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NONPARAMETRIC NONLINEARITY IDENTIFICATION WITH AN UPDATED EKF APPROACH USING DYNAMIC RESPONSE FUSION

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Most parametric nonlinear behavior identification methods require an assumed mathematical model to describe the hysteretic behavior of structural members or substructures. Due to the individuality of various construction materials and structural systems, it is challenging to forecast the real nonlinear performance of a structure member or a substructure under dynamic loadings with a general parametric model in prior. In this paper, a nonparametric nonlinear restoring force (NRF) identification approach with limited output and unknown input is proposed by employing a double Chebyshev polynomial combined with an updated Extended Kalman filter (U-EKF) approach, where the observation equation is updated without using external excitation information. Moreover, data fusion is used to deal with the drift problem in dynamic response forecasting. The proposed approach is validated numerically with multi-degree-of-freedom (MDOF) structures equipped with various nonlinear members, including MR damper (damping-dominant) and SMA damper (stiffness-dominant) employed to mimic different structural nonlinear behavior. Moreover, a four-story shear frame model, including MR damper on the fourth floor, is employed to experimentally validate the approach. Identified results show that the proposed algorithm can identify structural nonlinear behavior in a nonparametric way without using excitation as an input, which is helpful for structural damage diagnosis where nonlinearity and loading profile should be considered.

Keywords: Nonlinear restoring force, Double Chebyshev polynomial model, SMA dampers, MR dampers, Model-free, Data fusion.

1 INTRODUCTION

Structural damage identification for structural health monitoring (SHM) based on vibration measurements has been an active research field in the past decades due to the common concern on the structure serviceability and safety affected by unavoidable materials deterioration, harsh environment, and various man-made or natural hazards. The main purpose of SHM is to identify the occurrence of damage, locate the damage, determine the degree of damage, and estimate the remaining service life and remaining load-carrying capacity of structures, which are the major content of damage prognosis (DP) for engineering structures (Doebling *et al*. 1998, Olivier and

Smyth 2017). Strictly speaking, most of the vibration-based damage detection and identification methods using eigenvectors and/or eigenvalues and/or their derivatives extracted from dynamic response measurements are only suitable for linear systems, where the structural stiffness employed to mimic damage is a constant during the whole course of the application of dynamic loadings, only. However, nonlinearity exists widely in nature, especially when infrastructures suffer from strong dynamic loadings. It is of great scientific significance to develop efficient identification approaches for nonlinear systems where structure stiffness is a variable.

Even studies on the identification of nonlinear structures have been implemented for a long time, the identification of different types of civil engineering structures composed of different materials is still a challenging task. Moreover, it is usually impossible to get the displacement, velocity, acceleration response of structures at all DOFs and external excitation for identification. Therefore, it is essential to develop nonlinear structure identification approaches using partially available dynamic response measurements, and unknown external excitation. Yang *et al*. (2007) presented an adaptive extended Kalman filter (EKF) approach with unknown input to identify time-varying parameters and unknown external excitations for structures. A general EKF with unknown inputs (EKF-UI) approach was proposed by Pan *et al*. (2016) to meet the challenges coming with the availability of dynamic response and excitation information. Liu *et al*. (2016) proposed an EKF-UI approach with a data fusion method for simultaneous real-time identification of structural systems and unknown external. By using fused partially acceleration and strain responses, Huang *et al*. (2019) effectively identified structural parameters and unknown inputs in real-time with an EKF-UI approach and experimentally validated it with a five-story shear model. Li *et al*. (2020) proposed an earthquake ground motion identification method using incomplete modal information and limited measurements through the standard Kalman filter and demonstrated the effectiveness and accuracy of the proposed method results numerically with a two-dimensional numerical frame model and experimentally with a five-floor structure on shaking table. Zhang and He (2019) proposed structural parameters and unknown excitations identification algorithm using EKF and verify the proposed approach through three numerical examples. To address the problem associated with the difficulty of assuming a general parametric model describing various nonlinear behavior of civil engineering structures, He *et al*. (2019) presented a model-free nonlinear restoring force (NRF) identification method with a modified observation equation based on EKF and the NRF was identified by means of least squares estimation (LSE) at each time step.

