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RELIABILITY BASED MULTI-OBJECTIVE OPTIMIZATION OF CONCRETE MIX PARAMETERS USING NSGA II

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This article presents a method to optimize concrete mix proportions with respect to different goals of economy and reliability or, equivalently, probability of failure. This method is based on a quadratic generalized ridge regression model to predict compressive strength of concrete for 28 days curing period and a linear regression model to predict cost of concrete. NSGA II is used to obtain reliable Pareto-optimal fronts with non-dominated solutions for different compressive strength requirements. Pareto-optimal fronts evolved by varying compressive strength requirements and probability of failure are analyzed. It is found that there is a nominal rise in cost as probability of failure decreases up to a certain limit for a given compressive strength requirement. However, there is a sharp rise in cost of concrete below that limit.

Keywords: Compressive strength, Regression model, Probability of failure.

1 INTRODUCTION

Reliability based design techniques are getting more acceptance for practical optimization in civil engineering, due to uncertainties involved in design processes (Dimou and Koumousis 2009, Behnam and Eamon 2013, Torii *et al*. 2012, Tao and Tam 2013, Barone and Frangpol 2014). These uncertainties owe themselves to inherent randomness, limited information, imperfect knowledge, human errors, structural idealizations in formulating the mathematical model of the structure to predict its response or behavior and the limitations of numerical techniques. In Reliability Based Design Optimization (RBDO) technique, some constraints are replaced by probabilistic constraints allowing these constraints to be violated within the given limits of failure. RBDO sacrifices true optimum in order to find a reliable optimum corresponding to given probabilities of failure. However, studying the impact of probability of failure on optimal solutions can be of great practical importance as it gives some idea regarding how the design variables change so as to make corresponding solution more and more reliable. This can be realized by making optimization problem multi-objective by considering minimization of probability of failure of one or more constraints as additional objectives and obtaining a set of non-dominated solutions, *i.e.*, a Pareto-optimal front.

In this study, focus is on optimizing concrete mix designs as concrete plays a major role in the performance of a structure. Much work has been reported in literature to find optimal mixture composition of concrete satisfying specific performance. Yeh (1999, 2003, 2007, 2009) developed computer-aided design system to find optimum concrete mix compositions based on artificial neural networks and non linear optimization techniques. Karihaloo and Kornbak (2001) employed nonlinear mathematical programming technique in design of fibre reinforced concrete mixes which have both high ductility and high tensile strength. Lim *et al*. (2004) proposed a method to design high performance concrete mix based on genetic algorithms. Lee *et al*. (2009) proposed a design methodology based on artificial neural network, convex hull and genetic algorithms for optimal mixture proportioning of concrete composition. Jayaram *et al*. (2009) developed elitist genetic algorithm models for the optimization of high volume fly ash concrete.

The aim of this study is to find reliable concrete mix compositions, for normal strength concretes, meeting a specific compressive strength requirement and to study the impact of reliability level on optimal solution in multi-objective environment. However, a multi-objective problem demands to find as many Pareto-optimal solutions as possible. Over last two decades, a number of multi-objective Evolution Approaches [EA] have been suggested which are capable of finding multiple Pareto-optimal solutions in one single simulation run (Deb 2010). Elitist Non-Dominated Sorting Algorithm (NSGA-II) proposed by Deb *et al.* (2002) is one of such approaches that has low computational requirements, elitist approach, parameter less niching approach and simple constraint handling strategy. This technique is used to obtain reliable Pareto-optimal front in this article.

2 EXPERIMENTAL DATASET

The Compressive strength data explored in this study was generated by Kumar (2002) by conducting experiments under controlled laboratory conditions. The concrete mixes were proportioned using four basic ingredients, namely, water, cement, coarse aggregate and fine aggregate. A set of 49 normal strength concrete mixes were prepared by varying water-cement ratio, cement contents and aggregates fractions. Water-cement content ratio was kept between 0.42 and 0.55. For each mix, 15 cubes of 150 mm size were cast and were tested at 28 days of curing period. Thus, a sufficiently large data bank was generated and the same has been used in the present work for analyzing compressive strength of concrete. Also, unit cost of each material is determined by taking into account the price rates in India. Based on the prices, cost of 1 m^3 of concrete is calculated for each mixture and is measured in Indian rupees.

3 MATHEMATICAL MODELS FOR CONCRETE MIX PARAMETERS

There are four design variables, namely, water content (w), fine aggregate content (fa), coarse aggregate content (ca) and cement content (c). All the design variables are measured in kg/m^3. There are two response variables, namely, cost of concrete ($cost$) and 28 days compressive strength ($st28$). Compressive strength of concrete is measured in MPa and cost of concrete is measured in Indian rupees.

