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YIELD SURFACE SHAPE ESTIMATED BY USING NATURAL STRAIN UNDER PRE-DEFORMATION OF TENSION AFTER SIMPLE SHEAR

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Since the rigid body rotation can be clearly removed from the shearing strain component, and the additive law of strain can be satisfied, the Natural Strain is effective strain representation for expressing the stress-strain behavior under large elasto-plastic deformation. Based on the Natural Strain theory, the purpose of this study is to clarify the mechanism of development of anisotropy in the yield surface that occurs with the progression of deformation by examining the shape of yield surface obtained under large deformation. In our previous studies, using test pieces that the large pre-deformation of uniaxial tension, simple shear and the proportional loading of them are applied, proportional loading tests are carried out again by changing the ratio of tension and shear. In the present paper, another type of the pre-deformation that the deformation path is different but the final deformation state is the same as the pre-deformation of proportional loading of tension and shear, is chosen as the subject of this study. In particular, the deformation history of tension after simple shear is examined in this study. Then, the relationship between the development of anisotropy in yield surface and the deformation history is revealed in this paper.

Keywords: Finite strain, Anisotropy, Deformation history, Elasto-plastic analysis, Ductile materials, Yield criterion, Proof stress.

1 INTRODUCTION

The Natural Strain theory proposed in the present paper is the strain representation, which is obtained by integrating an infinitesimal strain rate on an identical line element along the whole process of deformation path. Since the associative law of strain on an identical line element can be satisfied, this strain can be represented from anywhere with reference to the intermediate deformed configuration. Moreover, since the shearing strain component of this strain is defined as a pure angular strain, the rigid body rotation can be clearly removed from the shearing strain component. Therefore, the Natural Strain theory proposed in this paper is effective strain representation.

On the basis of the Natural Strain theory, the purpose of this study is to reveal the mechanism of the development of anisotropy in yield surface by examining the shape of yield surface obtained after applying a large deformation. In a series of our previous study (Kato and Kazama 2013), using test pieces that the large pre-deformation of uniaxial tension, simple shear are applied, proportional loading tests have been carried out again by changing the ratio of tension and shear. Then, the shape of yield surface has been estimated by investigating the slope of the tangent in those deviatoric stress and deviatoric strain curves. Consequently, it was revealed that the shape of yield surface in a stress space becomes convex at the side of pre-deformation and becomes flat at the opposite side. Moreover, in the case of simple shear, it became clear that if the pre-deformation is applied in the opposite direction after it was applied in the forward direction, the anisotropy in the yield surface develops in the opposite direction. However, in these studies, the pre-deformation was limited to the uniaxial state of tension or shear. Hence, in subsequent work (Kato 2014), the shape of yield surface formed under the pre-deformations of combined loading of tension and shear with proportional loading state has been investigated.

However, as for the pre-deformation of combined loading of tension and shear, the experiments changing the loading path of pre-deformation had not performed yet in our previous studies. Hence, as the next step of this research, another type of the pre-deformation that the deformation path is different but the final deformation is the same as the pre-deformation of the proportional loading of tension and shear, is chosen as the subject of this research. In particular, the shape of the yield surface, which is obtained after applying the pre-deformation of tension after simple shear, is examined in this study. The effect of the deformation history on the development of anisotropy in the yield surface is revealed by comparing the results of this study with the previous study.

2 THE DEFORMATION HISTORY OF PRE-DEFORMATION

Figure 1 shows the shape of yield surface obtained by pre-deformation of the proportional loading of tension and shear that had been revealed in the previous study. In the present paper, the deformation, which the deformation path is different but the final deformation state is the same as the pre-deformation of proportional loading of tension and shear, is chosen. Especially, the pre-deformation of tension after simple shear is examined as the deformation history of pre-deformation. Figure 2 (a), (b) shows the differences in the deformation history between the pre-deformation of the present paper O→A→B and the pre-deformation of proportional loading O→B. As for the deformation path of this study, in the first stage, the simple shear is applied during O→A, and after that, in the second stage, tension is applied during A→B.

