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UNIVERSAL SIZE EFFECT STUDIES USING THREE POINT BEAM TESTS

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The universal size effect law for concrete is a law that describes the dependence of nominal strength of specimen or structure on both its size and the crack (or notch) length, over the entire of interest, and exhibits the correct small and large size asymptotic properties as required. The main difficulty has been the transition of crack length from 0, in which case the size effect mode is Type 1, to deep cracks (or notches), in which case the size effect mode is Type 2 and fundamentally different from Type 1. The current study is based on recently obtained comprehensive fracture test data from three-point bending beams tested under identical conditions. This paper presents a studying to improve the existing universal size effect law using the experimentally obtained beam strengths for various different specimen sizes and all notch depths. The updated universal size effect law is shown to fit the comprehensive data quite well.

Keywords: Concrete fracture, Scaling of strength, Fracture testing, Statistical testing, Notch depth effect, Flexural strength, Quasi-brittle fracture.

1 INTRODUCTION

The preceding conference article (Şener *et al.* 2014a) presented an introduction to the problem and reported comprehensive test data for fracture of concrete specimens. The experimental program, also described in (Şener *et al.* 2014b), consisted of 80 three-point bend beams with 4 different depths 40, 93, 215 and 500mm, corresponding to a size range of 1:12.5. Five different relative notch lengths, a/D = 0, 0.02, 0.075, 0.15, 0.30 were cut into the beams. A total of 20 different geometries (family of beams) were tested. The present paper will use these data to analyze the effects of size, crack length.

The Scientific and Technological Research Council of Turkey (TUBITAK) provided funding to carry out comprehensive fracture tests of beam specimens made from the almost the same age and same concrete mix to investigate the influence of size and notch length on specimen strength.

2 REVIEW OF SIZE EFFECT AND CRACK LENGTH EFFECT

The nominal strength of geometrically similar structures, defined with Eq. (1) is

$$\sigma_N = c_N \frac{P_u}{bD} \tag{1}$$

independent of structure size D (P = maximum load; b = structure width; and c_N = dimensionless constant chosen for convenience). Size effect – defined as any dependence of σ_N on D – is a phenomenon typical in fracture or damage mechanics.

According to linear elastic fracture mechanics (LEFM) theory, which applies to homogeneous perfectly brittle materials, and for geometrically similar structures with similar cracks, $\sigma_N \propto D^{-1/2}$, which is the strongest possible size effect. For quasi-brittle materials such as concrete, two simple types of size effects can be seen in Eq. (2).

$$\boldsymbol{\sigma_N} = \frac{\boldsymbol{Bf_t}}{\sqrt{\boldsymbol{1 + D/D_0}}} \tag{2}$$

Here B and the transitional structure size D_0 are empirical parameters to be identified by data fitting and f_t = tensile strength of concrete introduced for convenience. Eq. (2) was derived (Bazant 1984) by simple energy release analysis and later by several different approaches such as by asymptotic matching based of the asymptotic power scaling laws for very large and very small D (Bazant and Planas 1998). In the standard size effect plot of log σ_N versus log D, Eq. (2) gives a smooth transition from a horizontal asymptote to an inclined asymptote of slope -1/2 (Figure 1).

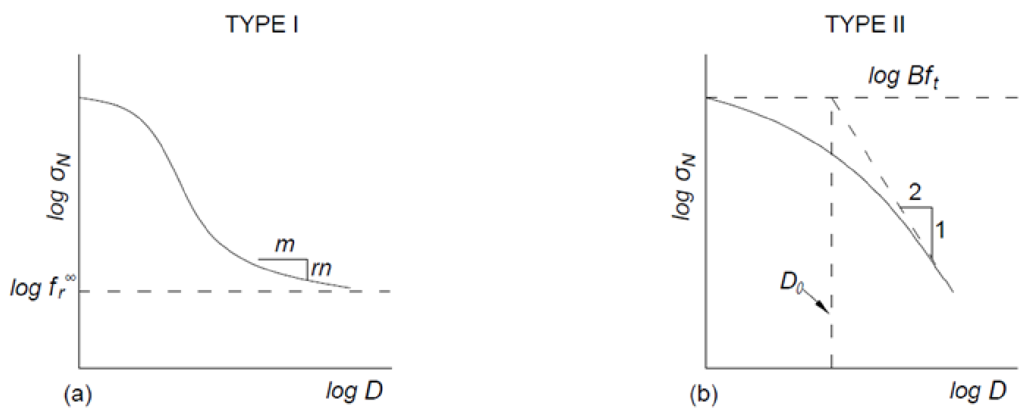

Figure 1. Dependence of □$_N$ on structure size D of beams with (a) no notched and (b) deep notch.

