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ENGINEERING PROBABILISTIC ANALYSIS USING MODEL REDUCTION AND AUGMENTED RADIAL BASIS FUNCTIONS

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Probabilistic analysis of practical engineering problems has long been based on traditional sampling-based approaches, such as Monte Carlo Simulations (MCS) and gradient-based first-order and second-order methods. Since the finite element (FE) or other numerical methods are required to evaluate engineering system responses, such as forces or displacements, it is not efficient to directly integrate FE and sampling-based analysis approaches. Over the years, various approximate methods have been developed and applied to the reliability analysis of engineering problems. In this study, an efficient model reduction technique based on high-dimensional model reduction (HDMR) method has been developed using augmented radial basis functions (RBFs). The basic idea is to use augmented RBFs to construct HDMR component functions. The first-order HDMR model only requires sample points along each variable axis. The HDMR model, once created and used to explicitly express a performance function, is further combined with MCS to perform probabilistic calculations. As test problems, a mathematical problem and a 10-bar truss example are studied using the proposed reliability analysis approach. The proposed method works well, and accurate reliability analysis results are found with a small number of original performance function evaluations, i.e., FE simulations.

Keywords: Finite element (FE), Monte Carlo Simulations (MCS), Numerical methods, High-dimensional model reduction (HDMR) method.

1 INTRODUCTION

The reliability analysis of an engineering problem is to calculate the probability of failure, P_f, as shown in Eq. 1.

$$P_f \equiv P(g(\mathbf{x}) \leq 0) = \int_{g(\mathbf{x})\leq 0} p_X(\mathbf{x})d\mathbf{x} \tag{1}$$

where $g(\mathbf{x})$ is a performance function, and $\boldsymbol{x}$ is a vector of random variables. To calculate the integration in Eq. (1) for practical engineering problems, two types of methods can be used. There is plenty of literature in the first-order and second-order reliability methods (FORM/SORM) (Hasofer and Lind 1974, Hohenbichler *et al.* 1987, Low and Tang 2007, Li and Low 2010). Other methods are available using sampling techniques (Rubinstein 1981, Au and Wang 2014). The sampling methods, such as Monte Carlo simulations (MCS), can be applied in a very straightforward manner when combined with existing analysis packages.

For practical applications of engineering reliability analysis, FE or other simulation methods are routinely used to evaluate responses of engineering systems or components. Therefore, the analysis software of FE code shall be coupled with a reliability analysis method so that the function values can be called repeatedly by MCS or FORM/SORM. For problems requiring expensive FE analyses, this may be computationally prohibitive. Therefore, approximate models or surrogate models have been developed and used in engineering reliability analysis and other applications. Quadratic polynomial functions and other more complicated functions are routinely used to replace implicit functions in metamodels (Faravelli 1989, Wu 1995, Hassing *et al.* 2010, Bai *et al.* 2012, Zhao *et al.* 2014, Yin *et al.* 2016, Wang *et al.* 2016, Wang *et al.* 2020). Dimension reduction technique using high-dimensional model representation is also available in literature (Tunga and Demiralp 2005, Chowdhury *et al.* 2009, Chen *et al.* 2016). Accurate and efficient approximation models shall be investigated and applied to complex engineering problems.

This work investigates a reliability analysis approach integrating HDMR, augmented RBF, and MCS. The explicit RBF-HDMR models are constructed based on augmented RBF component functions and used to replace implicit functions in a reliability analysis. MCS can be applied efficiently to find the failure probability using the explicit RBF-HDMR models of the performance functions. In Section 2, the proposed reliability analysis approach is first introduced, including HDMR, augmented RBF, and MCS. Two numerical examples are introduced and solved in Section 3. Section 4 presents other applications of the proposed approach, and a summary is given in Section 5.

