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SHEAR BEHAVIOR OF REINFORCED CONCRETE SLENDER BEAMS

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Major factors contributing to the shear behavior in reinforced concrete (RC) beams have been identified as compressive strength of concrete, shear span to effective depth ratio, and longitudinal reinforcement. Though significant, few of these factors are not fully incorporated in ACI code provisions for design of minimum shear reinforcement. To investigate the effect of these parameters, an analytical and experimental study was undertaken on the shear behavior of ordinary strength RC slender beams with moderate longitudinal reinforcement. The experimental program consisted of testing of eight simply supported RC slender beams subjected to two concentrated loads at a shear span to depth ratio (*a/d*) of 2.5 and equipped with varying shear reinforcement according to four different criteria. Ultimate shear strengths obtained in this experimental program are compared to the analytical shear strengths calculated according to existing as well as proposed equations. Test results exhibit that, the modified equation proposed in this work gives more accurate prediction of shear capacity of RC beams.

Keywords: Minimum shear reinforcement, Shear strength prediction, Shear span, Analytical equation.

1 INTRODUCTION

Reinforced concrete (RC) flexural members are, almost in all cases, primarily designed for flexure to decide on the cross section and longitudinal reinforcement. However, in the absence of adequate shear reinforcement, members may fail in shear before attaining their flexural capacity. Theories and relationships for predicting shear behavior have been extensively studied; however, consensus over a single theory to predict the response of members under shear does not exist. Although ACI code provisions on shear have been developed on the basis of years of research, yet all significant factors have not been fully incorporated. Major parameters influencing the shear strength of beams have been identified by many researchers as; shear span to depth ratio (a/d), depth of members and shape of cross section, presence of axial force, type and ratio of longitudinal reinforcement and concrete compressive strength. However, ACI (ACI Committee 318-11 2011) equations for shear capacity and minimum shear reinforcement do not take into account all of these factors.

1.1 Zararis Theory of Critical Diagonal Cracks

Zararis (2003) and Zararis and Papadakis (2001) carried out analytical research on shear behavior of RC slender beams with and without shear reinforcement and evolved a theory which describes the diagonal shear failure in slender beams. In this theory, reason for shear failure has been identified as the loss of sheer force of main tension reinforcement, which occurs due to a horizontal splitting of concrete cover along the reinforcement (Zararis 2003). Failure in slender beams without shear reinforcement, loaded under two or single point loading, occurs due to critical diagonal crack which is composed of two distinct branches (Zararis 2003). First branch is an inclined shear crack having height almost similar to flexural cracks. The second branch initiates from the tip of first branch and propagates towards the load point crossing the compression zone, with its line meeting the support point. Figure 1 illustrates the forces acting in the cracked portion of the beam:

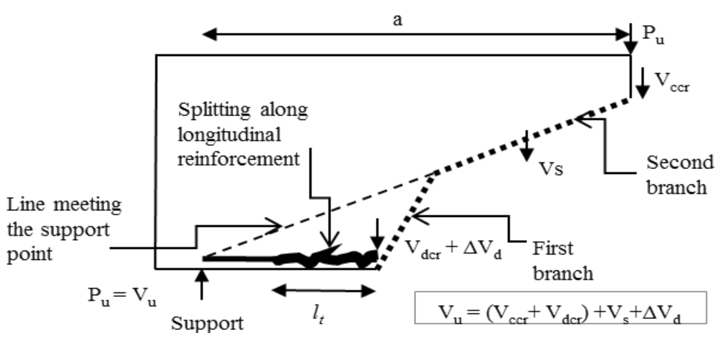

Figure 1. Distribution of forces in beam (based on Zararis theory).

By considering all these forces, ultimate shear force at the failure of a beam with stirrups can be given as under:

$$V_u = V_{cr} + V_s + \Delta V_d \tag{1}$$

The force V_d (in case of beams without stirrups) and ΔV_d (in case of beams with stirrups) are responsible for horizontal splitting between concrete and steel along the main reinforcement. Preventing this splitting can be helpful in avoiding the shear failure. It has been assumed that splitting is caused when the tensile stresses developed along the reinforcement in a distance l_t from the point of initiation of critical crack exceeds the tensile strength, f_t of concrete. It is believed (Zararis 2003, Zararis and Papadakis 2001) that splitting length, l_t has a constant value in any case which is about 0.5d. The equation proposed by Zararis (2003) for ultimate shear capacity (V_u) of beams with stirrups is as under:

$$V_u = [(1.2 - 0.2(\frac{a}{d})d)(\frac{c}{d}) f_{ct} + (0.5 + 0.25(\frac{a}{d}))\rho_v f_{yv}]bd \tag{2}$$

Minimum shear reinforcement should be calculated in such a manner that it satisfies the following relationship:

$$\frac{\rho}{\rho_v} \leq 1.75(\frac{a}{d}) \tag{3}$$

When the condition mentioned in Equation 3 is not met, the shear failure is likely to be accompanied by quick and extensive splitting along longitudinal reinforcement and significant widening of critical diagonal crack (Zararis 2003).