In Eq. (2),

$$\boldsymbol{Bf_t} = \sqrt{\frac{\boldsymbol{E'G_f}}{\boldsymbol{g_0'c_f}}}, \boldsymbol{D_0} = \frac{\boldsymbol{c_f g_0'}}{\boldsymbol{g_0}} \tag{3}$$

where $g_0 = g(\alpha_0)$; $g_0' = g'(\alpha_0)$; $\alpha = a/D$ = relative crack length; $\alpha_0 = a_0/D$ = initial value of α; $g(\alpha) = k^2(\alpha)$ = dimensionless energy release rate function $g(\alpha)$ of LEFM; $k(\alpha) = b\sqrt{D}K_I/P$ where K_I = stress intensity factor, P = load; $g'(\alpha) = \mathrm{d}g(\alpha)/\mathrm{d}\alpha$; $E' = E$ = Young's modulus for plane stress and $E' = E/(1-\nu^2)$ for plane strain (where ν = Poisson ratio); G_f = initial fracture energy = area under the initial tangent of the cohesive softening stress-separation curve; c_f = characteristic length, which represents about a half of the FPZ length. Eq. (2) may be rewritten as shown in Eq. (4).

$$\boldsymbol{\sigma_N = \sqrt{\frac{E^1 G_f}{g_0 D + g_0{}^1 c_f}}} \tag{4}$$

Because function $g(\alpha)$ or $k(\alpha)$ embodies information on the effects of crack length and structure geometry, Eq. (4) is actually a size effect law for Type 2 failures.

The Type 1 size effect, $\boldsymbol{\sigma_N}$ approaches, for large D, a constant value (a horizontal asymptote in the size effect plot), since the Weibull statistical size effect (Weibull 1939) is unimportant. For three point bend beams, it is indeed unimportant. Because the zone of high stresses is rather concentrated, even do not exist along a notch. This prevents the critical crack from forming at widely different locations of different random local strength (for the same reason, the statistical size effect is negligible in Type 2 failures also).

The large size asymptote for Type 1 size effect is, in the log-log plot, a downward inclined straight line of a slope $-n/m$, which is much milder than the slope of -1/2 for LEFM (Weibull 1939) (Figure 1); here m = Weibull modulus and n = number of spatial dimensions of fracture scaling (n = 2 for the present tests). The small size asymptote is also a horizontal line and, for medium sizes, the size effect is a transition between these two asymptotes. In absence of the statistical size effect, Eq. (5) was used by Hoover and Bazant (2014).

$$\boldsymbol{\sigma_N = f_r{}^{\infty}\left(1 + \frac{rD_b}{D + l_p}\right)^{1/r}} \tag{5}$$

Here f_r^{∞}, D_b, l_p, and r are empirical constants to be determined from tests; f_r^{∞} = nominal strength for very large structures, assuming no statistical size effect (in the special case of very large beams, f_r^{∞} represents the flexural strength, also called the modulus of rupture); and D_b = depth of the boundary layer of cracking (roughly equal to the FPZ size). In all previous works, D = same characteristic structure size as used for the Type 2 size effect [Eq.(4)]. Furthermore, l_p = material characteristic length, which is related to the maximum aggregate size d_a. If the structure is larger than $10l_p$, one can set $l_p \approx 0$, which corresponds to the original formulation of the Type I law.

It was further shown that the Type 1 and 2 SELs satisfy the large-size and small size asymptotic properties of the cohesive crack model applied to Type 1 and 2 failures. Furthermore, it was experimentally confirmed that, within the range of inevitable experimental scatter, the SEL of Type 2 gives about the same values of fracture energy G_f when applied to notched fracture specimens [e.g., compact compression test (Barr *et al.* 1998)].

