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ASSESSMENT OF SHEAR ANALOGY AND TIMOSHENKO METHOD FOR ANALYZING HYBRID CLT UNDER OUT-OF-PLANE LOADING

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Timber is a natural material which offers superior mechanical properties in parallel to fiber direction when compared against those in perpendicular to the fibers. Cross-laminated timber (CLT) is made up of layers of structurally graded timber, orthogonally oriented in layers whereby it can sustain loading in both directions. CLT is often used as floor panels, and hence, its performance under out-of-plane loading is of significant interest. Low rolling shear modulus resulting in higher shear flexibility of the cross-layers tend to decrease the effective bending stiffness of CLT sections. Developing hybrid CLT using timbers with higher rolling shear modulus as cross-layers in CLT is considered a viable option to improve its performance under out-of-plane loading. The present study investigates the performance of shear analogy and Timoshenko methods in predicting the deflection of hybrid CLT panels while considering different span-to-depth ratios and various combinations of rolling shear modulus. Numerical models were developed to conduct a parametric study and obtained deflection results were compared against those calculated from the shear analogy method and Timoshenko method. It was observed that for CLT with a small span-to-depth ratio and cross-layers made from material with higher rolling shear modulus, the deflection calculated from the analytical methods deviates from the values obtained from the numerical model.

Keywords: Timber engineering, Rolling shear, Deflection, Shear deformation, Shear stiffness, Engineered wood products.

1 INTRODUCTION

The inclination of the construction industry to move towards the use of environment-friendly, cost-effective, and renewable material has contributed to the rapid development of engineered wood products (EWPs), such as laminated veneer lumber (LVL), glued-laminated timber (GLT), Plywood, and cross-laminated timber (CLT) (Brandner 2013). Cross-laminated timber is a relatively new product is distinguished from the rest of the EWPs in its ability to withstand bi-axial loading due to its orthogonal make-up; layers are oriented in a perpendicular direction along the thickness direction (Harris 2015). In contrast to LVL and GLT, which are uni-directional structural elements used predominantly as beams, CLT can be used as panels subjected to out-of-plane bending, such as floors or roofs. The majority of commercially available CLTs are made of three to five layers of structurally graded timber laminas and is, at present, entirely manufactured from softwoods; according to recently reported investigations, Australian CLT panels are also produced using locally available softwood species radiata pine (Li *et al* 2020, 2019a, 2019b).

Rolling shear is a unique property of wood, defined as shear in plane perpendicular to plane of fiber direction in wood and is generally given minimal consideration as both structural timber and EWPs except for CLT, which is almost never subjected to this (Erhart and Brandner 2018). When subjected to out-of-plane bending, the cross-layers—layers with transversely oriented laminas—is subjected to rolling shear, and this has both serviceability limit state (SLS) and ultimate limit state (ULS) implications in terms of deflection due to significant shear deformation of the cross-layer and failure governed by the low rolling shear strength of the cross-layer, respectively (Brandner *et al.* 2016). Hardwoods generally tend to have higher strength and modulus for all mechanical properties compared to softwoods, and in congruence to that, research in recent years have indicated that hardwoods of lower structural grades have higher rolling shear strength and modulus compared to superior grades of softwoods (Aicher *et al.* 2016b).

The use of hardwoods in EWPs is limited due to the slower growth rate of hardwood and its higher value on its own as sawn structural timber board by virtue of its superior mechanical properties. In recent years, research towards developing hybrid CLT from the combination of softwood and hardwood has intensified to make use of hardwoods of lower structural grades. The higher rolling shear properties characteristic to hardwood incentivized the use of hardwood as cross-layers. To that endeavor, Aicher *et al.* (2016b) tested CLT made from C24 grade as longitudinal layers and Beech wood (hardwood) as the cross-layer, under the four-point bending test. The rolling shear modulus and strength for Beech wood were characterized by Aicher *et al.* (2016a) to be 370 MPa and 4.7 MPa, respectively. This is significantly higher than the rolling shear modulus of most softwoods, which ranges between 50 MPa and 100 MPa, with rolling shear strength ranging between 1 MPa and 1.5 MPa (Bendtsen 1976, Erhart *et al.* 2015). The study concluded that higher rolling shear properties result in significantly higher bending stiffness of the CLT section. The deflection under out-of-plane loading for short-span CLT panels is determined by the Shear analogy method and Timoshenko method. In this study, the applicability of these methods for hybrid CLT with higher rolling shear modulus of the cross-layer is investigated through parametric study using numerical simulations.

