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BUILDING STRUCTURE OPTIMIZATION BASED ON A HIGH-DIMENSIONAL MODEL REPRESENTATION APPROACH

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In this study, a model reduction technique based on a high-dimensional model representation (HDMR) approach was investigated and applied to design optimization of building structures. Those structures have long been designed using engineering intuition and an iterative trial-and-error method. In order to evaluate structural responses, a finite element (FE) analysis code is generally required. Gradient-based numerical algorithms and evolutionary algorithms are widely available and can be adopted to the design optimization of structures. An alternative category of optimization methods relies on approximate objective or constraint functions that can be created using various interpolation or regression techniques. In this work, the model reduction was achieved using augmented radial basis functions (RBFs) as component functions of HDMR. After sample points were generated along each variable axis, detailed FE analyses were conducted to evaluate building responses, which were used for constructing RBF-HDMR models of structural responses. The optimization was performed using a standard gradient-based numerical method. The accuracy of the RBF-HDMR could be improved if the optimal design point was added as an additional sample point. One advantage of the proposed optimization approach was that the interface programming with any existing FE code was not necessary. To illustrative the application of the method, a high-rise building was studied and optimized in order to reduce the building's global torsional responses. The proposed optimization method worked well for the example.

Keywords: Design optimization, High-rise building, Model reduction, Radial basis functions (RBFs).

1 INTRODUCTION

Structural optimization is an important and useful technique that structural engineers can use to create safe and efficient design solutions (Arora and Wang 2005, Arora 2017). To find optimal solutions, the design problem shall be formulated mathematically. It starts with an objective function $C(\mathbf{x})$ to be minimized, as can be seen in Eq. (1).

$$C(\mathbf{x}) \tag{1}$$

subject to constraint function $\boldsymbol{g}(\mathbf{x})$ given in Eq. (2), and lower and upper variable limits in Eq. (3), as

$$\boldsymbol{g}(\mathbf{x}) \leq \mathbf{0} \tag{2}$$

$$\mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U \tag{3}$$

The constraints are usually formulated according to applicable design codes or requirements. For numerical implementations, Eq. (1) and Eq. (2) shall be expressed in terms of the vector of design variables, $\boldsymbol{x}$. For practical applications of building structures, structural analyses using FE simulations and iterative procedures are usually required (Chan and Wang 2005, Zou *et al.* 2018). Therefore, the FE code has to be integrated with a numerical optimization algorithm, and this is usually an expensive process due to the extensive coding and program interface (Arora and Wang 2005). An alternative method is to explore an approximate model, which refers to as surrogate or metamodels. The basic concept of metamodels is to create an approximate function representing the actual response function of a structure and use it in numerical optimization or other analyses (Jin *et al.* 2001, Bi *et al.* 2010, Yin *et al.* 2016, Wang and Fang 2018). HDMR is a framework to express any function using a combination of component functions, which has been shown to be very efficient for many problems (Li *et al.* 2001, Chowdhury *et al.* 2009, Chen *et al.* 2016).

Although metamodeling methods have been developed and applied to various engineering optimization problems, their applications to structural optimization of practical large-scale civil engineering projects are still very limited. In addition, research is needed to study more efficient and accurate metamodels, especially those based on the HDMR framework, and application aspects of these metamodeling approaches. In this work, an alternative metamodeling approach based on the HDMR framework was studied. In this approach, the HDMR component functions were expressed using augmented RBFs. After some sample points were selected, the FE models of these points were constructed, and their responses were analyzed and obtained. Once the RBF-HDMR metamodel was created, the FE software was no longer needed. A traditional gradient-based nonlinear programing method was applied to find the optimal design. A three-dimensional high-rise building example was studied and the thicknesses of shear walls in the structure were optimized for torsional-resistant designs of the structure. After the optimal designs were found, the accuracy of the HDMR models was checked, and improved RBF-HDMR models were further created to improve the accuracy of the metamodel. The numerical results showed that fair accurate approximations of real response functions were achieved.

2 OPTIMIZATION SOLUTION TECHNIQUE

This section presents a brief introduction of the proposed optimization technique using HDMR framework and RBFs.

