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PROBABILISTIC BEHAVIOR OF JOINT FORCES IN MECHANISMS

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This paper discusses the influence of the clearance in joints on the joint reaction forces in mechanisms. By using mathematical programing, the optimal parameters of kinematic chains can be efficiently obtained by using the deterministic approach. However, the situation becomes more sophisticated if random effects of tolerances of the arm lengths and the random pin positions have to be considered. In this work the influence of clearances on joint forces is calculated by using the Taylor approximation and the Monte Carlo method. Using the two methods was necessary, because the Taylor approximation usually yields satisfying results only for small values of clearances and for this reason it makes sense to compute the required quantities also by another independent method. The implementation of the model is illustrated with two examples. The first example considers a closed loop chain, representing a four-bar mechanism being an actual part of a hydraulic support, employed in mining industry. The hydraulic support must fulfill some requirements so it's very important to have influence of clearances on joint forces in control. The second example considers joint reaction forces of car wiper mechanism. It will be shown, that the clearance in joints have some influence on the joint reaction forces.

Keywords: Kinematic chain, Stochastic model, Mathematical programming.

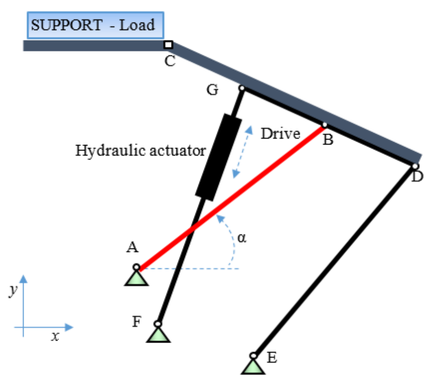
1 INTRODUCTION

Kinematic chains are widely used in industry, which is also reflected in extensive literature and studies available, see e.g.,,, Yoshikawa (1985), Conkur and Buckingham (1997). For the analysis and synthesis of kinematic chains, optimization methods have proved to be a valuable tool. Consequently, since the 1960s, various formulations for optimization of problems related to kinematic chains have been developed, see e.g.,, Arora and Wang (2005). Some good examples are the optimal track problem of a closed kinematic chain Oblak *et al.* (2000) and the problem related to the impact of tolerances of arm lengths and random pin positions, discussed in Dhande *et al.* (1973), Lee and Gilmore (1989).

In this paper, the study is conducted on two examples. The first one is the kinematic chain with the optimal length of the links, obtained in Oblak *et al.* (2000) by employing optimization methods, and the second example is a car wiper mechanism. For both examples the influence of the clearances in the joints on the joint reaction forces is studied. This is achieved by an adequate mathematical model that reflects the effects of tolerances of arm lengths and random pin positions. The influence of clearances on joint forces is calculated by the Taylor approximation and the Monte Carlo method.

The first example is a hydraulic support, Figure 1a, that is a part of the mining industry equipment in the mine Velenje - Slovenia, used for protection of working environment in the gallery. It consists of two four-bar mechanisms *FEDG* and *AEDB* as shown on Figure 1b. The mechanism *AEDB* defines the path of the coupler point C and the mechanism *FEDG* is used to drive the support by a hydraulic actuator. The original support design exhibited large transversal displacements of the point C and relatively very high joint forces, which reduced its employability. This was the reason to determine the best values for the most problematic parameters of leading four-bar mechanism *AEDB* by employing the methods of mathematical programming Kegl *et al.* (1991). In this work, the influence of the clearance in joints on the joint reaction forces will be studied for the four-bar mechanism *AEDB* of the support, Figure 1b.

(a)



(b)

Figure 1. (a) Hydraulic support engagement in a mine;
(b)The schematic model of the hydraulic support.

The car wiper mechanism consists of two four-bar mechanisms *ADCB* and *DGFE* as shown in Figure 2. The clearance in joints on the joint reaction forces will be studied for the *ADCB* mechanism.

