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A UNIFIED INELASTICITY AND DAMAGE MECHANICS FOR RUBBER-LIKE MATERIALS

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The nonlinearities observed in the behavior of rubber and polymeric materials are influenced not only by the far-field stress conditions but also by the morphology, microstructures, and changes at the meso-level such as kinks, crosslinking, and micro tearing. Considering constants temperature applications, the occurrence of micro-tearing has a significant contribution on the performance of polymers, soft tissues and other rubber-like materials. In this paper, a unified damage mechanics and nonlinear elasticity approach is presented to model nonlinear behavior of rubber-like materials under constant temperatures. The theory is cast within the general framework of the internal variable theory of thermodynamics with large deformations where the Clausius-Duhem inequality is provoked to develop general damage potential. The strain energy density function is formulated in terms of an effective Lagrangian strain tensor that evolves with cumulative damage as cracking and micro-tearing take places. Piola-Kirchhoff (PK) stress tensor is presented and a new form of the damage response tensor is proposed. The model prediction is illustrated against experimental results with good agreement.

Keywords: Continuum mechanics, Response tensor, Plasticity, Large deformation.

1 INTRODUCTION

Rubbery materials such as polymers, general elastomers, and soft tissues exhibit certain deformational characteristics that require the use of finite deformation theories in their modeling. These materials are generally considered incompressible, materially nonlinear, and non-homogenous. Experiments have shown that they exhibit hysteresis under cyclic loading and could experience microstructural tearing under higher levels of stresses. The micro-tearing causes the material to be more compliant and leads to further anisotropy. Even though a considerable literature can be found on the constitutive modeling of rubber-like materials (Mullins 1969, Simo 1987, Oktay 1993, Le Cam 2010, Volokh 2013, Wang *et al.* 2014, Armstrong and Spontak 2017) the development of a unified approach to rubber inelasticity with damage is surprisingly lagging the other modeling efforts.

The nonlinear deformational characteristics of rubber like materials stem from their microstructural properties. These properties exhibit a threshold by which when exceeded, micro-tearing and hence damage is initiated rendering the material more compliant. It is also shown that when rubber is stretched to certain strain level, the material cannot be damaged any further until the maximum previously experienced strain is reached again. This behavior is best demonstrated in the cyclic loading shown here schematically in Figure 1. The load-unloading curve shows the phenomenon known as the “Mullin’s effect”.

In this paper, a unified damage mechanics and nonlinear elasticity approach is combined and presented within the internal variable theory of continuum thermodynamics with finite deformations. The Lagrangian strain tensor along with a cumulative damage parameter representing the effects of micro-tearing is utilized. Using the spectral decomposition of tensors, the Lagrangian strain tensor is then decomposed into its positive and negative cones. This would allow for the decoupling of damage in tensile and compression modes. A form of strain energy density function is then postulated serving as a potential function for the Piola-Kirchhoff (PK) stress tensor. Finally, the paper proposes new anisotropic response tenors for certain elastomers to account for potential cross micro-tearing during large deformations.

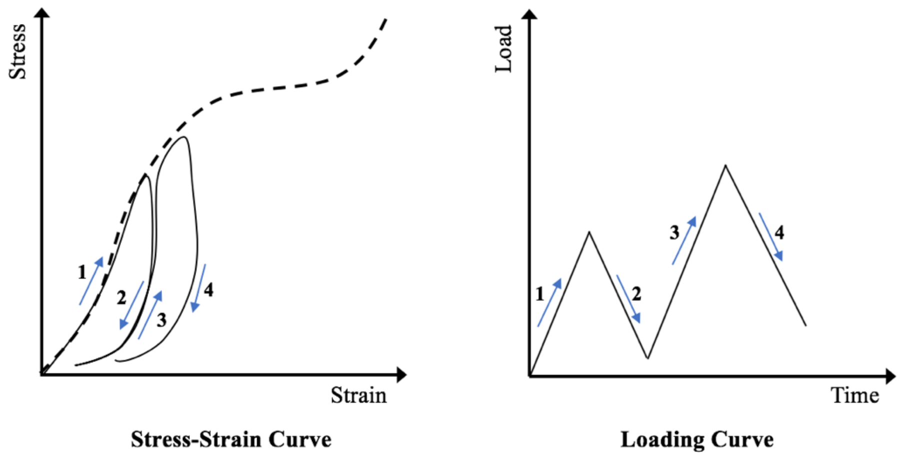

Figure 1. Schematic representation of Mullin's effect in uniaxial tension.