2 THE PROBABILISTIC ANALYSIS APPROACH

The HDMR method is to express a function using component functions, as can be seen in Eq. (2).

$$g(\mathbf{x}) = g_0 + \sum_{i=1}^{m} g_i(x_i) + \sum_{1\le i_1<i_2\le m} g_{i_1 i_2}(x_{i_1}, x_{i_2}) + \sum_{1\le i_1<i_2<i_3\le m} g_{i_1 i_2 i_3}(x_{i_1}, x_{i_2}, x_{i_3}) + \cdots + g_{12\ldots m}(x_1, x_2, \ldots, x_m) \quad (2)$$

where g_0 is a zeroth-order HDMR component function, which represents the function value at a reference point, i.e., $\boldsymbol{x} = \boldsymbol{c}$. A first-order HDMR component function, $g_i(x_i)$, is a function of only one variable, x_i, and can be either linear or nonlinear. The zeroth and first-order functions are written as in Eq. (3) and Eq. (4).

$$g_0 = g(\mathbf{c}) \quad (3)$$

$$g_i(x_i) = g(x_i, \mathbf{c}^i) - g_0 \quad (4)$$

where $(x_i, \boldsymbol{c}^i) = (c_1, \ldots, c_{i-1}, x_i, c_{i+1}, \ldots, c_m)$. To include only the zeroth and first-order HDMR component functions and neglect any higher-order component functions in Eq. (2), we have the first-order HDMR model as in Eq. (5).

$$g(\mathbf{x}) = \sum_{i=1}^{m} g(x_i, \boldsymbol{c}^i) - (m-1)g(\mathbf{c}) \quad (5)$$

Based on the augmented RBF metamodeling method, a function $g(\mathbf{x})$ can be expressed using an approximation function, as in Eq. (6) (Fang and Horstemeyer 2006).

$$\tilde{g}(\mathbf{x}) = \sum_{i=1}^{n} \lambda_i \phi(\|\mathbf{x} - \mathbf{x}_i\|) + \sum_{j=1}^{p} c_j f_j(\mathbf{x}) \quad (6)$$

where $\sum_{i=1}^{n} \lambda_i \phi(\|\mathrm{x} - \mathrm{x}_i\|)$ is a basic RBF, and $\sum_{j=1}^{p} c_j f_j(\mathrm{x})$ represents the augmented polynomial functions. Augmented RBFs in Eq. (6) are more accurate than basic RBFs for some functions, especially low order functions.

Substituting RBFs in Eq. (6) into the HDMR framework in Eq. (5), the first-order augmented RBF-HDMR metamodel can be created, as in Eq. (7).

$$\tilde{g}(\mathbf{x}) = \sum_{i=1}^{m}\left[\sum_{j=1}^{n_i}\lambda_{i_j}\phi\left(\left\|x_i - x_{i_j}\right\|\right) + \sum_{k=1}^{p_i} r_{i_k} f_k(x_i)\right] - (m-1)g(\mathbf{c}) \tag{7}$$

In Eq. (7), n_i sample points are generated to construct the first-order RBF-HDMR metamodel. MCS can be applied to the first-order RBF-HDMR metamodel, $\tilde{g}(\mathbf{x})$, to estimate the failure probability, P_f, as in Eq. (8).

$$P_f \equiv P(g(\mathbf{x}) \leq 0) = \frac{1}{N}\sum_{i=1}^{N}\Gamma[\tilde{g}(\mathbf{x}^i) \leq 0] \tag{8}$$

3 NUMERICAL EXAMPLES

In this section, one mathematical function and one truss example are presented in order to study the accuracy and numerical efficiency of the proposed reliability analysis method.

3.1 A Mathematical Function

This is a mathematical function studied in literature (Tan *et al.* 2011). The explicit performance function is written as in Eq. (9).

$$g(\mathbf{x}) = (x_1 + 2)^3 - (x_2 - 1)^2 \tag{9}$$

The two independent random variables, x_1 and x_2, follow a standard normal distribution. The direct MCS is applied to the example using 10^6 realizations based on the original explicit function in Eq. (9), and the probability of failure is P_f = 0.1980. The RBF-HDMR metamodel is created using 13 sample points, with seven sample points along each variable axis. The failure probability is estimated to be P_f = 0.1985, representing an error of 0.25%, when compared with P_f = 0.1980, i.e., the solution estimated using the direct MCS method and Eq. (9).