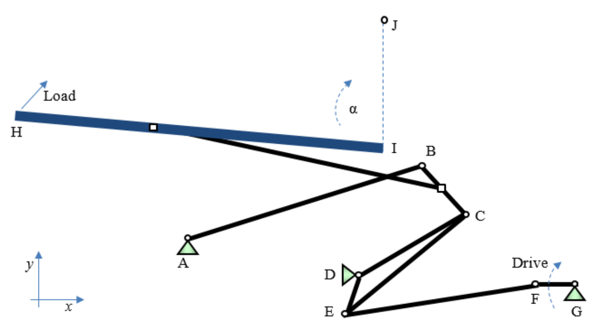


Figure 2. Schematic model of the car wiper mechanism.

2 THE STOCHASTIC MODEL OF THE MECHANISM

The foundation of our considerations is an appropriate mechanical model of the mechanical system – kinematic chain, which will enable the computation of the kinematic and dynamic quantities Harl *et al.* (2004). We assume here that this model is given by a system of m response equations, written as

$$h_i(\mathbf{b},\mathbf{u}) = 0, \quad i = 1,2,\cdots,m \tag{1}$$

The response equations allow us to compute the vector of response variables (u) in dependence on the vector of some design variables (b). This implies that $\mathbf{u} = \tilde{\mathbf{h}}(\mathbf{b})$, where $\tilde{\mathbf{h}}$ denotes a vector function establishing the relationship between the vector of design variables and the response of our mechanical system.

In a stochastic model, the vector b is treated as a random vector $\mathbf{B} = [B_1, B_2, \ldots, B_n]^T$, meaning that the vector u of response variables is also a random vector $\mathbf{U} = [U_1, U_2, \ldots, U_m]^T$, where is $\mathbf{U} = \tilde{\mathbf{h}}(\mathbf{B})$. It is supposed that the design variables $B_1, B_2, \ldots, B_n$ are mutually independent, at least from the probability point of view.

The probability distribution function of the random vector U, that is searched for, depends on the probability distribution function of the random vector B and it is mostly practical impossible to compute. Therefore, the random vector U will be described either by the help of "number characteristics", that can be estimated by a Taylor approximation of the function $\tilde{\mathbf{h}}$ at the point $\mathbf{b}$, or by using the Monte Carlo method described in Harl *et al.* (2015).

3 CLEARANCE IN JOINTS

In this work, the effective link length model is used Lee and Gilmore (1989). This model considers the uncertainties due to pin location, link length tolerance, and radial clearance tolerance as an effective length variation of the link. Therefore, the local configuration of each pin joint does not affect the model. The effective link length model is shown in Figure 3.

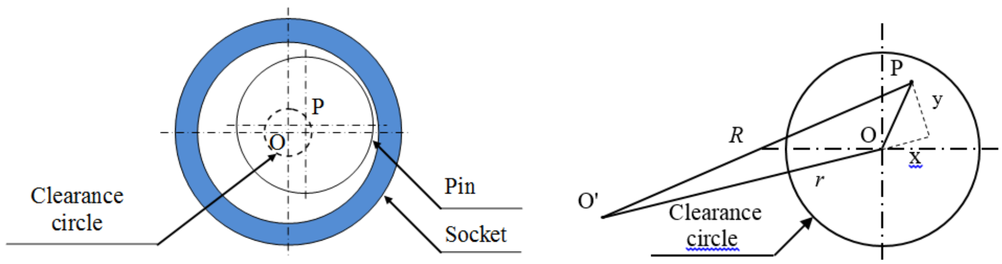


Figure 3. Mechanical model of the pin clearance.

As the pin moves inside the inner circle of the socket, the pin center moves on or inside the clearance circle. The radius of the clearance circle, i.e., the radial clearance, is determined by the tolerances of the pin and socket diameters. Since most tolerances follow a normal distribution at the worst case, the probability distribution of the pin position is assumed to follow such distributions in order to analyze the pin joint successfully. This model, the clearance model, readily accounts for uncertainties in a pin joint such as tolerance of the radial clearance and random location of the pin. Hence, the origin of such a frame coincides with the socket center *O* and the axis *x* is parallel to the nominal link direction. The line $\overline{O'P}$ represents the effective link and its length is given as

$$R = \sqrt{(r+x)^2 + y^2} \tag{2}$$

where *x* and *y* are the coordinates of the random pin location in the 1ocal coordinate frame and r is random link length.