2 FORMULATION

For isothermal processes and utilizing the internal variable theory of thermodynamics, the Clausius-Duhem inequality can be shown to yield the following dissipation inequality (Eq. (1)) as (Lubliner 1972):

$$\frac{\partial}{\partial D}\psi(\boldsymbol{E},D)\dot{D} \geq 0 \tag{1}$$

where, D represents a cumulative damage parameter, the superdot "." signifies the time rate of deformations, and the function $\psi(\boldsymbol{E},D)$ represents the strain energy density function for the material. The Lagrangian strain tensor is given by $\boldsymbol{E}$. This is a standard approach in solid mechanics where Lagrangian strain measure is used for large deformations. The dependence of the strain energy density function on $\boldsymbol{E}$ and $\boldsymbol{D}$ allows for the description of the current state of the material as deformation evolves with damage. It is further postulated that the current state of the deformation is related to the undamaged material state through energy equivalence (Eq. (2)) as:

$$\psi(\boldsymbol{E},D) = \psi^0\{\boldsymbol{E}^*(\boldsymbol{E},D)\} \tag{2}$$

where, $\boldsymbol{E}^*$ is defined as an effective Lagrangian strain tensor and Ψ^0 represents the equivalent undamaged strain energy density function. Specific forms of $\boldsymbol{E}^*$ would define different models in the literature. The standard thermodynamic argument leads to obtaining Piola-Kirchhoff stress tensor, $\boldsymbol{S}$, as thermodynamic forces conjugate to fluxes $\dot{\boldsymbol{E}}$ as:

$$\boldsymbol{S} = \frac{\partial \psi(\boldsymbol{E}, D)}{\partial \boldsymbol{E}} \tag{3}$$

and,

$$\boldsymbol{S} = \frac{\partial \psi^0\{\boldsymbol{E}^*(\boldsymbol{E}, D)\}}{\partial \boldsymbol{E}} \tag{4}$$

Eq. (3) and Eq. (4) are the same but in terms of current and undamaged elastic strain energy density functions, respectively. To simplify the analysis, it is customary to decouple damage that may occur into tension and compression modes. There is an elegant mathematical decomposition to accomplish this known as the “spectral decomposition” and can be found elsewhere in the literature. With this, the Lagrangian strain tensor can be decomposed into its positive and negative cones (Eq. (5)) as:

$$\boldsymbol{E} = \boldsymbol{E}^+ + \boldsymbol{E}^- \tag{5}$$

where, $\boldsymbol{E}^+$ and $\boldsymbol{E}^-$ are the positive and the negative cones of the strain tensor $\boldsymbol{E}$. The nature of the decomposition is such that all the eigen-values of $\boldsymbol{E}^+$ are positive and that all the eigen-values of $\boldsymbol{E}^-$ are negative. This decoupling would significantly simplify damage formulations. To progress further, it is assumed that the damage is irreversible, i.e. $\dot{D} \geq 0$. The dissipation inequality is then expressed (Eq. (6)) as:

$$\frac{\partial \psi}{\partial \boldsymbol{E}} : \dot{\boldsymbol{E}} + \frac{\partial \psi}{\partial D}\,\dot{D} \geq 0 \tag{6}$$

It is accepted in the literature and evidenced by experimental work that damage in rubber-like materials would only occur under tensile stress states and not in compression. With this and using the positive cone of the strain tensor, the general form of the damage surface $\boldsymbol{\phi}(\boldsymbol{E}^+, \boldsymbol{D})$ is obtained (Eq. (7)) by postulating a positive square function $\boldsymbol{G}^2(\boldsymbol{D})$ such that,

$$\phi(\boldsymbol{E}^+, D) = \boldsymbol{E}^+ : \boldsymbol{R} : \boldsymbol{E}^+ - G^2(D) = 0 \tag{7}$$

where G is considered as the hardening-softening function and must be determined experimentally for specific materials and where $\boldsymbol{R}$ is defined as the response tensor. The response tensor $\boldsymbol{R}$ records the directionality of damage and could lead to anisotropic or isotropic formations. If R is formulated to be an isotropic fourth-order tensor, then the process will remain isotropic. Isotropic formulations are popular in the literature because it would simplify the analysis. However, it is not realistic. Micro-tearing and cracking have a strong directionality and hence requires an anisotropic approach. In this paper a new damage function is postulated as outlined below. Let, the maximum eigen-value of $\boldsymbol{E}^+$ be given by α^* and the associated eigen-vector as $\boldsymbol{q}^*$. With this, a modified Lagrangian strain tensor, as a sub-set of total Lagrangian strain tensor, is identified (Eq. (8)) as

$$\tilde{\mathbf{E}}^+ = \alpha^* \mathbf{q}^* \otimes \mathbf{q}^* \tag{8}$$

With this, a fourth-order and new anisotropic response tensor is postulated to be:

$$\boldsymbol{R} = \frac{\widetilde{\boldsymbol{E}}^+ \otimes \widetilde{\boldsymbol{E}}^+}{\widetilde{\boldsymbol{E}}^+ : \widetilde{\boldsymbol{E}}^+} \tag{9}$$