2 ANALYTICAL METHODS

Even though CLT is a bi-axial element, for analysis and design purposes, under out-of-plane loading conditions, they are generally considered as beam elements. This simplification is justified as the width of the CLT and is generally limited to a maximum of 2.5 m to facilitate transportation and fabrication (Christovasilis *et al.* 2016), whereas the length of CLT is generally over 5m. However, for short-span CLT panels with a length to depth ratio below 15, shear deformation becomes significant, and consequently, the shear analogy method developed by Kreuzinger (1999) and the modified Timoshenko method proposed in (Schickhofer 2009) are found to be the most suitable. Both of these methods consider the thickness and grain direction of each layers and their consequent mechanical properties in determining the bending stiffness and shear stiffness of a CLT section.

The deflection, w, of the CLT under uniform loading conditions is then calculated according to Eq. (1) (Bajzecerova 2017), where q is the uniformly distributed load in N/mm, and L is the length in mm, when the beam is simply supported.

$$w = \frac{5\, qL^4}{384\left(EI_{eff}\right)} + \frac{qL^2}{8(GA_{eff})/k} \tag{1}$$

where k is the shear correction factor, and GA_{eff} and EI_{eff} are the effective shear and bending stiffness, respectively.

The shear analogy method uses Eq. (2) to determine GA_{eff}, whereas the Timoshenko method uses Eq. (3) to calculate the same. It can be noted here that the shear analogy method uses $k = 1.2$ as the shear connection factor, while for the Timoshenko method, k can be determined using Eq. 3(b).

$$GA_{eff} = \frac{a^2}{\left[\left(\frac{h_1}{2G_1 b}\right) + \left(\sum_{i=2}^{n-1} \frac{h_i}{G_i b}\right) + \left(\frac{h_n}{2G_n b}\right)\right]} \tag{2}$$

$$GA_{eff} = \sum (G_i A_i) \tag{3a}$$

$$k = \frac{\sum G_i A_i}{(EI_{eff})^2} \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{[E(z).S(z)]^2}{G(z).b} dz \tag{3b}$$

where E_i, G_i, is the elastic modulus and shear modulus in MPa of the i^{th} layer; b, h_i, z_i, and a, are the width of the panel, height of the i^{th} layer, distance of the neutral axis of the i^{th} layer to the neutral axis of the whole CLT cross-section in mm, and the distance between the neutral axis of the outer layers in mm, respectively. Consequently, h_n and z_n represent the thickness of the n^{th} layer and distance of the neutral axis of the n^{th} layer from the neutral axis of the CLT cross-section, respectively. A_i is the cross-sectional area of the i^{th} layer.

Details on the determination of (EI_{eff}) can be found in (Gagnon and Pirvu 2011) for the shear analogy method and in (Brandner *et al.* 2018) for the Timoshenko method.

3 NUMERICAL MODELING

A three-dimensional model of CLT panel of varying length to depth ratio and rolling shear modulus of cross-layer subjected to uniformly distributed load was developed in ABAQUS 2016 as the contribution of shear deflection to the total deflection of CLT is directly related to these parameters. The deflection recorded from the model was compared against those calculated from both shear analogy method and Timoshenko method.

