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EFFECTS OF A CENTRAL HOLE ON COMPRESSIVE BEHAVIOR OF CIRCULAR FIBER-REINFORCED RUBBER BEARINGS

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The aim of this study is to investigate the effects of the existence of a central hole on the main design parameters of a circularfiber-reinforced rubber bearing; namely compression modulus and shear strain due to compression. Since the compressive behavior of a multi-layered rubber bearing is mainly governed by the behavior of a single interior rubber layer in the bearing bonded to the reinforcing sheets at its top and bottom faces, the study is concentrated on uniformly compressed “bonded” rubber layers. The related compression problem is formulated using the “pressure” method. After deriving the closed-form expressions for compression modulus and shear strain, the effects of the hole on compressive behavior are investigated for fiber-reinforced bearings with different initial shape factors (a kind of aspect ratio for individual rubber layers) and rubber compressibility. It is shown that the compression modulus of a fiber-reinforced bearing may decrease considerably as the size of the central hole or the flexibility of the reinforcement increases, especially if the shape factor of the bearing is high and the compressibility of the rubber is not negligible. The study also shows that the effect of the hole on maximum shear strain reaches its most striking value when the hole size is smaller than 10% of the outer radius of the bearing.

Keywords: Compression modulus, Elastomer, Hollow circular bearing, Isolator, Pressure method, Seismic isolation.

1 INTRODUCTION

Multi-layered steel-reinforced rubber bearings have widely been used in various engineering applications, including isolation of machines from detrimental effects of vibrations, of bridges from adverse effects of thermal expansion and contraction, or of buildings from devastating effects of earthquakes. Rubber is, indeed, an ideal material for such isolation applications since it inherently has considerably low shear modulus, which ranges from 0.30 to 2.22 MPa (Kelly 1997). However, if a rubber bearing is subjected to high compression or bending loads, the excessive bulging occurred at the bulge-free surfaces of the bearing can lead to the failure of rubber under hydrostatic tension, which has to be prevented to fully utilize the favorable mechanical properties of rubber. The bulging of rubber, in turn, the hydrostatic tension in rubber, can be reduced if it is layered and reinforced using relatively rigid plates. In fact, the primary function of thin steel plates bonded to top and bottom faces of thin rubber layers in a multi-layered elastomeric bearing is to enhance the compressive and bending behavior of the bearing by controlling the bulging of interior rubber layers.

In the last three decades, many structures with sensitive equipment, historical value or post-earthquake importance have been isolated using multi-layered steel-reinforced rubber bearings. On the other hand, the researchers have realized that the use of rubber bearings for isolation of regular (residential, office, etc.) buildings or of structures in developing countries is rather limited. This phenomenon is mostly attributed to the fact that the conventional steel-reinforced bearings are usually considerably large, heavy and expensive due to the existence of steel plates as reinforcing elements. Kelly (1999, 2002) verified through analytical and experimental studies that both the cost and weight of the bearings can be reduced if fiber-reinforcement (in the form of two-directional sheets with large openings) is used instead of steel reinforcement. In the last fifteen years, many studies have been conducted on fiber-reinforced bearings (e.g., Kelly and Takhirov 2001, Mordini and Strauss 2008, Pinarbasi and Mengi 2008, Toopchi-Nezhad *et al.* 2008, Osgooei *et al.* 2014). However, most of these studies have been concentrated on either long rectangular strips or solid circular bearings. On the other hand, the use of hollow-circular isolators is also very common in practice. For this reason, it is essential to investigate the effects of the existence of a central hole on behavior of a fiber-reinforced bearing under compression/bending.

The main objective of this study is to investigate the effects of the existence of a central hole on the main design parameters of a circular fiber-reinforced rubber bearing; namely compression modulus and shear strain due to compression. Since the compressive behavior of a multi-layered rubber bearing is mainly governed by the behavior of a single interior rubber layer in the bearing bonded to the reinforcing sheets at its top and bottom faces, the study is concentrated on uniformly compressed "bonded" rubber layers. The related compression problem is handled using the well known "pressure" method. After deriving the closed-form expressions for compression modulus and shear strain, the effects of the hole on compressive behavior are investigated for fiber-reinforced bearings with different shape factors (a kind of aspect ratio for individual rubber layers) and rubber compressibility.