In this paper, a nonparametric NRF and unknown excitation identification method based on a double Chebyshev polynomial model and an updated EKF is proposed using partially available acceleration responses. The data fusion technique is used to avoid the drift problem in structural response forecasting. The proposed algorithm was numerically verified with MDOF structures involving different dampers mimicking different nonlinear behavior and experimentally with a four-story shear frame model, including MR damper on the fourth story.

2 NONPARAMETRIC IDENTIFICATION WITH AN UPDATED EKF AND CHEBYSHEV POLYNOMIAL

In physical science and mathematics, the Chebyshev polynomial is composed of a system of complete and orthogonal polynomials, with obvious advantages and applications. Given any piecewise continuous function $z(x,y)$ with finitely many discontinuities in the interval [-1,1], the sequence of sums can be shown as in Eq. (1),

$$z(x,y) \approx \sum_{n=0}^{N}\sum_{m=0}^{M} c_{nm} T_n(x) T_m(y) \tag{1}$$

where c_{nm} is the coefficient of the Chebyshev polynomial $T_n(x)$ and $T_m(y)$ (Xu *et al*., 2019).

In this paper, the parametric model does not need to be known in advance for structural nonlinear behavior identification. The NRF could be expressed using the double Chebyshev polynomial, which contains structural relative velocity and relative displacement as shown in the Eq. (2),

$$R_{i,i-1}[\dot{x}(t), x(t), \theta] \approx \sum_{h=0}^{k} \sum_{j=0}^{q} c_{i,i-h,h,j}^{non} T_h(v'_{i,i-1}) T_j(s'_{i,i-1}) \tag{2}$$

in which $R_{i,i-1}[\dot{x}(t), x(t), \theta]$ is the nonlinear restoring force between the i^{th} and i-1^{th} DOF, $v'_{i,i-1}$ and $s'_{i,i-1}$ are relative velocity and relative displacement vectors respectively, $c_{i,i-1,h,j}^{non}$ represents the polynomial coefficient, k and q are integers which depend on the nature and extent of the nonlinearity, and $T_{\mathrm{h}}(\mathrm{v}'_{i,i-1})$ as well as $T_j(\mathrm{s}'_{i,i-1})$ are Chebyshev polynomial.

In this study, an updated EKF method with data fusion is adopted to identify the NRF using Chebyshev polynomial, unknown response, and unknown excitation. The observation equation in the proposed method is changed into the following form (Eq. (3)) compared with the conventional EKF method.

$$\begin{bmatrix} \boldsymbol{\Phi} & 0 \\ 0 & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{y}_a \\ \boldsymbol{y}_s \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi} & 0 \\ 0 & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{D}_1 \boldsymbol{q}(\boldsymbol{Z}_k) \\ \boldsymbol{D}_2 \boldsymbol{x}_k \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Phi} & 0 \\ 0 & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_k^1 \\ \boldsymbol{v}_k^2 \end{bmatrix} \tag{3}$$

in which $\boldsymbol{y}_a$ represents the acceleration observation and $\boldsymbol{y}_s$ represents the displacement observation. $\boldsymbol{\Phi} = \boldsymbol{I} - \boldsymbol{D}_1 \left(\boldsymbol{D}_1^{\mathrm{T}} \boldsymbol{D}_1 \right)^{-1} \boldsymbol{D}_1^{\mathrm{T}}$ $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ are the matrix associated with the locations of observed accelerations and displacements, respectively; $\boldsymbol{v}_k^1$ and $\boldsymbol{v}_k^2$ represent measurement noise vectors assumed to be Gaussian white noise vectors with zero mean and covariance matrices $E\left(\boldsymbol{v}_k^1 \boldsymbol{v}_k^{1\mathrm{T}}\right) = \boldsymbol{R}_k^1$, $E\left(\boldsymbol{v}_k^2 \boldsymbol{v}_k^{2\mathrm{T}}\right) = \boldsymbol{R}_k^2$. Associated with the updating of the observation equation, the identification approach is updated accordingly and shown in detail in Figure 1.