Eq. (9) would predict that under the multi-axial stress states, the damage would occur only in the direction of maximum stress as supported by experiments. It is of interest to note that under equal triaxial extension where all three stress components in the eigen-space are equal, the formulation would assign equal damage levels to each direction. No damage would be predicted under any compressive stress state as discussed before. Using the Ogden strain energy function for the first two terms utilizing stretch ratios, $\boldsymbol{\lambda_i}$, the strain function is expressed (Eq. (10)) as:

$$\psi = \sum_{n=1}^{2} \frac{\mu_n}{\alpha_n} . \left(\lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + \lambda_3^{\alpha_n} - 3\right) \tag{10}$$

To complete the theory, a specific damage function $\boldsymbol{G(D)}$ is required. In this paper we have used the damage function as proposed by (Hokanson and Yazdani 1997) and given by Eq. (11):

$$g(D) = 1.0 - \left(\alpha . \exp\left(\frac{\beta G(D)}{\varepsilon_f}\right)\right) \tag{11}$$

where, α and β are material parameter and ϵ_f represents a strain measure below which no damage takes place.

The model with the new damage response tensor is tested for bovine artery soft tissue data as shown below in Figure 2. Values of $\boldsymbol{\alpha}$ = 0.004, $\boldsymbol{\beta = 5.5}$, and $\boldsymbol{\varepsilon_f = 0.60}$ were used. In the Ogden function, values of $\boldsymbol{\alpha_1 = -\alpha_2 = 7.6}$ and $\boldsymbol{\mu_1 = -\mu_2 = 0.65}$ were utilized. The general agreement between the experiment and theoretical results are well considering the simplicity of the new damage response tensor used.

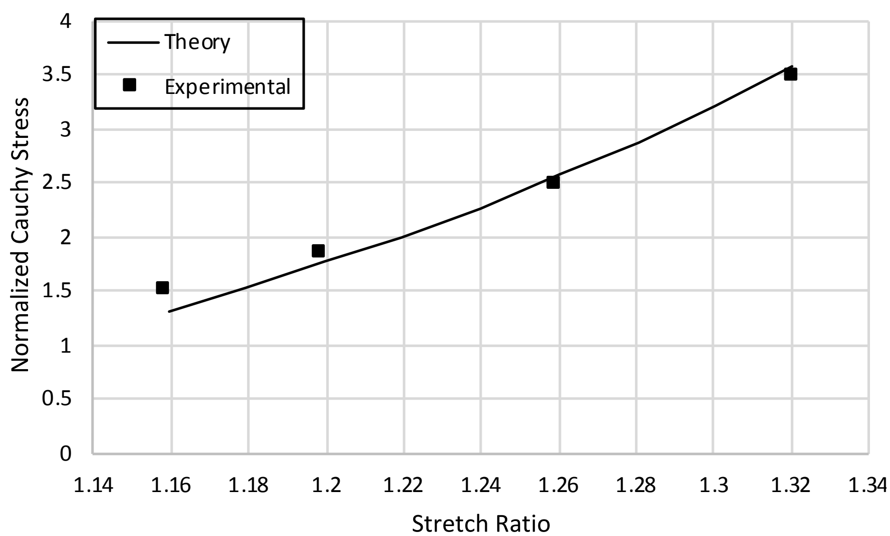

Figure 2. Experimental and theoretical results for Bovine artery soft tissue data.

3 CONCLUSION

It was shown in this paper that a unified damage and nonlinear elasticity can effectively be used for rubber-like materials with large deformations. The model represents a rigorous mathematical formulation with dissipation inequality to form a damage surface. The decomposition of the damage modes into negative and positive cones allows for the decoupling of the damage modes which simplifies the analysis. At the same time, it allows for the formulation of damage functions in both tensile and compression modes, if warranted. For polymeric materials, the compression stress state does not generally lead to any damage and hence validates the assumptions made in this paper. The model is robust and could easily be incorporated in computational codes.

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