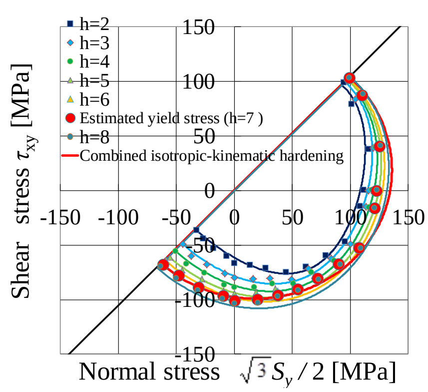
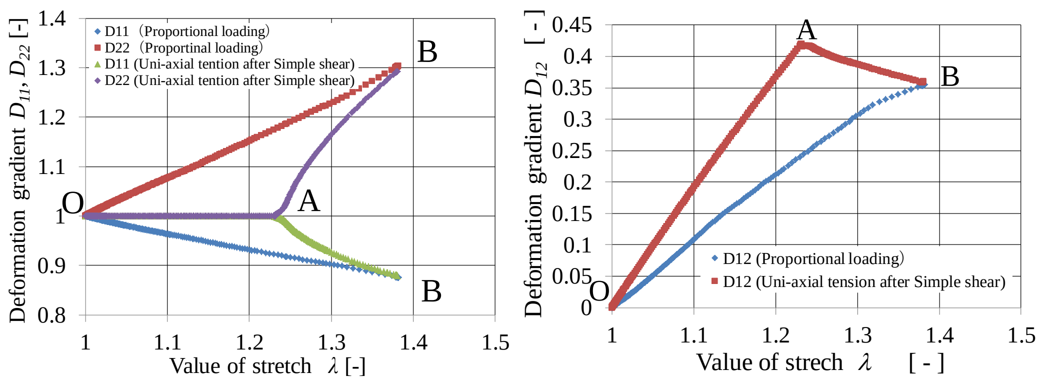


Figure 1. Distribution of h and yield surface in the case of proportional loading of tension and shear (the principal axis of pre-deformation; $\theta = 65$ [deg.] and the final stretch is $\lambda = 1.38$ [-]).



(a) Normal components D_{11}, D_{22} (b) Shearing component D_{12}

Figure 2. Deformation path during pre-deformation.

3 ESTIMATION METHOD OF YIELD STRESS

The method for determining the yield stress based on the slope of the tangent in the deviatoric stress and deviatoric strain curve is described in this chapter. Since the yield stress has been already found from the final stress condition of the pre-deformation, the slope of the tangent at the yielding point can preliminarily be determined. First, the experimental equation for the principal deviatoric stress, which becomes gentle curve around yielding point, is formulated as:

$$S_1 = a\left(1 - exp\left(b e_1\right)\right) + c e_1 + d \tag{1}$$

where, *a*, *b*, *c*, *d* are coefficients determined by using the *Levenberg-Marquardt Method*, which is one of the non-linear least-squares method. Moreover, in order to derive the slope of the tangent in the principal deviatoric stress-deviatoric strain curve, the following equation is obtained by differentiating Eq.(1), i.e.,

$$\frac{dS_1}{de_1} = -a\,b\,exp\left(b e_1\right) + c \qquad (2).$$

Hence, the slope of the tangent at yielding can be specified by using Eq.(2). On the other hand, the strain hardening modulus *h* is derived as follows.

$$h = \frac{2G}{\dfrac{dS_1}{de_1}} - 1 \tag{3}$$

As previously mentioned, since the slope of tangent at yielding is already specified, the value of the strain hardening modulus *h* at yielding can also be specified. Therefore, the shape of the yield surface can be estimated from the distributions of *h*.

4 EXPERIMENTAL METHOD

In the experiments, in order to apply a large pre-deformation to the specimens, the cylindrical specimens made of pure copper so called tough pitch copper, i.e., the gauge length 30 [mm] and purity 99.99%, are adopted. As for the experimental equipment, the multi-axial testing machine, which can be applied tension and torsion at the same time, is used. The displacement meters are used for measurements of longitudinal displacements, and the rotary encoders are used for measurements of twisting angles.

In the present study, the experiments are composed of two stages. Namely, the experiments, which apply the pre-deformation of tension after torsion to the specimens, are firstly conducted. Then, the experiments of the proportional loading for tension and torsion are conducted in arbitrary direction in the stress space. As for the experimental conditions of pre-deformation, the torsion, i.e., the principal stretch λ = 1.16 (shear deformation D_{12} = 0.3), is firstly applied to the specimens, then tension, i.e., the principal stretch in tension m = 1.28, is applied. However, the final deformation of each pre-deformation is the same value in all experiments, i.e., the principal stretch λ = 1.38 [-]. On the other hand, as for the experimental conditions for proportional loading tests, attaching the tri-axial strain gages to the specimens that the pre-deformation was already applied, the experiments are conducted with conditions that the principal axis of stress is fixed in arbitrary angle, i.e., 0, 10, 25, 35, 45, 55, 65, 70, 80, 90, 100, 110, 115, 120, 130, 135, 145, 150, 155, 170 [deg.]. Here, the principal strain in the proportional loading tests is measured from the extensional strain components of the tri-axial strain gauge in consideration of the Natural Strain theory, and the principal deviatoric stress-deviatoric strain curve in each tests are drawn. Then, according to the method explained in the previous chapter, the value of yield stress is determined. Moreover, the shape of the yield surface is estimated from the distribution of the yield stress.