3 APPLICATION OF USEL BY FRACTURE TESTS

To calibrate the deterministic USEL, the mean of the data was computed separately for each family of identical specimens from comprehensive fracture tests (Şener *et al.* 2014a, 2014b, Çağlar and Şener 2015). The surface of the optimized Universal Size

Effect Law (USEL) is plotted in Figure 2. In this Figure 2 size effect curves were given for only α= 0, 0.15 and 0.3. Transition from these curves for calibrating USEL is just used with smooth curves. The studies on these transition curves are still on going.

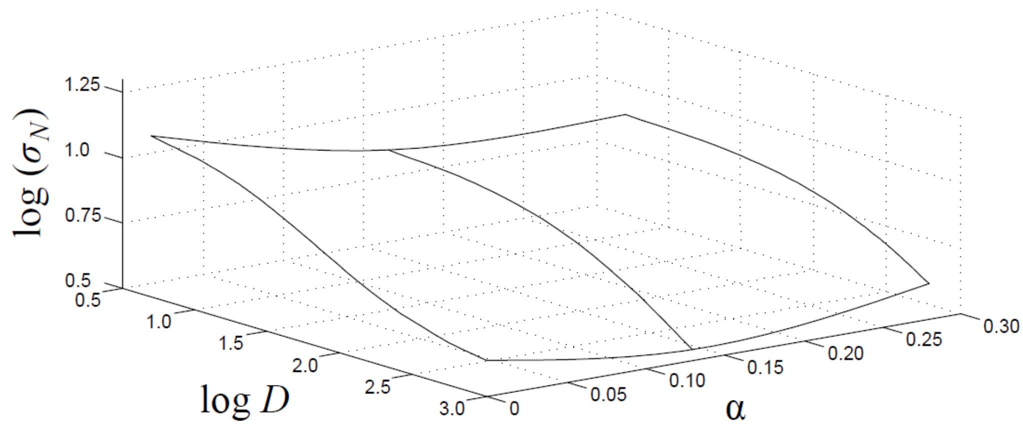

Figure 2. Entire Universal Size Effect law surface.

In particular, the fracture parameters G_f and c_f should not be influenced by the data for beams with no notches (Type I data) or shallow notches and f_r^{∞}, D_b, l_p, and r should not be influenced by the data for deep notches. Therefore, these parameters were determined first by separate fitting of specimens with deep notches ($\alpha = 0.30$ or 0.15) and specimens with shallow or no notches (α =0). Only the nonstatistical USEL in Eq.(6) was considered. Nonlinear fitting of the Type I SEL [Eq.(5)] to the notchless ($\alpha = 0$) beams gave (Şener *et al.* 2014a, b) values in Eq.(6) with coefficient of variation of fit 9.4%.

$$D_b = 90 \text{ mm}, \; l_p = 50 \text{ mm}, \; f_r^{\infty} = 4 \text{ MPa}, \; r = 1/2 \tag{6}$$

These values are different from than the studies by Hoover and Bazant's (2014) D_b=73.2 mm, l_p=126.6 mm, $\square_{\square}^{\infty}$ =5.27 MPa. The differences between some of the parameters were in the order of two for especially l_p value. The size range 1:12.5 was large enough to identify all the fracture parameters in Eq.(5). The USEL can be drawn for a fixed α, which gives a size effect plot of log(σ_N) versus log D (Figure 2).

In Figure 3, this plot is created and compared with the data from Şener *et al.* (2014a, b). Also in Figure 3, for α=0.3 and 0.15, Type II (at top) size effect was used, for unnotched specimen α=0, Type I (at bottom) size effect was used. For the α=0.02 and 0.075 USEL (at center) should proposed.

4 CONCLUSIONS

(1) The Type 2 size effect in specimens with deep notches or cracks does not give a correct transition to of Type 1 in specimens with no notch or crack.