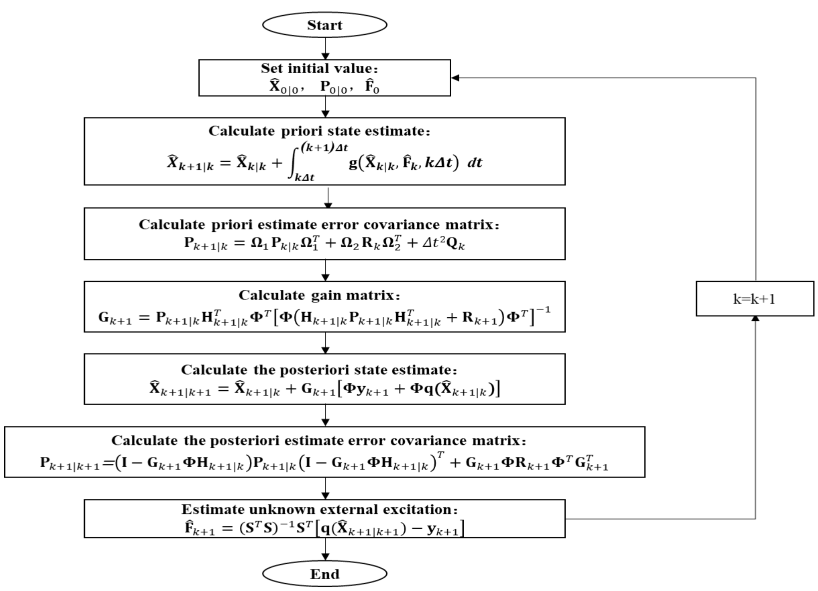

Figure 1. Identification flowchart of structural NRF and unknown excitations.

3 NUMERICAL VALIDATION FOR NONLINEAR STRUCTURES WITH THE PROPOSED APPROACH

To validate the applicability of the proposed method, a six-story shear frame model shown in Figure 2 is investigated numerically. The mass, stiffness, and damping of the corresponding linear structure are m_i=400kg, c_i=240$N\cdot s/m$, and k_i=320kN/m (i=1, …, 6), respectively. To mimic various nonlinear behavior, an SMA damper and an MR damper are used, which were installed on the 2nd floor and the 6th floor of the structure, respectively. Acceleration signals of the first to the fifth floors and the displacement signals for the second and fifth floors are simultaneously used for data fusion. The acceleration response measurements are all mixed with 5% Gaussian white noise. In the iterative identification process, the initial value of stiffness and damping coefficients are assumed to be 80% of the real value. The shear frame model is exerted on the third floor using a wide-band random excitation with frequencies ranging from 0.1Hz to 30Hz.

The NRF of the used MR damper for the Dahl model can be expressed in the Eqs. (4) and (5) (Zhou and Qu 2002).

$$f_{non}^{MR} = K_0 s_{i,i-1} + C_0 v_{i,i-1} + F_d Z - f_0 \quad (4)$$

$$\dot{Z} = \sigma v_{i,i-1}(1 - Z\, sgn(v_{i,i-1})) \quad (5)$$

in which f_{non}^{MR} is the NRF of MR damper of Dahl model, K_0, C_0, F_d, f_0 and σ are parameters of MR damper for Dahl model and Z is a constant which is coulomb dry friction. In this example, the parameters take the following values: $\sigma = 470s/m$, $K_0 = 20N/\mathrm{m}$, $C_0 = 400N\cdot\mathrm{s/m}$ and $f_0 = 0$.

The hysteretic force model shown in Figure 3 is employed to model the SMA damper and the corresponding mathematical expression is shown in Eq. (6). In this study, the following values are taken for the parameters, $S_a = 0.005m$, $S_b = 0.012m$, $k_1^{SMA} = 100kN/m$, $k_2^{SMA} = 50kN/m$.

$$F_{non}^{SMA} = \begin{cases} k_1^{SMA} \times S & (oab, ao, o'a'b', a'o) \\ (k_1^{SMA} - k_2^{SMA}) \times S_b \times \mathrm{sgn}(S) + k_2^{SMA} \times S & (bc, b'c') \\ (k_1^{SMA} - k_2^{SMA}) \times (S_b - S_c) \times \mathrm{sgn}(S) + k_1^{SMA} \times S & (cd, c'd') \\ (k_1^{SMA} - k_2^{SMA}) \times S_a \times \mathrm{sgn}(S) + k_2^{SMA} \times S & (da, d'a') \end{cases} \quad (6)$$

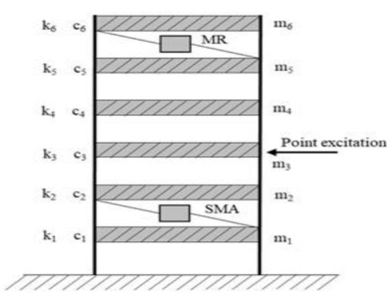

Figure 2. Numerical model.