5 EXPERIMENTAL RESULTS

Figure 3 shows the locus of stress points in each proportional loading tests, which are conducted after applying the pre-deformation to the specimens. Here, ① is the result of 0 [deg.], i.e., the case of uni-axial compression, ② is 10[deg.], ③ is 25[deg.], ④is 35[deg.], ⑤ is 45[deg.], i.e., the case of the simple shear, ⑥ is 55[deg.], ⑦ is 65[deg.], ⑧ is 70[deg.], ⑨ is 80[deg.], ⑩ is 85[deg.], ⑪ is 90[deg.], i.e., the case of uni-axial tension and it is the final direction of the pre-deformation, ⑫ is 100[deg.], ⑬ is 110[deg.], ⑭ is 120[deg.], ⑮ is 135[deg.], i.e., the case of the simple shear in the reversed direction, ⑯ is 145[deg.], ⑰ is 155[deg.] and ⑱ is 170[deg.]. As an example, the principal deviatoric stress- deviatoric strain curves in case of ①, ⑤, ⑧, ⑪, ⑭ and ⑮ are described in Figure 4 (a), (b), (c), (d), (e), (f), respectively. Here, the experimental equation, i.e., Eq. (1), is described by the solid curve in each figure. As is obvious from these figures, the stress and strain relation is a sharp curve in tension side of ⑪, and it becomes gradually gentle curve in order of ⑧, ⑭ ,⑤, ⑮, ①. Hence, in the case by proof stress, the yield stress is estimated smaller as the loading direction approaches compression side, i.e., the opposite direction to the final pre-deformation.

Figure 5 (a) shows the results of the estimated yield stress in all directions in a stress space. In this figure, the circular plots (●) indicate the estimated yield stress. Moreover, the distributions of the strain hardening modulus h near the yield surface are also described in this figure. As the loading direction approaches to the tension side that is the second stage of the pre-deformation, the distance of curves in h becomes narrow, however, it becomes larger as the loading direction approaches the compression side. Therefore, in contrast to the result in Figure 1, it can be found that the anisotropy in the yield surface is formed near the tension side.

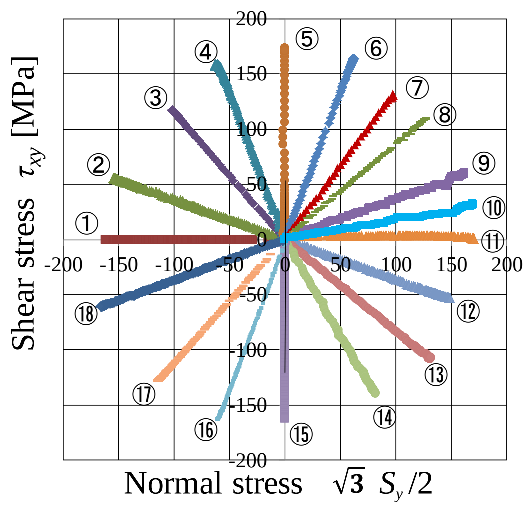


Figure 3. Stress points in each proportional loading tests.

