

FLEXURAL STRENGTH OF SQUARE CFT BEAM-COLUMNS WHEN DAMAGED WITH CONCRETE CRACKS

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The lateral stiffness of CFT beam-columns is considered to decrease because of the damage by the concrete crack or yielding of the materials however the relationship between two phenomena is not clear. It is necessary to clarify the strength when the CFT columns damaged by the concrete crack for evaluating the lateral stiffness of CFT beam-columns properly. The purposes of this study are to calculate the flexural crack strength of a square CFT section and to compare them with the yield strength. The analysis is carried out as follows; 1) assuming the stress-strain relationships of materials and the strain and stress distribution of the cross-section, 2) setting the limit of the tensile strain of concrete and 3) calculating the relation between the axial force and the bending moment when the strain of concrete reaches the limit tensile strain. Parameters are width-thickness ratio, the yield strength of a steel tube and compressive strength of concrete. As a result of the analysis, we showed that the axial force ratio is $0.2 \sim 0.3$ when the flexural crack strength and the yield strength are the same. It is concluded that the lateral stiffness decreases because of the crack of concrete – and not from the yielding of materials -- when the axial force ratio is smaller than $0.2 \sim 0.3$.

Keywords: Axial force ratio, Bending analysis, Lateral stiffness, Restoring force characteristics, Steel-concrete composite structures, Tensile strength, Yield strength.

1 PURPOSE

The skeleton curve of concrete-filled steel tube (hereafter CFT) members are shown in AIJ CFT Recommendations (AIJ 2008). The first break point is the yield strength, and the second break point is the ultimate bending moment. Until the first break point, the initial lateral stiffness is calculated on the condition that all cross sections are effective and elastic in the CFT Recommendations. However, the effect of the concrete cracks is not considered and there is a possibility that the deformation of the CFT beam-column is evaluated conservatively.

Regarding a decrease of the lateral stiffness, the following can be considered. When the CFT columns are subjected to the axial force and the flexural moment, and the axial force ratio is relatively small, firstly the concrete cracks occur, and then the lateral stiffness decreases due to concrete cracking. As the bending moment further increases, a part of the cross section yields and the lateral stiffness further decreases. Where "a part of the cross section yields" means a steel tube yields or the stress of concrete reaches two-thirds of the compressive strength. On the other hand, when the axial force ratio is relatively large before the cracks occur, a part of the cross

section yields, and the lateral stiffness decreases. As described above, it is presumed that the factors resulted in the decrease of lateral stiffness depends on the axial force ratio. However, it is not clear that the strength when the CFT columns are damaged by the concrete cracks, and moreover the relationship between two strengths, when the CFT beam-columns are damaged by the concrete cracks and yield. For the proper evaluation of the lateral stiffness of the CFT beam-columns, it is necessary to clarify the relationship between the axial force and flexural moment of the CFT columns damaged by the concrete cracks, and the relationship between the crack moment and the yield moment. In the previous research, it has been shown that the parameters such as the material strength and width-thickness ratio affect the yield strength of the CFT columns (Liu *et al.* 2011).

The purposes of this study are to calculate the flexural moment when the square CFT beamcolumn is damaged by the concrete cracks and to show the crack flexural moment on the M-Nrelationships. The effects of the material strength and width-thickness ratio on the crack flexural moment are clarified. The correspondence of the crack flexural strength to the yield strength is shown. The range of the axial force ratio when the two strengths are the same is clarified.

2 METHOD

2.1 Analysis Model and Analysis Condition

Figure 1 shows the CFT cross-section used in the analysis. The cross-section consists of the rectangular steel tube whose cross section (width) is ${}_{s}D$ and the plate thickness is *t*, and it is subjected to the axial force *N* and the flexural moment *M*. Figure 2 shows the strain distribution and the stress distribution of the concrete portion when the tensile edge stress of the concrete portion has reached the tensile strength ${}_{c}\sigma_{cr}$ of concrete. ${}_{c}D$ is the depth/width of the concrete part.



Figure 1. Beam-column cross-section.



Figure 2. The strain distribution and stress distribution of the concrete part.

In this study, we use the tensile strength $_c\sigma_{cr}$ recommended by JSCE (2017).

$$c\sigma_{cr} = 0.23 \cdot c\sigma_B^{\frac{2}{3}} \tag{1}$$

The tensile strength of concrete is ${}_c\sigma_{cr} = 3.53$ N/mm², when concrete compressive strength ${}_c\sigma_B = 60$ N/mm². The strain at the tensile strength ${}_c\varepsilon_{cr}$ is about 0.01% although it depends on Young's modulus. In addition, the stress-strain relationship of concrete was assumed to be linear in both compression and tension for simplifying the analysis. Figure 3 shows the linear stress-strain relationship when the compressive strength of the concrete ${}_c\sigma_B = 60$ and 100N/mm². The Young's modulus ${}_cE$ of concrete was calculated by using the equation shown in AIJ RC Standard (AIJ 2010). The value of Young's modulus ${}_cE$ is also shown in Figure 3.