2 UNIFORM COMPRESSION OF A RUBBER LAYER BONDED TO FIBER-REINFORCED SHEETS AT ITS TOP AND BOTTOM FACES

The uniform compression of a typical interior rubber layer in a multi-layered circular fiber-reinforced rubber bearing with a central hole is shown in Figure 1. The disc has an inner radius of a, outer radius of R, rubber thickness of t and equivalent reinforcement thickness of t_f. Under a concentric compressive load of P, the top surface of the disc approaches to the bottom with a relative vertical displacement of Δ. This compression problem is formulated using the pressure method modified to include the extensibility of the flexible reinforcing sheets (Kelly 1999). It is convenient to define a cylindrical coordinate system (r, θ, z) with its origin located at the center of the disc (Figure 1).Since the deformed shape of the disc is axisymmetric, the displacement component in θ direction vanishes and those in r and z directions (i.e., u and w) are independent of θ. Using the pressure method, the nonzero displacement components can be expressed as $u(r,z) = u_0(r)(1\text{-}4z^2/t^2)+u_1(r)$ and $w(r,z) = w(z)$. The constitutive equation for the rubber disc can be written, in terms of the strain components (ε_{rr}, $\varepsilon_{\theta\theta}$and ε_{zz}), mean pressure (p) and bulk modulus (K) of rubber as follows:

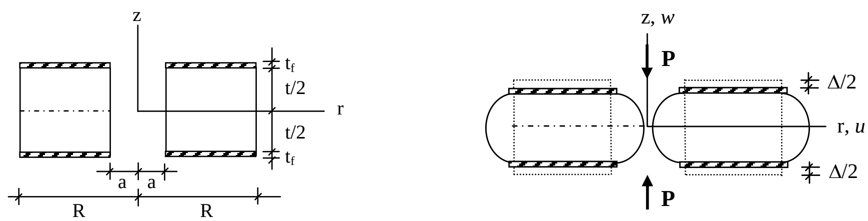

Figure 1. Uniform compression of a circular fiber-reinforced rubber layer with a central hole.

$$\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz} = -p / K \tag{1}$$

ε_{rr}, $\varepsilon_{\theta\theta}$and ε_{zz} can be expressed in terms u_0, u_1 and w,using axisymmetric strain-displacement relations. Substituting these relations into Eq. (1), then integrating the resulting equation through the rubber thickness, Eq. (1) can be rewritten in terms of the three unknowns of the problem: u_0, u_1 and p. The second governing equation for the problem, the equilibrium equation in radial direction for the rubber disc, can be written, in terms of the stress components σ_{rr}, $\sigma_{\theta\theta}$,$\sigma_{zz}$and τ_{rz}, as follows:

$$\sigma_{rr,r} + \tau_{rz,z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \tag{2}$$

Using the "pressure" assumption and stress-strain relation for shear stress and strain (i.e., $\tau_{rz}= \gamma_{rz}G$, where G is the shear modulus of rubber), Eq. (2) can be expressed in terms of the unknown. Finally, the third governing equation is obtained from the equilibrium of the reinforcing sheets, which are subjected to bonding shear stresses (τ^{-}_{rz} and τ^{+}_{rz}, see Figure 2), in radial direction. Assuming that the sheets are in the state of plane stress, the equilibrium equation can be written, in terms of the internal forces per unit lengthin the sheet(N_{rr} and $N_{\theta\theta}$) and bonding shear stresses, as follows:

$$N_{rr,r} + \frac{N_{rr} - N_{\theta\theta}}{r} + \tau_{rz}^{-} - \tau_{rz}^{+} = 0 \tag{3}$$

Using the linear stress-strain relations, N_{rr} and $N_{\theta\theta}$can be expressed in terms of u_1, elasticity modulus$E_f$and Poisson's ratio ν_f of the reinforcement. Similarly, bonding shear stresses can be expressed in terms of u_1 and u_0.Thus, Eqs. (1) to (3) constitute three governing equations for the studied problem. Since there is no applied load/stress at the inner/outer lateral faces of the layer or the edges of the sheets, the boundary conditions for the problem can be expressed as $p = 0$ and $N_{rr} = 0$ at $r = a, R$. Using these conditions, the unknown displacement functions (u_0 and u_1) and the mean pressure (p) can be obtained. For details, Pinarbasi and Okay (2011) is to be referred.