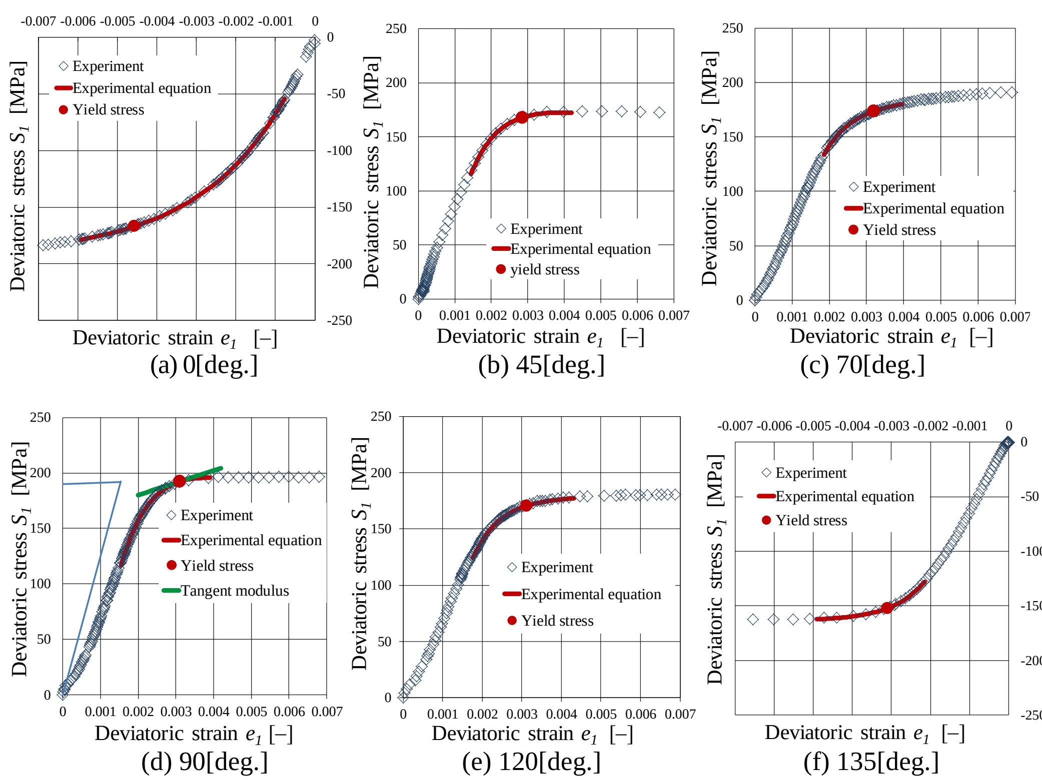
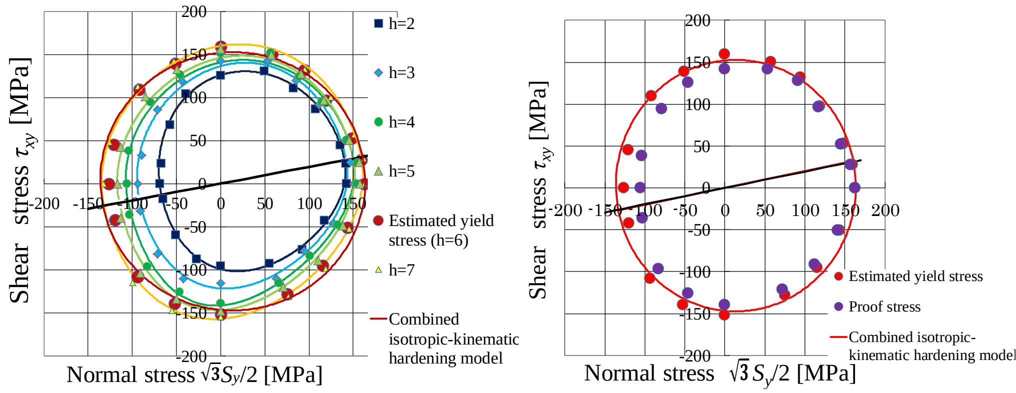


Figure 4. Principal deviatoric stress – deviatoric strain diagram.



(a) Distribution of *h* and yield surface . (b) Comparison of yield surface.

Figure 5. Shape of yield surface and distributions of strain hardening modulus *h*.

Lastly, Figure 5 (b) shows the results of comparison between the estimated yield surface and conventional proof stress. The shapes of both yield surfaces almost coincide in the neighborhood of the tension side of 90 [deg.]. However, as approaches the compression side of 0 [deg.], the yield surface estimated by proof stress becomes smaller compared with the yield surface estimated in this study. Hence, in the case of conventional proof stress, the flattening tendency of yield surface is seen more compared to the result in this study.

6 CONCLUSIONS

In the present study, in order to examine the effect of deformation history on the anisotropy of yield surface, the yield stress was estimated by examining the slope of tangent for the principal deviatoric stress- deviatoric strain curve on the basis of the Natural Strain theory. As the result, the shape of the yield surface has a tendency to become convex near the tension side and become flat at the compression side. Hence, it was revealed that the anisotropy of the yield surface is formed closely related to the tension applied at the second stage of pre-deformation. As for the distributions of strain hardening modulus *h*, distance between adjacent curves of *h* is small at the tension side, however, at the compression side, the distance becomes large. Moreover, the shape of yield surface estimated in this study was compared with the shape estimated by conventional proof stress. Consequently, both yield surfaces almost coincide at the tension side. However, on the opposite side, the yield surface by proof stress becomes smaller compared with the yield surface estimated in this study.

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