(2) The size effect data from deeply notched specimens (α = 0.3 and 0.15), and parameters f_r^{∞}, D_b, l_p, and r were determined separately by fitting only the size effect data for un-notched specimens (α = 0).

(3) USEL fits the measured nominal strength quite well.

(4) Both Type I and II sizes were effect observed in this study and confirmed the need to be account for size effect in design codes.

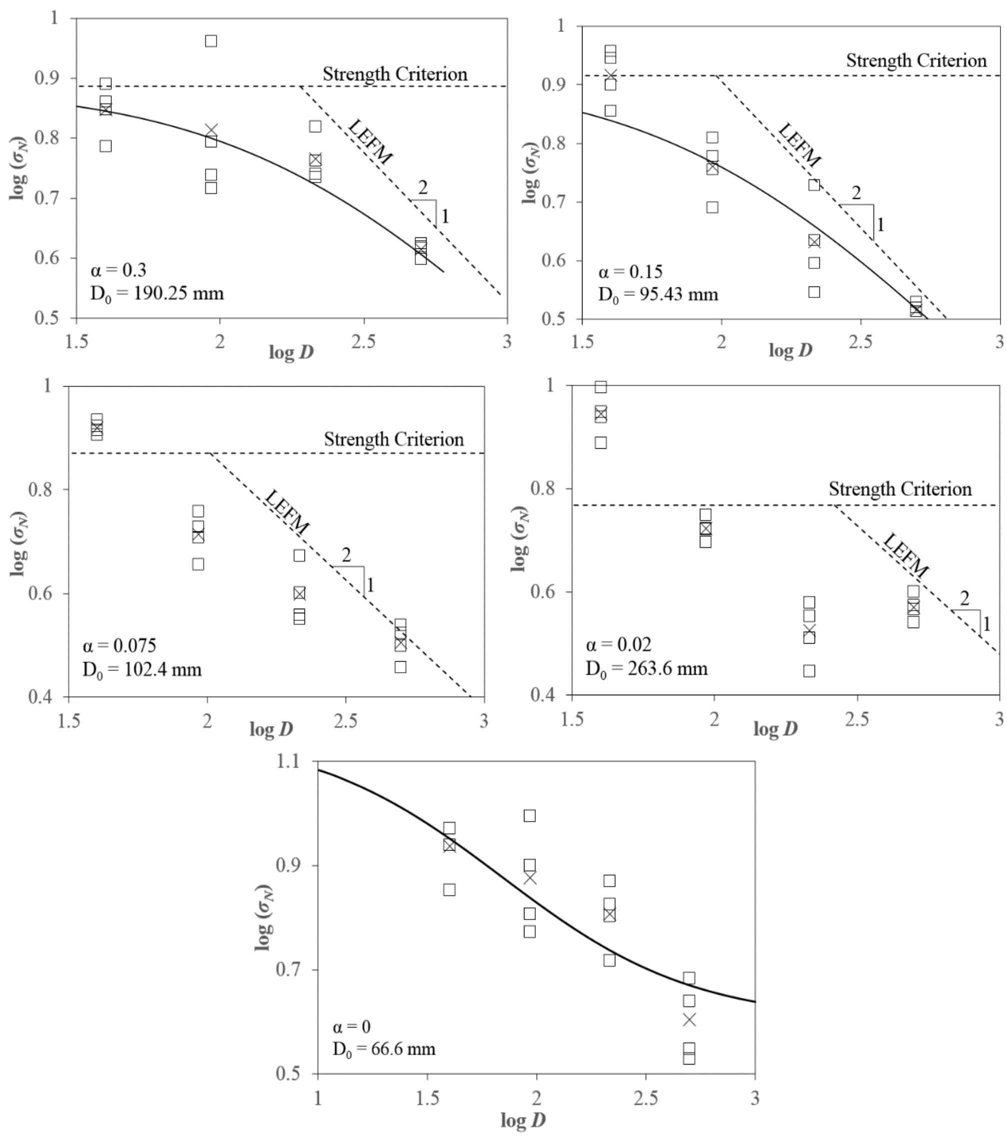

Figure 3. Effect of structure size on the nominal strength of the data from Şener *et al.* (2014a).

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