1.2 Proposed Modification to Zararis Theory

In this experimental study, theory developed by Zararis (2003) has been considered being concise and incorporating all major factors influencing the shear strength of RC beams. However, it is believed that splitting length, l_t is linked with the development length, l_d of the bars. This concept is based on the fact that the factors influencing the development length are similar to those linked with the splitting along main reinforcement. According to ACI Committee 318-11 (2011), the development length is influenced by size, location and number of bars, concrete cover, coating on the reinforcement, confining reinforcement, yield strength of steel and compressive strength of concrete. Therefore, instead of relating splitting length, l_t with the depth of beam only, it is felt more appropriate to relate it with a fraction (α) of the development length, l_d. Exact value of this fraction α may be found by experimental studies, however, for the purpose of this research, it was assumed to be 0.25 and was later confirmed during experimental work. By incorporating the above mentioned assumption, Equation 3 becomes:

$$\frac{\rho}{\rho_v} \leq 0.89(\frac{l_d}{d})(\frac{a}{d}) \quad (4)$$

Similarly, using the value of splitting length $l_t = 0.25l_d$, the Zararis equation for predicting ultimate shear strength of reinforced concrete slender beams can be modified as under:

$$V_u = [(1.2 - 0.2(\frac{a}{d})d)(\frac{c}{d}) f_{ct} + 0.25((\frac{l_d}{d}) + (\frac{a}{d}))\rho_v f_{yv}]bd \quad (5)$$

2 EXPERIMENTAL INVESTIGATION

Eight full scale beams were cast and tested to study the shear behavior of RC beams. Concrete strength (f'_c) was selected as 28 N/mm^2 and cross section was 254 mm x 457 mm which was selected so that minimum shear reinforcement requirement is governed. Details of all the beam specimens are given in Table 1.

Table 1. Details of all the beam specimens.

Beams	**f'_c, (N/mm^2)**	**Longitudinal reinforcement**		**Shear reinforcement**		**a/d**	**d, (mm)**
		Rebar No	**f_y, (kN/mm^2)**	**Rebar No**	**f_{yv}, (kN/mm^2)**		
N Series	28	3 # 8	414	-	-	2.5	406
A Series	28	3 # 8	414	# 2 @ 190 mm c/c	276	2.5	406
Z Series	28	3 # 8	414	# 3 @ 152 mm c/c	276	2.5	406
M Series	28	3 # 8	414	# 3 @ 203 mm c/c	276	2.5	406

2.1 Materials

Type I cement conforming to ASTM C 150 – 04 and locally available sand with fineness modulus of sand was calculated to be 2.51. Grade 60 and grade 40 steel was used for longitudinal and transverse reinforcement respectively. Table 2 gives the mix design details used for the beam specimens.

2.2 Test Specimens

Specimens were cast as per ASTM C 31 (ASTM Standard C31 2012). Details of reinforcement for each type of beams are mentioned in Table 1. The specimens were cast in 25 mm thick plywood formwork which was removed from beams after 48 hours.

2.3 Test Setup and Loading

The testing facility of Civil Infrastructure Laboratory at NUST, Islamabad was used. The load was applied through a hydraulic jack having 1200 kN capacity. The supports comprised of 102 mm diameter solid steel bars, making the beam simply supported. A steel girder was used to apply two point loading at predefined shear span of 1016 mm. Three linear variable displacement transducers (LVDTs) were placed under the beams at mid span and at quarter points to measure the deflections. Deflections were also measured and recorded through the structural load analysis and data logging system. The load was applied in increments of 25 kN, deflections recorded and cracks marked at each load increment.

Table 2. Details of concrete mix design

Description	Details
Cement, Type I, ASTM C150	450 kg/m^3
Fine Aggregate	760 kg/m^3
Coarse Aggregate	1040 kg/m^3
W/C ratio	0.41
Superplasticizer (Ultra Superplast 470), ASTM C494	0.9 % by weight of cement
Average compressive strength at 28 days	27.57 N/mm^2
Average compressive strength at test day (90 days)	28.71 N/mm^2

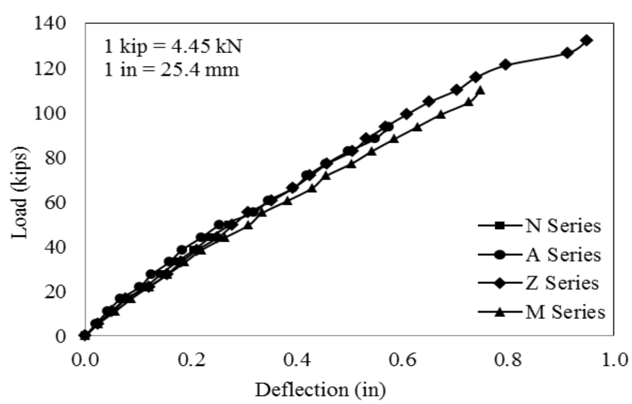

Figure 2. Average load – deflection plot for all beams.