3.2 A Ten-Bar Truss

Figure 1 shows a well-studied two-dimensional linear-elastic truss structure (Penmetsa and Grandhi 2002, Chowdhury *et al.* 2009). The truss has ten bars, as shown in Figure 1. Two concentrated loads are simultaneously applied in the vertical direction at nodes 2 and 3 (P_1 = 10^5 lb). The material's elastic modulus is 10^7 psi, which is treated as a deterministic variable. The cross-sectional areas of all bars, $\boldsymbol{x} = [x_1 \quad \cdots \quad x_{10}]^T$, follow a normal distribution with a mean value of 2.5 in^2 and standard deviation of 0.5 in^2. The ten cross-sectional areas are the only random variables in this example, and they are assumed to be independent. Based on the applied external loads, the critical vertical displacement of truss is $u_3(\boldsymbol{x})$ at node 3. If a vertical displacement limit $u_{\max}$ = 18 in. is required for design, the implicit performance function can be written as in Eq. (10).

$$g(\mathbf{x}) = u_{\max} - u_3(\mathbf{x}) \tag{10}$$

To compute the nodal displacements and evaluate the performance function values, an existing FE software package, SAP2000 (CSI 2011), is adopted. The use of commercial software in this example is to study the practical application aspects of the RBF-HDMR method. An RBF-HDMR metamodel is created using 41 sample points, with five sample points along each variable axis. In addition, a conventional RBF approach is also applied, and 41 samples are generated using the Latin hypercube sampling approach. For both the RBF-HDMR and RBF methods, there are 41 performance function evaluations and FE analyses required according to the number of sample points. Failure probability estimations and the corresponding numbers of original performance function evaluations using direct MCS, FORM, SORM, and RBF-HDMR and RBF methods are listed in Table 1. It is found that the failure probability P_f, calculated using direct MCS, is 0.1394, which is regarded as the true value. Compared with the solutions obtained using MCS, RBF-HDMR underestimates P_f by about 3.0% (P_f = 0.1352), while the conventional RBF method overestimates the P_f by about 16.7% (P_f = 0.1627). In addition, if a correlation coefficient of 0.1 is adopted among all random variables, the failure probability is estimated to be P_f = 0.1674. If the correlation coefficient increases to 0.2, the failure probability increases to 0.1912. The RBF-HDMR method is very efficient and is shown to be more accurate in this example.

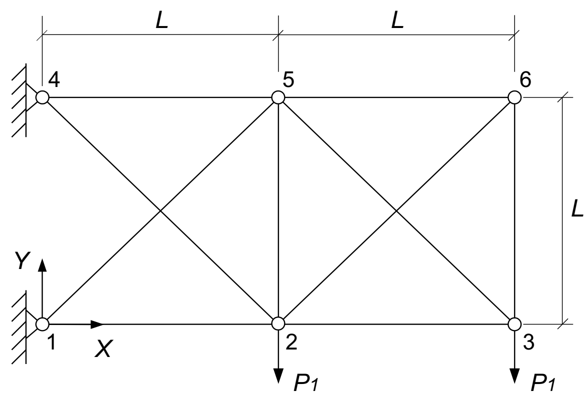

Figure 1. A ten-bar truss structure.

Table 1. Reliability analysis results.

Method	Failure probability	% error	Number of function evaluation
Direct MCS (Chowdhury et al. 2009)	0.1394	-	1,000,000
FORM (Chowdhury et al. 2009)	0.0894	35.9%	190
SORM (Chowdhury et al. 2009)	0.1571	12.7%	577
RBF	0.1627	16.7%	41
RBF-HDMR	0.1352	3.0%	41

4 OTHER APPLICATIONS

The proposed reliability analysis approach is a general method and can be applied to different types of engineering problems. In other engineering applications, including structural engineering, geotechnical engineering, and transportation infrastructure, problems have been successfully solved using the approach. It is particularly useful and efficient for problems involving implicit performance functions and expensive numerical simulations, such as FE, computational fluid dynamics, and heat transfer analyses.

5 SUMMARY

An engineering reliability analysis approach has been developed and tested using two example problems. Accurate reliability analysis results have been obtained with small sample sizes. The approach provides an efficient tool for analysis and design of complex engineering problems in which expensive response simulations are required. Future research work is needed to further develop the method and apply it to broader engineering applications.

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