Figure 3. The stress-strain relationship of concrete.

2.2 Flexural Moment and Axial Force

2.2.1 Concrete portion

When the coordinate axis is taken as shown in Figure 2(a) the strain at the position of y expressed by the Eq. (2) according to the Navier's hypothesis.

$${}_{c}\mathcal{E}(y) = {}_{c}\mathcal{E}_{0} + y_{c}\varphi \tag{2}$$

From the definition of the section force, the axial force $_cN$ and the flexural moment $_cM$ of the concrete part can be calculated. Furthermore, using the following various quantities and dimensionless quantities, the dimensionless axial force, and the dimensionless flexural moment $_cn$, $_cm$ are given by the Eq. (3) respectively. Where $_c\varepsilon_0$ is the strain of the concrete at the axis of the member, $_c\varepsilon_m$ is the strain at the compressive strength, $_c\varphi$ is the curvature and y_0 is the distance from the axis of the member to the neutral axis.

$$c\tilde{\epsilon}_{0} = c\epsilon_{0}/c\epsilon_{m}, \ c\tilde{\varphi} = c\varphi_{c}D/c\epsilon_{m}, \ cN_{0} = cD^{2} \cdot c\sigma_{B},$$

$$cM_{0} = cD^{3} \cdot c\sigma_{B}, \ cn = cN/cN_{0}, \ cm = cM/cM_{0}$$

$$cn = \frac{cE \cdot c\epsilon_{m} \cdot c\tilde{\epsilon}_{0}}{c\sigma_{B}}, \ cm = \frac{cE \cdot c\epsilon_{m} \cdot c\tilde{\varphi}}{12c\sigma_{B}}$$
(3)

2.2.2 Steel tube portion

The stress-strain relationship of the elastic-perfectly plastic model was used for the steel tube portion. The dimensionless axial force and the dimensionless flexural moment $_{s}n$ and $_{s}m$ of the steel tube portion are given by the Eq. (4).

$$sn = \frac{sN}{sNy} = \frac{c\mathcal{E}m}{s\mathcal{E}y}c\mathcal{E}0$$

$$sm = \frac{sM}{sMy} = \frac{1}{2}\frac{sD}{cD}\frac{c\mathcal{E}m}{s\mathcal{E}y}c\mathcal{P}$$
(4)

Where, ${}_{s}N_{y} = {}_{s}A\sigma_{y}$ (${}_{s}A$ is the sectional area of the steel tube), ${}_{s}M_{y} = Z\sigma_{y}$ (Z is the section modulus of the steel tube) and ${}_{s}\varepsilon_{y}$ is the yielding strain of the steel tube.

2.3 Crack Condition

Assuming that the flexural crack occurs in the CFT column when the strain at the tensile edge of the concrete reaches the strain $_{c}\varepsilon_{cr}$ at the tensile strength of the concrete, the following equation is derived by the Eq. (2).

$${}_{c}\mathcal{E}(-\frac{cD}{2}) = {}_{c}\mathcal{E}_{0} - \frac{cD}{2}{}_{c}\varphi = {}_{c}\mathcal{E}_{cr}$$

$$\tag{5}$$

Therefore, the crack condition is given by the following expression with dimensionless quantities.

$$c\tilde{\epsilon}_{0} - \frac{c\tilde{\varphi}}{2} = \frac{c\mathcal{E}_{cr}}{c\mathcal{E}_{m}}, \quad c\tilde{\varphi} = \frac{1}{c\tilde{\epsilon}_{0}/c\tilde{\varphi} - 1/2} \cdot \frac{c\mathcal{E}_{cr}}{c\mathcal{E}_{m}}$$
(6)

2.4 Flexural Crack Strength of CFT Column

The flexural crack strength of CFT column *M* and *N* are obtained by the following procedure; 1) calculating the value of $_{c\tilde{\epsilon}0}$ and $_{c\tilde{\varphi}}$ by the Eq. (6) with varying the value of $_{c\tilde{\epsilon}0}/_{c\tilde{\varphi}}$ from -0.5 to 0.5, which is the condition that the neutral axis position is in the concrete portion, 2) calculating $_{c}n$, $_{c}m$, $_{s}n$, and $_{s}m$ by substituting $_{c\tilde{\epsilon}0}$ and $_{c\tilde{\varphi}}$ into Eqs. (3) and (4). The dimensionless crack flexural moment *m* and the axial force *n* of CFT cross section is calculated by the following equation. $_{c}A$ is the sectional area of the concrete portion.