3.1 Model, Geometry, and Boundary Conditions

The model was developed from fully integrated solid elements (C3D20), where the timber material was modeled as transversely isotropic with perfect bonding in layer interfaces of the CLT. Based on convergence study, the cubic elements of length 8 mm was used. The width of all the models were kept constant at 200 mm. The length and thickness of the models were varied to investigate their implications to its performance under out-of-plane bending. The models were subjected to uniformly distributed load under simply supported boundary condition, as illustrated in Figure 1. It should be noted that post-failure behavior is not considered in this study as only the deflection is investigated.

Nine combinations of length-to-depth ratios (L/D) were investigated using the FE element software to investigate the scope of shear analogy method and Timoshenko method. The combination is tabulated in Table 1. The width of all the models were kept to 200 mm and a uniform load of 1.5 N/mm was used in the study.

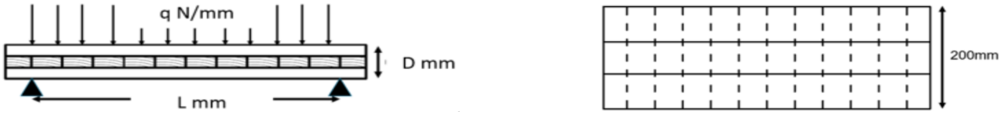


Figure 1. Geometric illustration of the model.

Table 1. Geometry and load combinations of models investigated.

L/D ratio	8.55	11.4	15	17.1	20	20.83	27.79	30	41.67
Length, L (mm)	1026	1026	1800	1026	1800	2500	2500	1800	2500
Depth, D (mm)	120	90	120	60	90	120	90	60	60

3.2 Material Properties

Timber is cylindrically anisotropic, so the mechanical properties vary along the longitudinal, radial and, tangential directions. The CLT was modeled with an assumption of timber being transversely isotropic, i.e., mechanical properties in radial, and tangential direction was assumed to be equal. Orthotropic modeling of CLT was also carried out to justify this assumption. However, rolling shear modulus (G_{RT}) was varied between 50 MPa, 100 MPa, 150 MPa, 200 MPa, and 250 MPa for each L/D ratio.

The mechanical properties used in the modeling of the CLT is tabulated in Table 2. The elastic properties are used with reference to EN 338 (2009), and Poisson's ratio is used from (Keunecke *et al.* 2008).

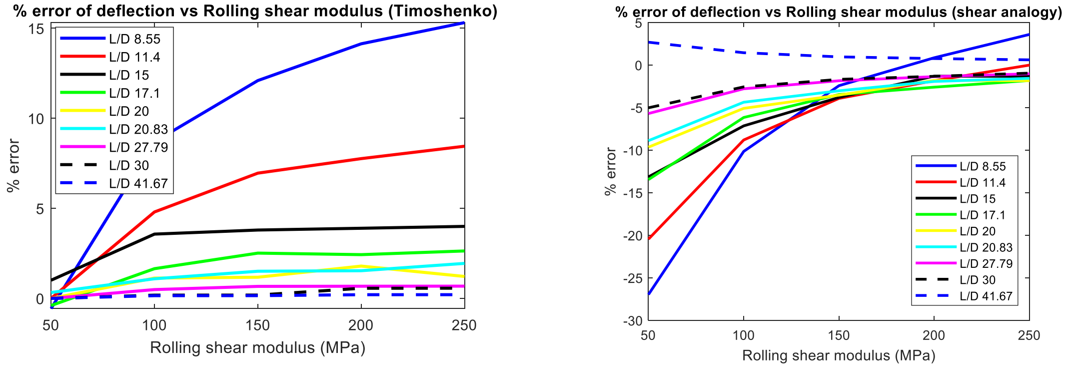
Table 2. Mechanical properties used for timber.

