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EFFECTS OF THE INITIAL DEFLECTION SHAPE ON THE MOMENT AMPLIFICATION FACTOR OF BEAM-COLUMNS

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In the design of slender steel beam-columns, the moment amplification factor is used to estimate the maximum moment along with the longitudinal direction. While formulas for evaluating the factor have been presented on the basis of elastic or elastic-plastic analysis, the initial deflection of the column is not considered. The effect that the initial deflection on the strength and behavior of the column has been shown only when the initial deflection shape is half sine wave. This paper discusses the effect of the initial deflection shape on the value of the moment amplification factor by performing the analytical work. The analytical model is the hinged-end beam-column subjected to constant axial compressive force and end moments. First of all, the equilibrium differential equation which governs the problem is solved and the formula for calculating the bending moment is presented. In the parametric study, magnitude of initial deflection, initial deflection shape, axial load ratio, slenderness ratio and end moment ratio are selected as the parameters. In this paper, we discuss the effects of the amount of the initial deflection and the initial deflection shape.

Keywords: Steel beam-columns, End moment ratio, Axial load ratio, Slenderness ratio, Strength formula, Analytical solution.

1 INTRODUCTION

In the design formulas of the steel beam-columns of AIJ Recommendation for Limit State Design of Steel Structures (hereinafter referred to as LSD guideline) (AIJ 2010), when the maximum value of the bending moment occurs between the point of supports, the bending moment is evaluated by using the moment amplification factor a_m. In LSD guideline, the moment amplification factor ${}_d a_m$ is adopted as the factor at the maximum strength in the inelastic state (Sakamoto *et al.* 1968). On the other hand, the moment amplification factor ${}_e a_m$ can be analytically calculated for a linear elastic body steel beam-columns. Moment amplification factor a_m, ${}_d a_m$ and ${}_e a_m$ have been obtained by analysis in the absence of the initial deflection. Regarding the effect of the initial deflection on the strength and behavior, we examined only when the initial deflection shape was sine half wave (Utsunomiya *et al.* 2016). This paper discusses the effect of initial deflection expressed by a half sine wave only or the sum of a half sine wave and one-cycle sine wave and the dimensionless factors on the value of moment amplification factor by performing the analytical work.

2 ANALYSIS

2.1 Problem Setting

As shown in Figure 1, the analytical model is the hinged-end beam-column subjected to constant axial compressive force N and end moments M_1 and $M_2(=\kappa M_1, |\kappa| \leq 1)$. The value of κ is the end moment ratio. The end moment M_1 is numerically larger than M_2. The value of κ is positive when the member is bent in reverse curvature. Boundary conditions are pin supports at both ends. In this paper, the moment amplification factor a_m (=$M_{\max}/M_1$:$M_{\max}$ is the maximum bending moment in the entire region of the member) is analytically calculated when the initial deflection y_0 is given.

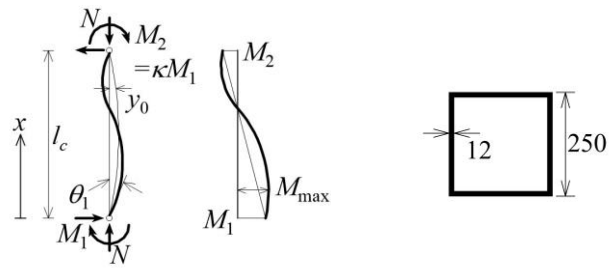

Figure 1. Analytical model.

2.2 Equilibrium Differential Equations and the Solutions

The moment considering the initial deflection y_0 is given in Eq. (1) (Utsunomiya *et al.* 2016).

$$\frac{M(\xi)}{M_1} = \cos\left(\pi\sqrt{n_y\lambda_c^2}\,\xi\right) - \frac{\kappa + \cos\left(\pi\sqrt{n_y\lambda_c^2}\right)}{\sin\left(\pi\sqrt{n_y\lambda_c^2}\right)}\sin\left(\pi\sqrt{n_y\lambda_c^2}\,\xi\right) + \sum_{j=1}^{n}\frac{Nc_j}{M_1}\frac{j^2}{j^2 - n_y\lambda_c^2}\sin(j\pi\xi) \tag{1}$$

In the above equation, ξ is $\xi \equiv x/l_c$ (x is the coordinate in the member length direction with the origin at the column base), the axial load ratio n_y and the normalized slenderness ratio λ_c in the Eq. (1) are defined by Eqs. (2) and (3), respectively.

$$n_y = \frac{N}{N_y} \quad \left(N_y = A\sigma_y\right) \tag{2}$$

$$\lambda_c = \sqrt{\frac{N_y}{N_e}} \quad \left(N_e = \frac{\pi^2 EI}{l_c^2}\right) \tag{3}$$

Where N_y is yield axial force, A is cross-sectional area, σ_y is yield stress, E is the Young's modulus (=205,000N/mm^2), I is the moment of inertia.