The compression modulus for the studied layer (see Figure 1) can be computed from $E_c = (P/A)/(\Delta/t)$, where P is obtained by integrating the mean pressure (p) over the bonded area and $A = \pi(R^2\text{-}a^2)$. Thus, the compression modulus for a multi-layered fiber-reinforced elastomeric bearing (denoted as $E_{c,HC}$) has the following form:

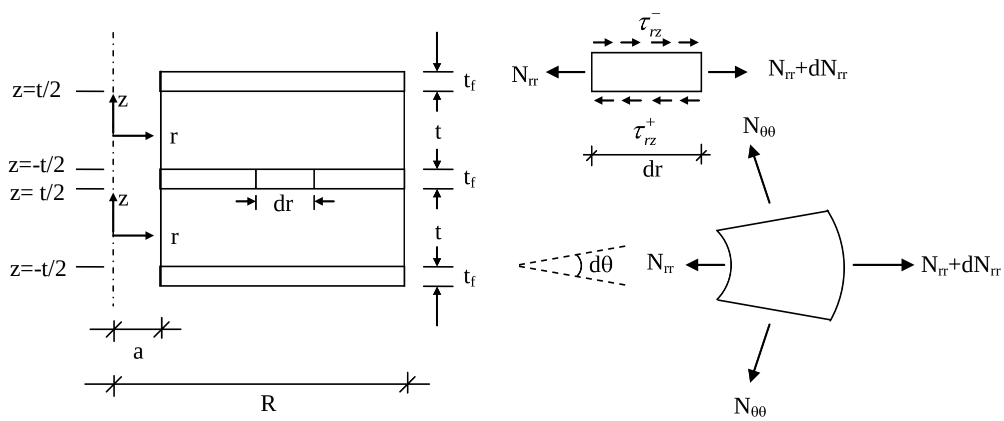

Figure 2. Forces on infinitesimal area of a reinforcing sheet (Pinarbasi and Okay 2011).

$$E_{c,HC} = \frac{k_f}{t}\frac{(1+\nu_f)}{2}\frac{\left\{\begin{array}{l}\left(1-\frac{a^2}{R^2}\right)\left[I_0\left(\beta^*R\right)-A_1K_0\left(\beta^*R\right)\right] \\ -2\left[\frac{I_1\left(\beta^*R\right)+A_1K_1\left(\beta^*R\right)}{\left(\beta^*R\right)}\right]+\frac{2a^2}{R^2}\left[\frac{I_1\left(\beta^*a\right)+A_1K_1\left(\beta^*a\right)}{\left(\beta^*a\right)}\right]\end{array}\right\}}{A_2-\frac{a^2}{R^2}A_3+\left(1-\frac{a^2}{R^2}\right)A_4} \tag{4}$$

$$\left(\beta^*\right)^2 = \alpha^2+\lambda^2 = \frac{12G}{k_f t}+\frac{12G}{Kt^2}, \quad k_f = \frac{E_f t_f}{1-\nu_f^2} \tag{5}$$

In Eq. (4), I_i and K_i (i=1,2) are the modified Bessel functions of, respectively, first and second kinds of order i.The closed-form expressions for A_i (i=1 to 4) are presented in Pinarbasi and Okay (2011). The other design parameter for the disc, namely, the maximum absolute shear strain in the rubber layer, has the following closed-form:

$$\gamma_{\max} = \frac{6\beta^*}{\alpha^2 t}\frac{\Delta}{t}\frac{(1+\nu_f)}{2}\left(1-\frac{a^2}{R^2}\right)\frac{\left[I_1\left(\beta^*r\right)+A_1K_1\left(\beta^*r\right)\right]}{A_2-\frac{a^2}{R^2}A_3+\left(1-\frac{a^2}{R^2}\right)A_4} \tag{6}$$