4 NUMERICAL EXAMPLES

Two numerical examples will be considered. The influence of the clearance in joints *A*, *E*, *D*, *B* of the hydraulic support (Figure 1b) and for joints *A*, *D*, *C*, *B* of the car wiper (Figure 2) on the joint reaction forces will be calculated.

The symbols F_j, where $j = 1, \ldots, 4$, represent the norms of the joint force vector. By using the Taylor approximation, the influence for all four joints can be estimated by the standard deviations

$$\sigma_j = \sqrt{\sum_{i=1}^{4} \left(\frac{\partial F_j}{\partial u_i}\right)^2 \left(\frac{\Delta r_i}{6}\right)^2 + \left(\frac{\partial F_j}{\partial x}\right)^2 \left(\frac{\Delta x}{6}\right)^2 + \left(\frac{\partial F_j}{\partial y}\right)^2 \left(\frac{\Delta y}{6}\right)^2} \tag{3}$$

In the above equations, Δr_i are the tolerances of the link lengths of *AEDB* and *ADCB* mechanisms, while Δx and Δy are the tolerances of the pin and the socket.

During numerical investigation, it turned out that the Taylor approximation usually yields satisfying results only for small values of Δr_i, Δx, and Δy. For this reason it makes sense to compute the required quantities also by another independent method. In our case the Monte Carlo method was engaged for this purpose. In this case the influence of clearances on joint forces can be computed as

$$\mu_j = \frac{\sum_{i=1}^{n} F_{j_i}}{n} \text{ and } \sigma_j = \frac{\sum_{i=1}^{n} (\mu_j - F_{j_i})^2}{n-1} \tag{4}$$

For the first numerical example - hydraulic support, the carrying capability is 1600kN. Both four-bar mechanisms *AEDB* and *FEDG* must fulfill the following requirements:

- They must exhibit minimal transversal displacements of the point *C*,
- They must provide sufficient side stability.

The starting parameters of both four-bar mechanisms (Figure 1b) were the values obtained in the earlier work by optimization. These values are (in millimeters)

$\left[\overline{AE}, \overline{AB}, \overline{BD}, \overline{DE}, \overline{EF}, \overline{FG}, \overline{GD}\right]^T = \left[676, 1361, 382, 1310, 400, (1325 + d), 1251\right]^T$. The parameter d is a displacement of the hydraulic actuator with a maximal value of 925 mm. During operation, the parameter d varies, so that the angle α between links $\overline{AB}$ and $\overline{AE}$ can is varied in the range from 76.8° to 94.8°. For the selected pin and socket, the maximal values of x and y are $x = y = 2\,\mathrm{mm}$. For calculating the clearances, the tolerances of the links of the four-bar mechanism *AEDB* are needed and the values, in millimeters, are $\left[\Delta r_1, \Delta r_2, \Delta r_3, \Delta r_4\right]^T = \left[0.01971, 0.00868, 0.001, 0.00933\right]^T$ meanwhile, the tolerances of the clearance are $\left[\Delta x, \Delta y\right]^T = \left[0.001, 0.001\right]^T$.

By using this data the deterministic joint forces in the hydraulic support, during the variation of *d*, can be computed. Their histories are shown in Figure 4a, where: F_1 (full line – blue), F_2 (dotted line – green), F_3 (dashed line – yellow) and F_4 (dash-dotted line - red). It is now possible to estimate the effect of clearances on joint reaction forces, obtained by the Taylor approximation. The variations of these forces are shown in Figure 4b, where: ΔF_1 (full line – blue), ΔF_2 (dotted line – green), ΔF_3 (dashed line – yellow) and ΔF_4 (dashed-dotted - red).