2.1 The HDMR Framework

The HDMR method is to express a function using component functions, as seen in Eq. (4) (Li *et al.* 2001, Chowdhury *et al.* 2009),

$$g(\mathbf{x}) = g_0 + \sum_{i=1}^{m} g_i(x_i) + \sum_{1\le i_1<i_2\le m} g_{i_1 i_2}(x_{i_1}, x_{i_2}) + \sum_{1\le i_1<i_2<i_3\le m} g_{i_1 i_2 i_3}(x_{i_1}, x_{i_2}, x_{i_3}) + \cdots \quad (4)$$

where g_0 is a zeroth order component function, which represents the function value at a reference point, i.e., $\boldsymbol{x} = \boldsymbol{c}$; $g_i(x_i)$ is a first-order component function in terms of a single variable, x_i. The zeroth and first-order component functions are seen in Eq. (5) and Eq. (6).

$$g_0 = g(\mathbf{c}) \quad (5)$$

$$g_i(x_i) = g(x_i, \mathbf{c}^i) - g_0 \quad (6)$$

where

$$g(x_i, \boldsymbol{c}^i) = g(c_1, \ldots, c_{i-1}, x_i, c_{i+1}, \ldots, c_m) \quad (7)$$

The HDMR model in Eq. (4) can be expressed as seen in Eq. (8).

$$g(\mathbf{x}) = g_0 + \sum_{i=1}^{m} g_i(x_i) + \mathbb{R}_2 \tag{8}$$

where $\mathbb{R}_2$ is a residual function representing the contributions from all higher-order HDMR component functions. A 1st-order HDMR model can be obtained by keeping only the zeroth and first-order HDMR component functions while neglecting the residual function $\mathbb{R}_2$ in Eq. (8).

2.2 Radial Basis Functions (RBFs)

RBFs have been used for complex optimization problems. They seek to define a function with the creation of a new function or metamodel, $\tilde{g}(\mathbf{x})$, as seen in Eq. (9).

$$\tilde{g}(\mathbf{x}) = \sum_{i=1}^{n} \lambda_i \phi(\|\mathbf{x} - \mathbf{x}_i\|) \tag{9}$$

The coefficients of the new model are dependent on the value of the unknown function at some sample points. The RBFs can be highly accurate with the addition of linear or quadratic functions. If an RBF implements with linear or quadratic functions to approximate the true function, it is considered augmented (Fang *et al.* 2005, Fang and Wang 2008).

2.3 HDMR Based on RBFs

Substitute RBFs in Eq. (9) into the HDMR model in Eq. (8), a 1st-order RBF-HDMR metamodel can be created, as seen in Eq. (10).

$$\tilde{g}(\mathbf{x}) = g_0 + \sum_{i=1}^{m} \left[\sum_{j=1}^{n_i} \lambda_{i_j} \phi\left(\left\| x_i - x_{i_j} \right\| \right) \right] \tag{10}$$

In Eq. (10), n_i sample points are generated to construct the 1st-order RBF-HDMR metamodel. The residual function $\mathbb{R}_2$ in Eq. (8) can be expressed using RBFs, as seen in Eq. (11).

$$\mathbb{R}_2 = \sum_{j=1}^{n_{\mathbb{R}}} \lambda_{\mathbb{R}_j} \phi(\|\mathbf{x} - \mathbf{x}_j\|) \tag{11}$$

where $n_{\mathbb{R}}$ is the sample size for the residual function, $\mathbb{R}_2$. Using the optimal design point based on the 1st-order RBF-HDMR model as an additional sample point, $\mathbb{R}_2$ in Eq. (11) can be generated. An improved 1st-order RBF-HDMR model is achieved by adding $\mathbb{R}_2$ to the 1st-order RBF-HDMR model.

2.4 Optimization Algorithm

Once an explicit RBF-HDMR function becomes available, any numerical optimization algorithm can be used, such as gradient-based methods (Arora 2017) or gradient-free methods (Goldberg 1989). In this work, the generalized reduced gradient algorithm in the Excel Solver (Microsoft 2013) was adopted.

3 A HIGH-RISE BUILDING EXAMPLE

In this section, a high-rise concrete building structure was optimized using the proposed method. Figure 1 shows a typical floor plan and three-dimensional model of the seventeen-story structure and the layout of shear walls in the structure. The structure consists of two separate buildings, which are connected on top of the structure. A large amount of torsion was observed when the structure was analyzed. For the structural optimization formulations, the thicknesses of all shear walls were treated as design variables, i.e., w_1, w_2, w_3, and w_4, as illustrated in Figure 1. Two separate formulations were used for numerical optimization. The first formation (Formulation 1) was to find the four shear wall thicknesses to minimize the ratio of the first torsional period (T_t) and first translational period (T_l), as seen in Eq. (12).

$$C(w_1, w_2, w_3, w_4) = \frac{T_t}{T_1} \tag{12}$$

subject to the constraints on the variable bounds, as seen in Eq. (13).

$$w^L \leq w_1, w_2, w_3, w_4 \leq w^U \tag{13}$$

The lower and upper bounds of the shear wall thicknesses were taken as w^L =0.4 and w^U =0.8 meters, respectively. A second optimization formulation (Formulation 2) was also adopted to minimize the total cross-sectional area of all shear walls, which was proportional to the concrete quantity in the building, as seen in Eq. (14).

$$C(w_1, w_2, w_3, w_4) = \sum_{i=1}^{4} L_i w_i \tag{14}$$

subject to a constraint on the $\frac{T_t}{T_1}$ ratio, as seen in Eq. (15).

$$g(w_1, w_2, w_3, w_4) = \frac{T_t}{T_1} - 0.85 \leq 0 \tag{15}$$

In addition, the shear wall thickness bounds defined in Eq. (13) were included in Formulation 2 as well. In order to obtain the structural responses, an existing FE analysis software, ETABS (CSI 2019), was used to analyze the building structure. To construct the 1st-order HDMR model, 17 sample points were generated and 17 FE analyses were conducted using different thicknesses of shear walls.