3 DISCUSSIONS AND CONCLUSIONS

Figure 3 shows the variation of $E_{c,HC}/G$ as a function of initial shape factor (defined as $S_o=R/2t$) for two different values of reinforcement rigidity, k_f/Gt=30000 (which represents CFRP sheets used by Kelly (2002)) and 300,for three different values of radius ratio, $\beta=a/R$=0, 0.01, 0.1 and 0.5 andrubber compressibility, ν=0.5, 0.4995 (which represents natural rubber) and 0.49.As it is seen, $E_{c,HC}$ increases as$S_o$increases until it reaches an asymptotic value, which increases as $k_f/Gt\to\infty$ and $\nu\to 0.5$. The presence of hole decreases$E_{c,HC}$ considerably if S_o, $k_f/Gt$and ν are large. For example, for

S_o=30 and k_f/Gt=30000, the ratio of $E_{c,HC}$ to $E_{c,C}$ (i.e., E_c for circular case) is 66% and 21% respectively when β=0.1 and 0.5 if ν=0.5, but 93% and 62% if ν=0.4995. Figure 4 shows the variation of $\gamma_{max}/\varepsilon_c$ (where $\varepsilon_c=\Delta/t$) as a function of β for various values of S_o, k_f and ν. As shown in the graphs, $\gamma_{max}/\varepsilon_c$ increases considerably as $\beta \to 0$. The increase is larger if S_o, k_f and ν are large. However, when β>0.1, $\gamma_{max}/6S_o\varepsilon_c$ (ratio to the incompressible circular steel-reinforced case) is less than 2.5 even when ν=0.5.