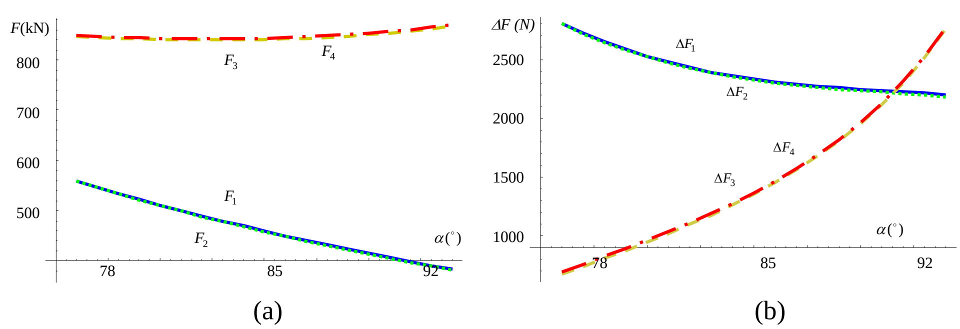
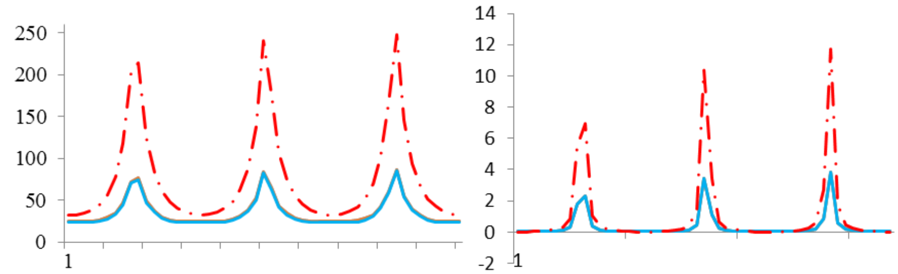


Figure 4. (a) Joint forces of the mechanism; (b) The effect of clearances on the joint forces of the mechanism *AEDB* obtained by the Taylor approximation.

The car wiper, Figure 2, is loaded by a force of 20N. The starting parameters, in mm, are $\left[\overline{AB}, \overline{BC}, \overline{CD}, \overline{AD}, \overline{DE}, \overline{EF}, \overline{FG}, \overline{DG}\right]^T = \left[200, 40, 100, 125, 42.5, 110, 11.5, 220\right]^T$. The drive of the car wiper mechanism is defined as $f(t)$ and can vary the angle α between the links $\overline{IH}$ and $\overline{IJ}$ from 10° to 90°. For the selected pin and socket, the maximal values of x and y are $x = y = 0.2\,\mathrm{mm}$. To compute the clearances, the tolerances of the links of the four-bar mechanism *ADCB* are needed. Their values, in millimeters, are $\left[\Delta r_1, \Delta r_2, \Delta r_3, \Delta r_4\right]^T = \left[0.01, 0.01, 0.01, 0.01\right]^T$. The tolerances of the clearance are $\left[\Delta x, \Delta y\right]^T = \left[0.001, 0.001\right]^T$.

The joint forces during the movement are shown in Figure 5a, where: $F_1 \approx F_2 \approx F_3$ (full line – blue) and F_4 (dashed-dotted - red). Figure 5b shows the effect of clearances on joint reaction forces, obtained by the Taylor approximation, where: $\Delta F_1 \approx \Delta F_2 \approx \Delta F_3$ (full line – blue) and ΔF_4 (dashed-dotted - red).



(a) (b)

Figure 5. (a) Joint forces of the mechanism *AEDB*; (b) The effect of clearances on the joint forces of the mechanism *AEDB* obtained by the Taylor approximation.

5 CONCLUSIONS

By utilizing an adequate mathematical model of the mechanical system the effects of random parameters on forces in a kinematic chain can be estimated. In this paper a hydraulic support and a car wiper were considered. It was shown that the clearance in joints have some influence on the joint reaction forces. T his influence varies as the kinematic chain operates. However, for the considered examples the magnitudes of these variations were rather minor except maybe at some position of the car wiper mechanism. So, one can say that for usual mechanisms with rather large security factors the influence of considered random parameters may probably be neglected. However, for highly loaded mechanisms the presented approach might provide valuable additional information on the actual loads emerging during operation.

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