Initial deflection c_j is the coefficient that defines the initial deflection y_0 expressed in Eq. (4).

$$y_0 = \sum_{j=1}^{n} c_j \sin(j\pi\xi) \tag{4}$$

$M_{\max}/M_1$ obtained by Eq. (1) gives the moment amplification factor a_m. From Eq. (1), a_m is related to the end moment ratio κ, $n_y\lambda_c^2$, and Nc_j/M_1. The value of Nc_j/M_1 is divided into the n_y, c_j, and η.

The dimensionless amount η is defined in Eq. (5) equation, where M_y is a yield moment.

$$\eta = \frac{M_1}{M_y} \tag{5}$$

2.3 Moment Amplification Factor (Elastic and Inelastic)

Equation (6) shows the moment amplification factor ${}_e a_m$ obtained by the elastic analysis in the case there is no initial deflection, and Eq. (7) is the moment amplification factor ${}_d a_m$ at the maximum strength in the inelastic state prescribed in LSD guideline.

$$\begin{aligned} {}_e a_m &= 1 && (\kappa \geq -\cos(\pi \sqrt{N/N_e})) \\ &= \frac{\sqrt{1+\kappa^2 + 2\kappa\cos\left(\pi\sqrt{N/N_e}\right)}}{\sin\left(\pi\sqrt{N/N_e}\right)} && (\kappa < -\cos(\pi \sqrt{N/N_e})) \end{aligned} \tag{6}$$

$$\begin{aligned} {}_d a_m &= 1 && \left(n_y \lambda_c^2 \leq 0.25\left(1+\kappa\right)\right) \\ &= \frac{1-0.5\left(1+\kappa\right)\sqrt{N/N_e}}{1-N/N_e} && \left(n_y \lambda_c^2 > 0.25\left(1+\kappa\right)\right) \end{aligned} \tag{7}$$

2.4 Analytical Parameters

We targeted two initial deflection shapes, 1) a half sine wave only, 2) the sum of a half sine wave and one-cycle sine wave.

We selected the value of initial deflection shape n, the coefficient of the initial deflection c_1 and c_2, end moment ratio κ, axial load ratio n_y, and η values in Eq. (5) as the analytical parameters, and they vary as follows.

1) n=1 (a half sine wave only)
   n=2 (the sum of a half sine wave and one-cycle sine wave)
2) $c_1=l_c/1500$, $c_2=l_c/15000$, $l_c/7500$, $l_c/3000$
   $c_1=l_c/1000$, $c_2=l_c/10000$, $l_c/5000$, $l_c/2000$
   $c_1=l_c/500$, $c_2=l_c/5000$, $l_c/2500$, $l_c/1000$
3) □□=-1.0, -0.5, 0, 0.5, 1.0
4) n_y=0.1, 0.3, 0.5, 0.7
5) □ =1, 2/3

The cross section used in the analysis is a square steel tube, the width of the cross section is 250 mm, and the plate thickness is 12 mm (see Figure 1). The yield stress σ_y is 325 N/mm^2.

3 RESULT AND DISCUSSION

3.1 $a_m/{}_e a_m$-$n_y\lambda_c^2$ Relations

3.1.1 *Influence of initial deflection*

Figure 2 shows the influence of the initial deflection c_1, c_2 on the moment amplification factor a_m when the axial load ratio n_y=0.3, η=1. In the figure, $c_1=l_c/500$, $l_c/1000$, and $l_c/1500$ are indicated by solid lines, ● mark, and ○ mark, respectively.

From Figure 2, it can be seen that the moment amplification factor a_m increases as the initial deflections c_1 and c_2 are larger.

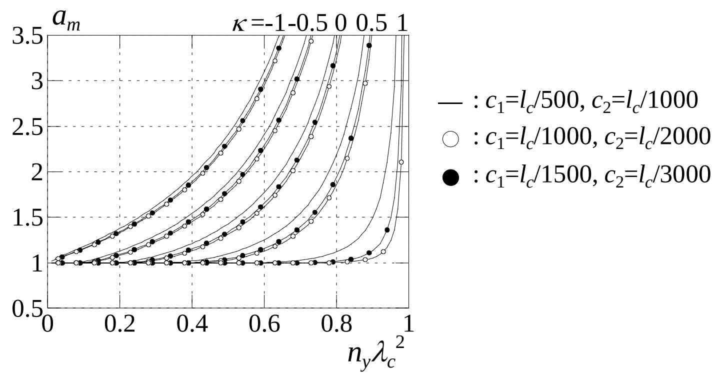

Figure 2. a_m-$n_y\lambda_c^2$ Relations (Influence of initial deflection c_1, n_y=0.3, η =1).