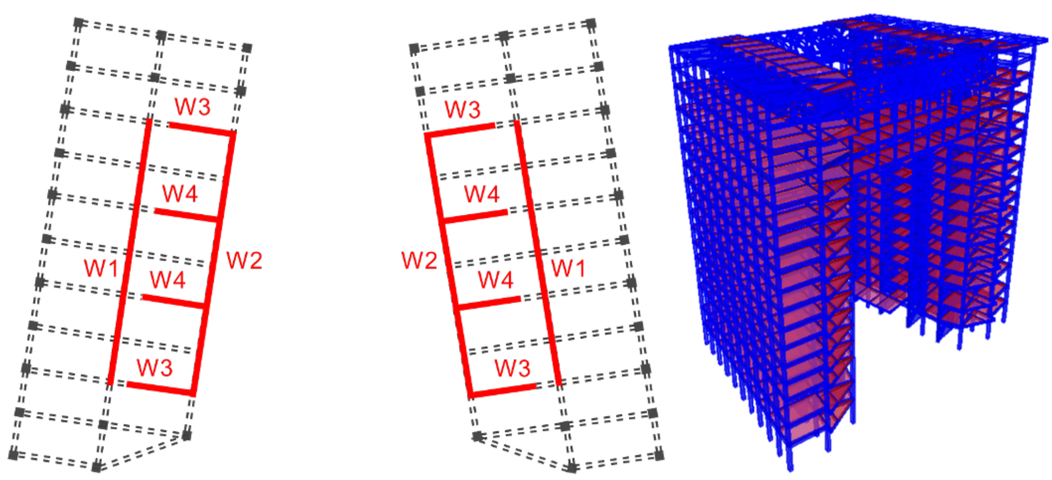

Figure 1. A typical floor plan and three-dimensional model.

Table 1 shows the optimal design results for the two optimization formulations. The optimal designs were obtained, which were further verified based on the FE analysis results at the optimal design points using the final shear wall thicknesses. The predicted errors of $\frac{T_t}{T_1}$ using 1st-order RBF-HDMR were 0.29% and 0.07% for Formulations 1 and 2, respectively. The 1st-order RBF-

HDMR models were shown to be very accurate. The optimal point was added as one additional sample, in order to generate the residual function, $\mathbb{R}_2$. The improved 1st-order RBF-HDMR models based on a total of 17+1=18 sample points were created and used again for numerical optimization. The optimal designs were obtained and listed in Table 1. Even smaller errors were observed for both optimization formulations. In this example, both the 1st-order and improved 1st-order RBF-HDMR models worked very well and proved to be both efficient and effective. Note that although it is possible to optimize the building by integrating a traditional optimization algorithm with the FE code, this was not performed in the study.

Table 1. Optimization results of two formulations.

	Formulation 1		Formulation 2	
	1st-order RBF-HDMR	Improved 1st-order RBF-HDMR	1st-order RBF-HDMR	Improved 1st-order RBF-HDMR
w_1 (m)	0.800	0.800	0.527	0.530
w_2 (m)	0.400	0.400	0.400	0.400
w_3 (m)	0.400	0.400	0.400	0.400
w_4 (m)	0.400	0.400	0.400	0.400
C(**w**) (m^2)	-	-	40.1	40.2
T_t/T_1 (RBF-HDMR)	0.565	0.564	0.600	0.600
T_t/T_1 (FE)	0.564	0.564	0.600	0.600
T_t/T_1 (% error)	0.29%	0.00%	0.07%	0.05%
No of samples	17	17+1=18	17	17+1=18

4 CONCLUDING REMARKS

An optimization technique combining the HDMR framework, RBF metamodels, and an existing nonlinear program solver was proposed and applied to building structure optimization**.** One major advantage of the approach was to avoid the direct coupling of complex FE models in optimization loops. The proposed approach was successfully applied to a high-rise concrete building structure. The RBF-HDMR models were shown to be highly efficient and accurate, and a small number of FE analyses of the building were needed. The new approach is especially attractive for large and complex structures requiring expensive FE analyses. Further research shall be conducted to study iterative or adaptive HDMR procedures to improve the accuracy and efficiency of the method in a dynamic manner.

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