In this study, no cost is associated with the water content and as such, cost of concrete is a linear function of fine aggregate content, coarse aggregate content and cement content. Linear regression model developed for cost of concrete is shown in Eq. (1).

$$cost = 236.461 + 0.629fa + 0.333ca + 4.892c \tag{1}$$

However, compressive strength of concrete may not be linearly related to the constituent materials of concrete. In the present study, compressive strength of concrete has been modeled as a quadratic function of w/c ratio, fa/c ratio, ca/c ratio and cement content c. These predictor variables have been selected on the basis of correlation analysis. Table 1 shows correlation matrix of w, fa, ca, c, w/c ratio, fa/c ratio, ca/c ratio and $st28$.

Table 1. Correlation matrix.

Parameter	w (kg/m^3)	fa (kg/m^3)	ca (kg/m^3)	c (kg/m^3)	w/c	fa/c	ca/c	$st28$ (MPa)
w (kg/m^3)	1.000	0.805	-0.305	0.541	0.171	0.261	-0.492	0.000
fa (kg/m^3)		1.000	0.102	0.026	0.617	0.751	0.046	-0.462
ca (kg/m^3)			1.000	-0.375	0.194	0.320	0.870	-0.214
c (kg/m^3)				1.000	-0.734	-0.637	-0.776	0.821
w/c					1.000	0.960	0.517	-0.968
fa/c						1.000	0.546	-0.900
ca/c							1.000	-0.581

All the four selected predictor variables have strong correlation with $st28$ in comparison to remaining variables. Further, it can also be observed from the table that data used for modeling suffers from multi-collinearity as correlation between each pair of predictor variables is numerically greater than 0.5. To tackle this problem, Generalized Ridge Regression (GRR) technique proposed by Hoerl and Kennard (1970) has been employed for model development. The quadratic GRR model developed for compressive strength of concrete is given in Eq. (2):

$$\begin{aligned} st28 = {} & -0.034569 - 0.110144\, w/c - 35.27014\, fa/c + 0.83211\, ca/c + 0.38787c - 0.11413(w/c)^2 \\ & - 0.39153(fa/c)^2 + 0.34431(ca/c)^2 - 0.00041c^2 - 0.20540(w/c * fa/c) \\ & - 35.14388(w/c * ca/c) + 11.14388(fa/c * ca/c) \end{aligned} \tag{2}$$

4 MULTI-OBJECTIVE PROBLEM FORMULATION

Multi-objective concrete mix optimization problem is formulated in Eq. (3):

$$\left.\begin{array}{r} \underset{\mu_w,\mu_{fa},\mu_{ca},\mu_c}{Minimize} \quad cost \\ \underset{\mu_w,\mu_{fa},\mu_{ca},\mu_c}{Minimize} \quad Pof\_st28 \\ where \;\; Pof\_st28 = 1 - Prob(st28 \geq f_c) \\ Subject\ to\text{:} \quad Pof\_st28 \leq P_f \\ 0.42 \leq w/c \leq 0.55 \\ w_l \leq w \leq w_u \\ fa_l \leq fa \leq fa_u \\ ca_l \leq ca \leq ca_u \\ c_l \leq c \leq c_u \end{array}\right\} \tag{3}$$

where $\mu_w, \mu_{fa}, \mu_{ca}\ and\ \mu_c$ are, respectively, the mean values of water content, fine aggregate content, coarse aggregate content and cement content; Pof_st28 denotes probability of failure of compressive strength constraint. f_c denotes the target 28 days compressive strength of concrete; P_f is the upper bound on probability of failure of compressive strength constraint. Water-cement content ratio w/c is kept between 0.42 and 0.55. w_l, fa_l, ca_l, c_l, respectively, are lower bounds for water, fine aggregate, coarse aggregate and cement content. w_u, fa_u, ca_u, c_u, respectively, are upper bounds for water, fine aggregate, coarse aggregate and cement content.

In this article, all the four design variables are considered as random design variables. There are no deterministic variables and random parameters. It is assumed that all the four design variables follow normal distribution with their respective means and standard deviations listed in Table 2. The lower and upper bounds for the design variables are also taken from Table 2. The constraint on w/c is taken as deterministic constraint.

Table 2. Descriptive statistics.

Variable	**Minimum (kg/m^3)**	**Maximum (kg/m^3)**	**Mean (kg/m^3)**	**Standard deviation (kg/m^3)**
w	180.00	230.00	202.44	12.69
fa	416.93	642.18	535.64	57.29
ca	798.48	1252.05	1064.85	133.42
c	350.00	475.00	424.49	37.32
$st28$	31.66	54.49	45.80	5.42

Since the two objectives given in (3) are of conflicting nature, therefore, the problem is to find a set of non-dominated solutions forming Pareto-optimal front depicting the tradeoff between the objectives efficiently. Here, NSGA II has been used to locate Pareto-optimal fronts.

5 RESULTS AND DISCUSSION

The optimization is run for target compressive strength of 25 MPa, 30 MPa, 35 MPa and 40 MPa, which are normal strength concretes used for general applications. The results are obtained for P_f as 0.05. Probability of failure is calculated using mean value approximation method. VisualDoc7.2 developed by Vanderplaats Research & Development, Inc. has been used to carry out reliability analyses and optimizations.