3.1.2 *Influence of initial deflection shape*

Figure 3 shows the influence of c_2 on the moment amplification factor a_m when the axial load ratio n_y=0.3, η=1. Fig. 3(a), (b) and (c) show the case of the initial deflections $c_1=l_c/500$, $l_c/1000$, $l_c/1500$ respectively. Regarding the initial deflection c_2, there is a relationship of $c_2=c_1/2$. In the figure, the initial deflection of a half sine wave only with n=1 is shown by the dotted line, and the initial deflection of the sum of a half sine wave and one-cycle sine wave is shown by the solid line.

According to Fig. 3, it can be seen that in the case of the initial deflection made of the half sine wave and one-cycle sine wave, the moment amplification factor a_m is larger than the case where the initial deflection is only the half sine wave.

3.2 Influence of Initial Deflection Shape

Figure 4 shows the relationship between a_{m2}/a_{m1} and $n_y\lambda_c^2$. a_{m2}/a_{m1} is the value obtained by dividing the moment amplification factor a_{m2} in the case of n=2 by the moment amplification factor a_{m1} in the case of n=1. Figs. 4 (a)-(c) show the cases where the initial deflection c_1 is $l_c/500$

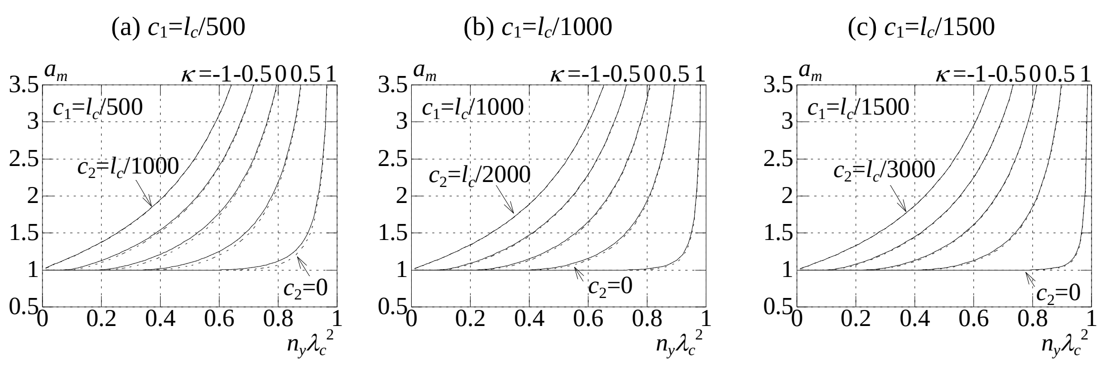

Figure 3. a_m-$n_y\lambda_c^2$ Relations (Influence of initial deflection shape, n_y=0.3, η =1).

and c_2 is $l_c/1000$, $l_c/2500$ and $l_c/5000$ respectively. Figs. 4 (d)-(f) show the case of initial deflection c_1 is $l_c/1000$ and c_2 is $l_c/2000$, $l_c/5000$ and $l_c/10000$ respectively. Figs. 4 (g)-(i) show the case of initial deflection c_1 is $l_c/1500$ and c_2 is $l_c/3000$, $l_c/7500$ and $l_c/15000$ respectively. In the figure, ○, dotted line, ●, Dashed line, and ■ marks in the figure indicate cases where end moment ratio κ is -1, -0.5, 0, 0.5, 1, respectively.