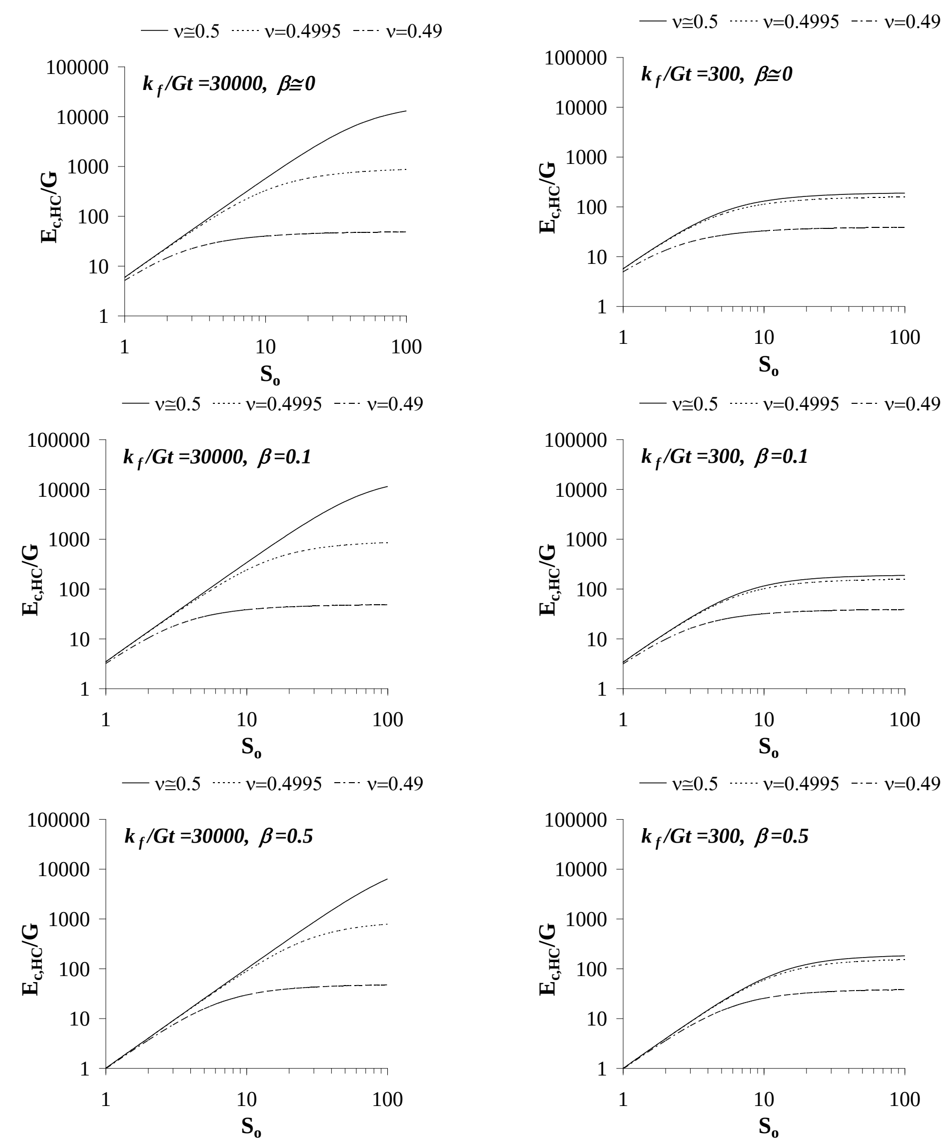

Figure 3. Effect of the existence of a central hole on compression modulus.

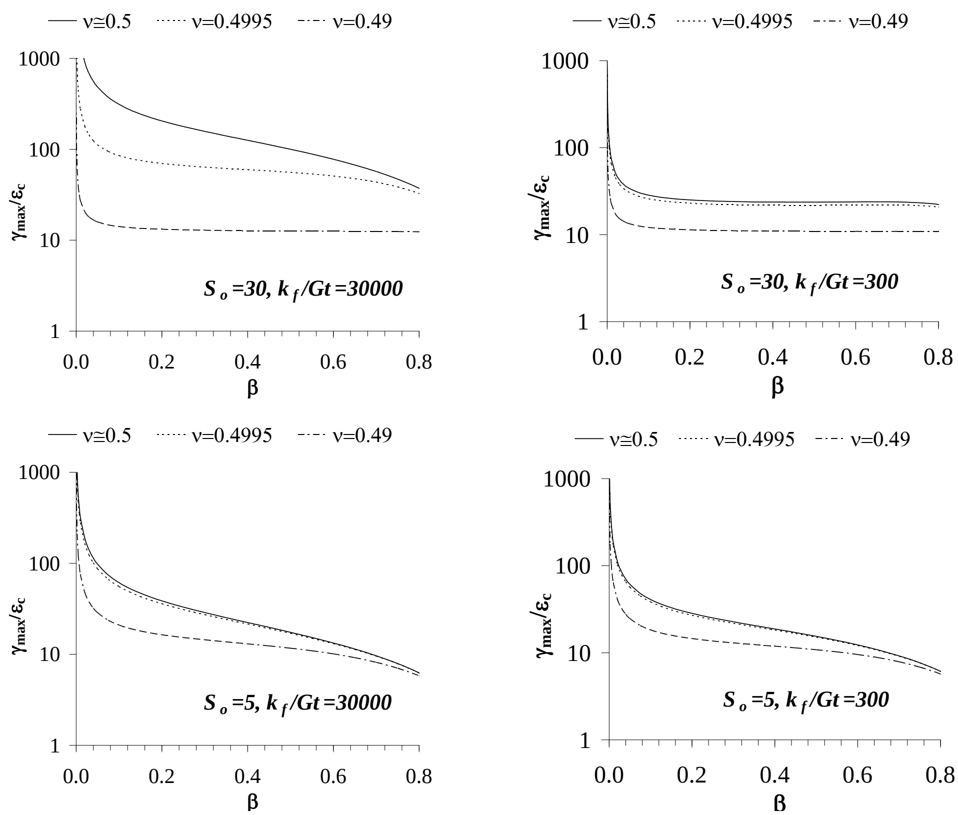

Figure 4. Effect of the existence of a central hole on maximum shear strain due to compression.

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