$$n = \frac{N}{N_0} = \frac{cn \cdot cN_0 + sn \cdot sN_y}{sA \cdot \sigma_y + cA \cdot c\sigma_B}$$

$$m = \frac{M}{sM_y} = \frac{cm \cdot cM_0 + sm \cdot sM_y}{sM_y}$$
(7)

3 RESULTS AND DISCUSSION

3.1 Analytical Parameters

Analysis parameters are as follows:

- Width to thickness ratio of square steel tube ${}_{s}D/t = 20, 40$
- Yield strength of square steel tube $\sigma_y = 325, 440 \text{ N/mm}^2$
- Concrete compression strength $_c\sigma_B = 60, 100 \text{ N/mm}^2$

3.2 Results of Calculation

Figure 4 shows the crack strength on the *m*-*n* relationship. Also, the yield strength obtained by the method presented in the past research (Liu *et al.* 2011) is indicated by thin lines. The yield strength is determined by the lower of the strengths the stress of the concrete reaches $2/3_c \sigma_B$ or the steel tube yielding. When calculating the yield strength, the stress-strain relationship of the concrete is assumed by a quadratic function and the strain ${}_c\varepsilon_m$ at the compressive strength is 0.2% and 0.25%.



Figure 4. The bending moment-axial force relationship.

3.3 Effect of the Parameters on the Crack Flexural Strength

The effect of parameters on the crack strength is discussed. With regards to the effects of the width-thickness ratio ${}_{s}D/t$ and comparing Figures 4 (a) with (b), (c) with (d), (e) with (f) respectively, it is observed that the value of the crack flexural moment *m* at the same axial force ratio increases as ${}_{s}D/t$ increases. With regards to the effect of the compressive strength of concrete ${}_{c}\sigma_{B}$ and comparing Figures 4 (a) with (c), (b) with (d) respectively, as ${}_{c}\sigma_{B}$ increases, it is observed that the value of *m* at the same axial force ratio increases. As for the effect of the steel tube yield strength σ_{y} and comparing Figures 4 (c) with (e), (d) with (f) respectively, as σ_{y} increases, it is observed that the value of *m* at the same axial force ratio decreases.

3.4 Relationship between the Crack Flexural Strength and Yield Strength

According to the relation between the yield moment and the crack moment in Figure 4, the following can be considered. For example, in Figure 4(a), the crack moment is less than the yield moment at n = 0.1 and the yielding moment is reached after cracking occurs. On the other hand,

when n = 0.4, the crack moment is larger than the yielding moment, and cracks would not occur before yielding. In other words, in the case where the axial force is relatively small, it is considered that the lateral stiffness decreases due to the occurrence of cracks.

The point A in Figure 4 shows the point when the crack moment is equal to the yielding moment. The axial force ratio to be point A is approximately 0.2 to 0.3. Therefore, when the axial force ratio is less than or equal to this axial force ratio, cracks occur and the lateral stiffness would start decreasing. This result suggests that the concrete cracking should be considered when we evaluate the lateral stiffness of the CFT beam-columns.

4 CONCLUSIONS

The flexural crack strength of the CFT column was calculated by taking the width-thickness ratio of the steel tube, the yield strength of the steel tube and the compressive strength of the concrete as parameters. The flexural crack strength was shown on the m-n interaction diagram and the influence of each parameter was clarified. In addition, we compared the flexural crack strength with the yield strength calculated based on the method by the past research. The conclusions derived from this study are as follows:

- As the width-thickness ratio of the steel tube and the compressive strength of concrete increase, the value of the flexural crack moment at the same axial force ratio increases. As the yield strength of the steel tube increases, the value of the flexural crack moment at the same axial force ratio decreases.
- The axial force ratio when the flexural crack strength is equal to the yield strength is approximately in the range of 0.2 to 0.3.

It is considered that the lateral stiffness decreases due to the occurrence of the flexural cracks when the axial force ratio is less than 0.2~0.3. We would now like to go on to calculate by expanding the range of the parameters and present the skeleton curve of the CFT beam columns, which can consider the effect of the concrete cracking by using these results.

Acknowledgments

This study was supported by the Grant-in-Aid for Scientific Research of Japan Society for the Promotion of Science.

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