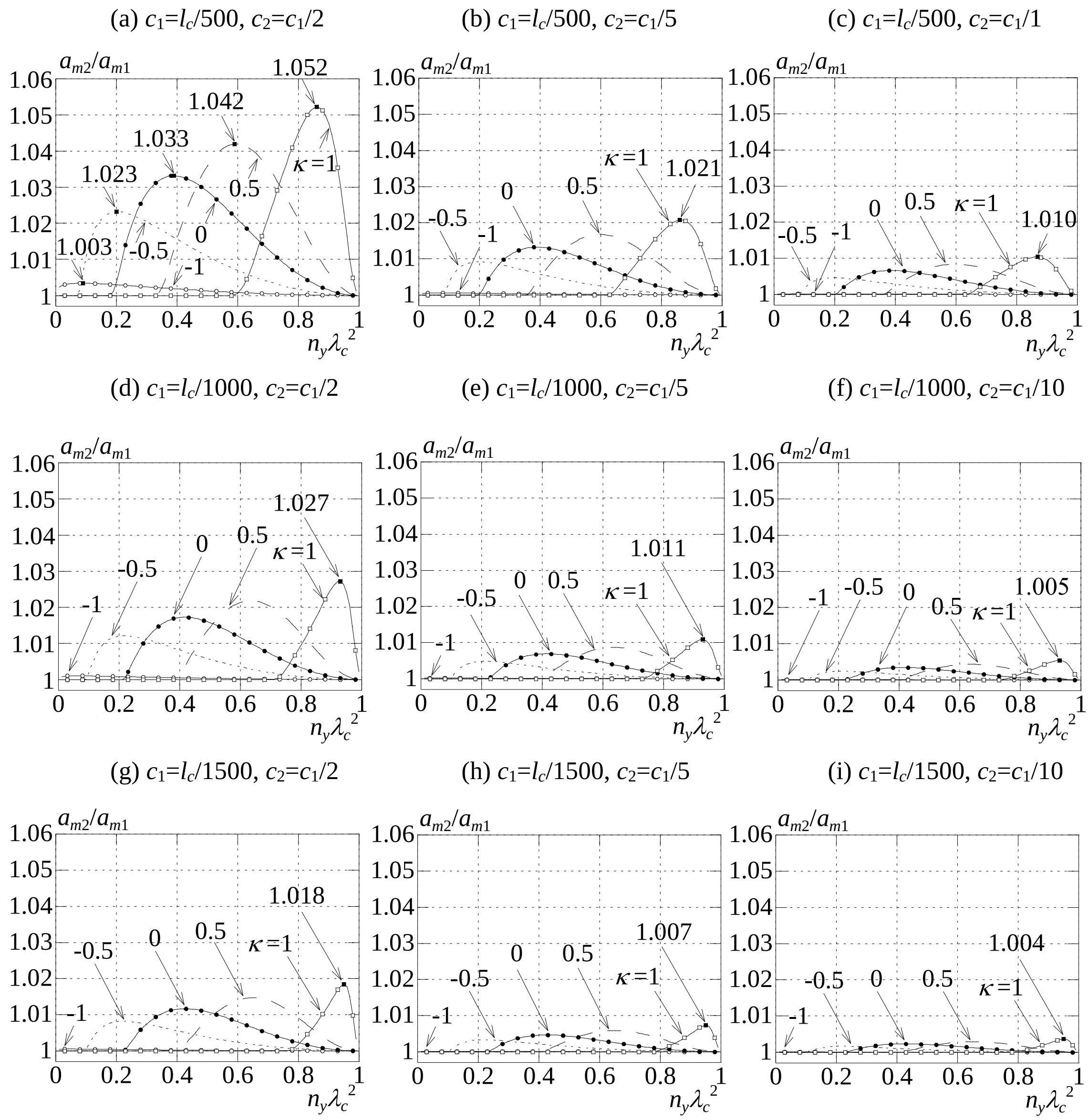

Figure 4. Influence of initial deflection shape *n*.

According to Figure 4, it is observed that the maximum value of a_{m2}/a_{m1} increases as the end moment ratio κ increases. Figs. 4 (a), (b), and (c), when the value of the initial deflection c_1 is constant, the maximum values of a_{m2}/a_{m1} are 1.052, 1.021, and 1.010, respectively, and the larger the initial deflection c_2 is, the maximum value of a_{m2}/a_{m1} is larger. Figures 4 (a), (d), and (g), when the value of the initial deflection c_2 is equal to $c_1/2$, the maximum value of a_{m2}/a_{m1} is 1.052, 1.027 and 1.018 respectively, the larger the initial deflection c_1 is, the larger the maximum value of a_{m2}/a_{m1}.

Also, when $c_1=l_c/500$ and $c_2=l_c/1000$ in Fig. 4 (a) where the effects of the initial deflection is the largest, the maximum values of a_{m2}/a_{m1} in the cases of the end moment ratio κ=-1, -0.5, 0, 0.5 and 1 are 1.003, 1.023, 1.033, 1.042 and 1.052, respectively, at most about 5%.

From the above, the effects of the initial deflection shape on the moment amplification factor increases as the value of c_2 increases. Even when c_2 is large as c_2=c_1/2 and the influence of one-cycle sine wave is at most about 5% larger than the case of an only half sine wave.

4 CONCLUSIONS

The conclusions derived from this study are as follows:

1) As the initial deflection c_1 and c_2 increase, the value of the moment amplification factor a_m increases when $n_y\lambda_c^2$ is equal.
2) In the case of initial deflection is the sum of a half sine wave and one-cycle sine wave, the moment amplification factor a_m is larger than the case where the initial deflection is only the half sine wave. The effects of the initial deflection shape composed of the sum of a half sine wave and one-cycle sine wave on the moment amplification factor becomes larger as the value of c_2 becomes larger, but even if c_2 is large as c_2=c_1/2, a_{m2}/a_{m1} is about 1.052 and 5% at the maximum, and the effects of one-cycle sine wave is not large (see Fig. 4).

Acknowledgments

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