Mechanical Property	**Elastic modulus, E_L (MPa)**	**Elastic modulus E_R and E_T (MPa)**	**Poisson's ratio ν_{LR}**	**Poisson's ratio ν_{LT}**	**Poisson's ratio ν_{RT}**	**Shear modulus G_L and G (MPa)**		
						G_{LR}	**G_{LT}**	**G_{RT}**
Characteristic value	11600	370	0.014	0.014	0.21	690	690	50

4 RESULTS AND DISCUSSION

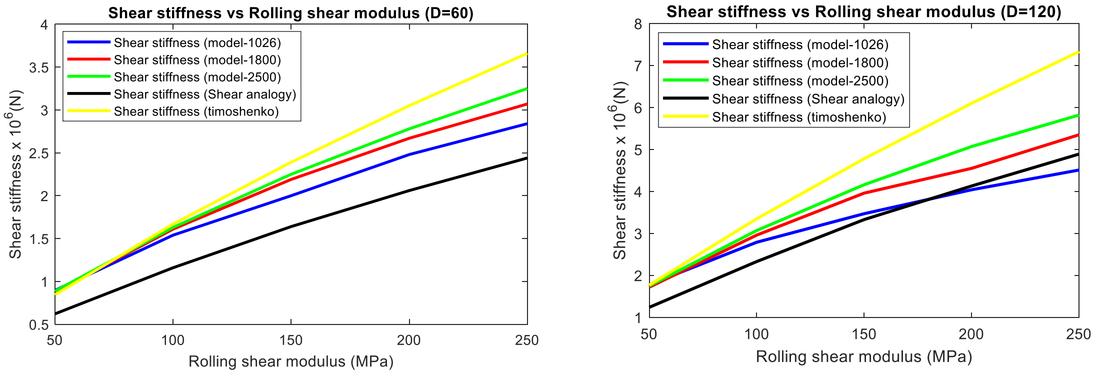
The deviation of deflection of CLT calculated using the shear analogy method and Timoshenko method against that estimated by the model for the range of rolling shear modulus is plotted in Figure 2.

It is observed from Figure 2 that as the length to depth ratio increases, the accuracy of both the Timoshenko method and shear analogy method increases. This is due to the reduction of shear deflection with an increase of rolling shear modulus of the cross-layer. The deviation of the analytical methods from the numerical model is attributed to the deflection due to the shear deformation, which is dependent on the shear stiffness of the sections. Figure 3 illustrates the variation of the shear stiffness calculated from the Timoshenko method, shear analogy method, and Numerical model for the CLT section while considering different magnitude of rolling shear modulus. Three different span lengths and two different widths (60 and 120 mm) are considered for this comparison.



(a) (b)

Figure 2. Deviation of the deflection calculated using the Timoshenko method (a) and the Shear analogy method (b) from model estimation.



(a) (b)

Figure 3. Shear stiffness for varying rolling shear modulus for section depth of (a) 60mm and (b) 120 mm.

Figure 2(a) illustrates that increasing the rolling shear modulus results in an increase in deviation between the numerical estimation and values calculated using the Timoshenko theory, whereas the shear analogy method tends to be more accurate as the rolling shear modulus of cross-layer increases as shown in Figure 2(b).

Below the L/D ratio of 20, the shear analogy method tends to be more accurate as the rolling shear modulus of cross-layer increases. In contrast, the Timoshenko methods tend to be quite accurate for the rolling modulus values ranging between 50 MPa and 100 MPa for all length to depth ratios. The shear stiffness for constant depth is independent of length, according to Eq. 2 and Eq. 3(a), but from Figure 3, it is observed that the shear stiffness increases with an increase in length of the panels even when the depth is constant. This is identified to be the source of discrepancy between the analytical and numerical solutions.

5 CONCLUSION

Increasing interest in improving the performance of CLT under out-of-plane bending loading is promoting the development of hybrid CLT using low quality hardwoods. With this trend, it is important to develop analytical methods that can precisely estimate the behavior of these hybrid CLTs. The study based on parametric study with a L/D ratio ranging between 8.55 and 41 and a rolling shear modulus ranging between 50 MPa and 250 MPa concluded that the Timoshenko method tends to be more accurate compared to that of the shear analogy method for CLT panels, especially as the length to depth ratio increases. From this study it is apparent that the effect of the L/D ratio and variation in shear modulus between layers should be incorporated in the available analytical method to predict the deflection of a hybrid CLT panel more accurately.

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