5.1 Selection of NSGA II Parameters

The maximum number of generations is varied up to 100 in steps of 25. The Pareto-optimal fronts obtained for different values of number of generations for f_c = 25 MPa, P_f = 0.05 are demonstrated in Figure 1. It is seen that difference between the graphs is not much beyond generation size of 75.

Similar observations are made in remaining cases also. Therefore, the maximum generation size is logically set as 75. Also, Population size is taken as 16, 18, 20, 22, 24 in each case under study. The Pareto-optimal fronts obtained for different population sizes for f_c = 25 MPa, P_f = 0.05 are shown in Figure 2. It can be seen from the figure that best Pareto-optimal front is obtained for population size of 20. Similar results are obtained in all the remaining cases. So, the optimal value of population size is taken as 20.

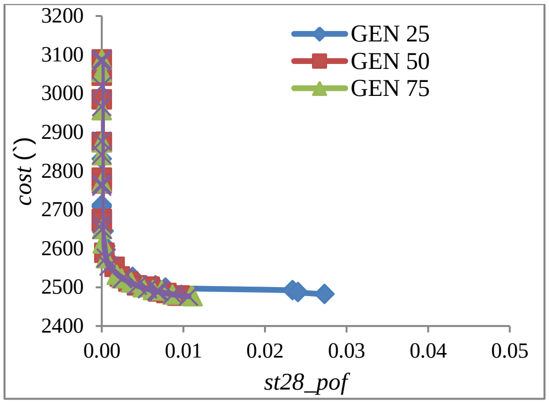

Figure 1. Pareto-optimal fronts obtained for 28 days compressive strength of 25 MPa and P_f of 0.05 for different number of generations.

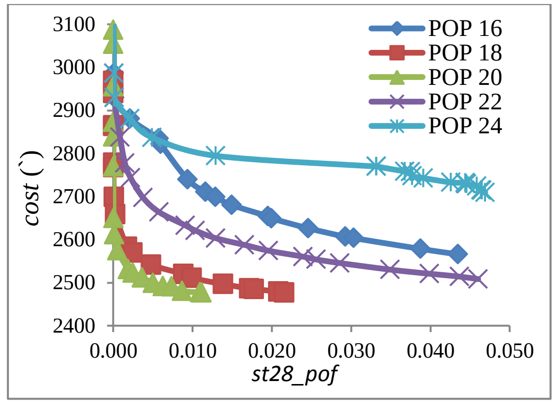

Figure 2. Pareto-optimal fronts obtained for 28 days compressive strength of 25 MPa and P_f of 0.05 for different population size.

5.2 Optimization Results

Figure 3 depicts the locations of Pareto-optimal fronts in all the four cases under study. It can be noted from Figure 3 that Pareto-optimal front for f_c of 25 MPa does not cover entire range of allowed probability of failure. This can be attributed to the fact that 25 MPa is a very small value of compressive strength and can be attained with a very high level of reliability for range of parameters under study.

Pareto-optimal fronts depict that the two objectives of cost and probability of failure are not linearly related to each other. It can be noted that Pareto-optimal fronts are divided into two parts. For very low values of P_f (10^{-5}or below for f_c = 25 MPa, 10^{-4}or below for f_c = 30 MPa, 35 MPa and 10^{-3} or below for f_c = 40 MPa), the rate of rise in optimal cost is more rapid. But, as the probability of failure increases, this rate becomes slow. This means lowering probability of failure below a certain level may not be cost effective and the designer should assess the required reliability level properly to get reliable economic design.

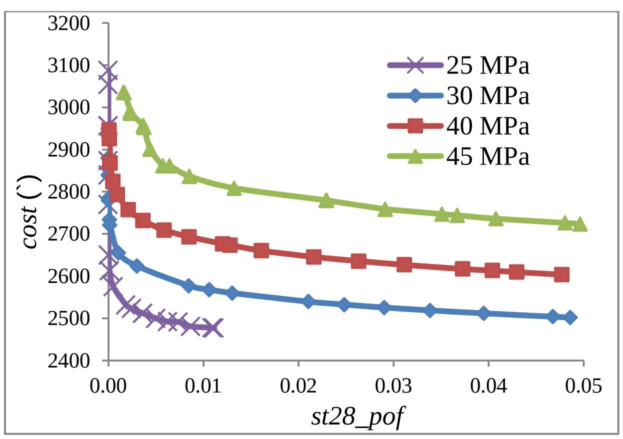

Figure 3. Pareto-optimal fronts for $st28\_pof \leq 0.05$.

6 CONCLUSIONS

A multi-objective optimization problem, where cost of concrete and probability of failure of compressive strength constraint are simultaneously minimized, is formulated. Pareto-optimal solutions are obtained using NSGA II. Pareto-optimal fronts evolved by varying compressive strength requirements are analyzed. It is found that rise in cost with decreasing pof_st28 is nominal up to a certain limit of P_f for a given compressive strength requirement. Below that limit, there is a sharp rise in cost for a small fall in P_f. It is also observed that cement content plays a major role in optimization of concrete mix parameters.

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