand.

Table 1. Constitutive model parameters for sand with different densities.

	C	D_r (%)	p_{cr} (MPa)	m	d_o	M	e_Γ	λ_{csl}	ξ	λ	A
Loose (e_o=0.907)	0.13	18.5	0.28	3.5	0.88	1.25	0.934	0.019	0.7	0.028	0.007
Medium-dense (e_o=0.833)	0.37	37.9	1.2	3.5	0.88	1.25	0.934	0.019	0.7	0.035	0.005
Dense (e_o=0.735)	0.47	63.7	3.0	3.5	0.88	1.25	0.934	0.019	0.7	0.050	0.002

4 CONCLUSIONS

A simple constitutive model was proposed based on some well-established relationships for sand obtained from drained shearing test to predict the undrain shear response and the static liquefaction of granular materials. The combined effects of density and mean stress level on the peak shear strength was considered and reflected in the relevant expressions. A rise in density and decline in the confining pressure lead to a marked dilation behavior of granular material. These features are reflected by the stress-dilatancy relationship considering the state of granular materials. The parameters of constitutive model can be determined form the triaxial shear test. The proposed simple constitutive model has the capacity of predicting density-dependent and stress-dependent undrained shear properties of granular material using a few parameters. The predicted results show good agreement with the measured values for sand at various densities and confining pressures.

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