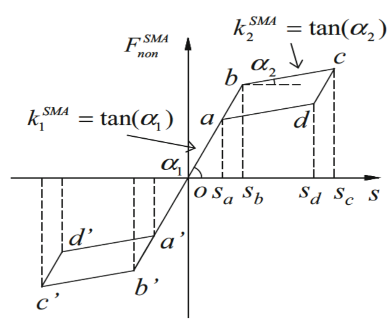

Figure 3. Hysteretic model of SMA damper.

Figure 4 shows the comparison of the identified unknown external excitation, the hysteretic force of SMA damper on the second floor, and that of MR damper on the sixth floor with their simulated results. Figure 4 shows that identified results are close to their true values with an acceptable range. The normal root-mean-square errors (NRMSE) for the excitation, SMA damper, and MR damper are 0.0036, 0.0135, 0.0034, respectively.

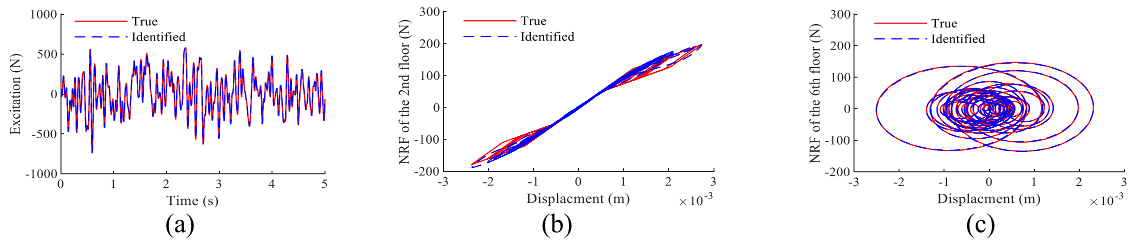

Figure 4. The comparison of identified results: (a) unknown external excitation, (b) the hysteretic of SMA damper on the second floor, (c) the hysteretic of MR damper on the sixth floor.

4 EXPERIMENTAL VALIDATION OF AN MDOF FRAME MODEL

To further validate the effectiveness of the proposed algorithm, a four-story shear frame model is established, which is shown in Figure 5. To simulate the nonlinear behavior of the structure, the shear frame model is equipped with an MR damper installed on the 4^{th} floor during the vibration test. A random excitation is exerted on the second story of the structure.

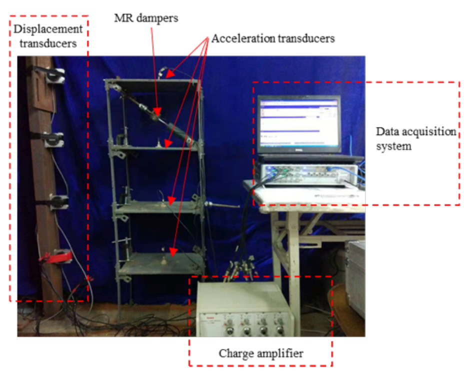

Figure 5. The experimental steel frame structure model.

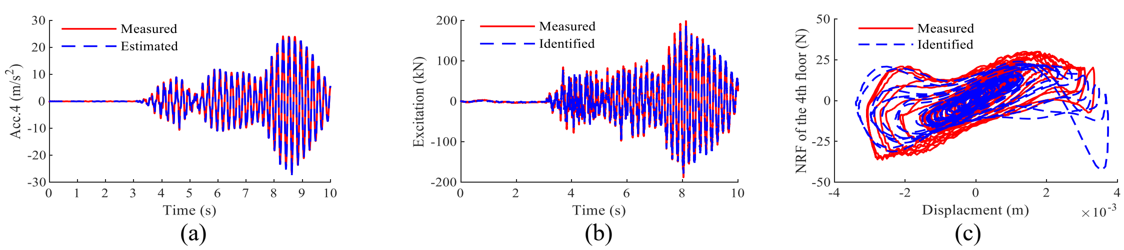

Figure 6. The comparison of identified results: (a) acceleration on the fourth floor, (b) excitation on the second floor, (c) the hysteretic of NRF on the fourth floor.

In the experimental validation of the presented method, acceleration signals at the 1^{st}, 2^{nd}, and 3^{rd} floors of the model are measured, and at the same time, displacement measurements at the 1^{st} and 3^{rd} floors are adopted for data fusion. The initial structural parameters, including stiffness and damping coefficients, are assumed as 80% of the reference values. Figure 6 shows the comparison between the acceleration on the 4^{th} floor, excitation on the second floor, and the NRF on the fourth floor and the test measurements. A good agreement can be observed and the effectiveness of the proposed method for unknown dynamic response, excitation, and NRF identification is experimentally validated.

5 CONCLUDING REMARKS

In this paper, an NRF and excitation identification approach based on a double Chebyshev polynomial model and an updated EKF with unknown input is proposed. The proposed method can identify structural responses, NRF and unknown excitation and the effectiveness of the proposed approach is verified via a numerical model with different dampers and an experimental model with MR damper. The data fusion technique shows a good ability to restrain the drift problem and noise influence in the U-EKF method. The proposed method is important for structural NRF identification with unknown inputs, which plays a vital role in post-event damage prognosis and remaining load-carrying capacity and remaining service life forecasting for engineering structures where structural nonlinear behavior and loading profiles should be taken into account.

References

Doebling, S. W., Farrar, C. R., Prime, M. B., *A Summary Review of Vibration-Based Damage Identification Methods,* Shock and Vibration Digest, EBSCO, 30(2), 91-105, 1998.

He, J., Zhang, X. X., Qi, M. C., and Xu, B., *Model-Free Identification of Nonlinear Restoring Force with Modified Observation Equation*, Applied Science, MDPI, 9(2), 306, 2019.

Huang, J. S., Li, X. Z., Yang, X. J., Zheng, Z. P., and Lei, Y., *Experimental Validation of the Proposed Extended Kalman Filter with Unknown Inputs Algorithm Based on Data Fusion*, Journal of Low Frequency Noise, Vibration and Active Control, SAGE, https://doi.org/10.1177/1461348419868860, 2019.

Li, Y., Luo, Y. Z., Wan, H. P., Yun, C. B., and Shen, Y. B., *Identification of Earthquake Ground Motion Based On Limited Acceleration Measurements of Structure Using Kalman Filtering Technique*, Structure Control and Health Monitoring, John Wiley & Sons, Ltd, 27(1), e2464, 2020.

Liu, L. J., Su, Y., Zhu, J. J., and Lei, Y., *Data Fusion Based EKF-UI for Real-Time Simultaneous Identification of Structural Systems and Unknown External Inputs*, Measurement, Elsevier, 88, 456-467, 2016.

Olivier, A., and Smyth, A. W., *Review of Nonlinear Filtering for SHM with an Exploration of Novel Higher-Order Kalman Filtering Algorithms for Uncertainty Quantification*, Journal of Engineering Mechanics, ASCE,143(11), 04017128, 2017.

Pan, S. W., Xiao, D., Xing, S. T., Law, S. S., Du, P. Y., and Li, Y. J., *A General Extended Kalman Filter For Simultaneous Estimation of System and Unknown Inputs*, Engineering Structures, Elsevier, 109, 85–98, 2016.

Xu, B., Deng, B., Li, J., and He, J., *Structural Nonlinearity and Mass Identification with A Nonparametric Model Using Limited Acceleration Measurements*, Advances in Structural Engineering, SAGE, 24(4), 1018-1031, 2019.

Yang, J. N., Pan, S., and Huang, H., *An Adaptive Extended Kalman Filter For Structural Damage Identifications II: Unknown Inputs*, Structural Control and Health Monitoring, John Wiley & Sons, Ltd, 14(3), 497-521, 2007.

Zhang, X. X., and He, J., *Identification of Structural Parameters and Unknown Excitations Based On the Extend Kalman Filter*, Engineering Mechanics, Chinese Academic Journal Electronic Published House, 36(4), 221-230, 2019.

Zhou, Q., and Qu W. L., *Two Mechanic Models for Magneto-Rheological Damper and Corresponding Test Verification*, Earthquake Engineering and Engineering Vibration, Chinese Academic Journal Electronic Published House, 22(4), 144-150, 2002.