3 EXPERIMENTAL RESULTS

Average load – deflection response of all series of beam specimens is shown in Figure 2. Initial flexural cracks in almost all cases occurred between 75 – 100 kN load. The location of significant diagonal crack was within the shear span. Both beams in N series exhibited sudden failure induced by diagonal cracking at 250 kN. In A-series beams A-1 and A-2, failure occurred at 414 kN and 424 kN respectively. The inclined cracks which lead to failure in Z-series beams appeared at relatively higher loads as compared to the other beams. Failure load was 589 kN and 586 kN for Z-1 and Z-2, respectively. Two distinct branches of the critical diagonal cracks were not witnessed, as was postulated by (Zararis 2003). Beams in M series also behaved much like those in Z-series; however, they failed at lesser loads as their failure occurred at 493 kN and 482 kN for M-1 and M-2, respectively. Since the longitudinal reinforcement and the a/d were kept constant, an increase in failure load and deflections was seen with the increase in amount of shear reinforcement.

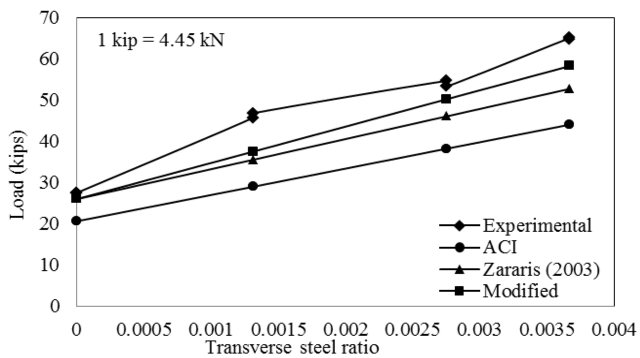

Figure 3. Comparison of shear strength predictions.

Table 3. Variation between experimental and ACI predicted shear strengths.

Beams	Experimental Strength, kN			ACI Predicted Strength, kN			$\gamma = V_u / f'_c$
	V_c	V_s	V_u	V_c	V_s	V_u	
N Series	123	0	123	92	0	92	2.67
A Series	169	37	206	92	37	129	3.67
Z Series	184	105	289	92	105	197	4.00
M Series	162	78	240	92	78	170	3.52

4 ANALYSIS AND DISCUSSION

4.1 Discussion on Shear Strength Calculation

Shear strength for each series of beams was calculated and predicted according to ACI Equation 11.2 (ACI Committee 318-11 2011), Zararis Equation 2 (Zararis 2003) and the modified form of Zararis Equation 5. Experimentally obtained ultimate shear strengths were compared to the theoretical predictions as illustrated in Figure 3. This shows that ACI Equation gives quite conservative results. Shear strength calculation by Zararis equation was claimed to be more accurate (Zararis 2003). However, present experimental study indicates that the modified form of Zararis equation is more appropriate in predicting the ultimate shear capacity of beams. Variation between experimental values and those predicted by ACI code Equation 11.2 is given in Table 3.

4.2 Proposed Modification to Zararis Equations

Equations developed by Zararis were modified to take into account the development length and value of α was assumed to be 0.25. This provided a value of 0.00275 for ρ_v and accordingly the M-Series beams were equipped with this amount of transverse steel. Experimental results revealed a value of α to be 0.21. By incorporating this value, Equations 6 and 7 represent modified Zararis equations as under:

$$\frac{\rho}{\rho_v} \leq 0.75(\frac{l_d}{d})(\frac{a}{d}) \tag{6}$$

and,

$$V_u = [(1.2 - 0.2(\frac{a}{d})d)(\frac{c}{d}) f_{ct} + 0.21((\frac{l_d}{d}) + 0.25(\frac{a}{d}))\rho_v f_{yv}]bd \tag{7}$$

5 CONCLUSIONS

Based on the results of this experimental study, following conclusions are drawn:

1. ACI equation for prediction of shear strength in reinforced concrete beams is more conservative, while Zararis equation appears to be better in predicting the ultimate shear capacity. The proposed modified Zararis equation gives a better shear strength prediction in RC slender beams.
2. Initiation and propagation of shear cracks in general and critical diagonal cracks in particular do not follow the pattern of two distinct branches as was claimed by Zararis. These cracks follow approximately the same orientation on which they initially appeared.
3. Failure in all test beams provided with the shear reinforcement occurred due to excessive diagonal crack widths. Failure due to splitting along longitudinal reinforcement, as assumed by Zararis, was not witnessed.

References

ACI Committee 318-11 (2011). Building code requirements for structural concrete and commentary, American Concrete Institute, Farmington Hills, MI.

ASTM Standard C31 (2012). "Standard Practice for Making and Curing Concrete Test Specimens in the Field." ASTM International, West Conshohocken, PA.

Zararis, P. D. (2003). "Shear strength and minimum shear reinforcement of reinforced concrete slender beams." ACI Structural Journal, 100(2), 203-214.

Zararis, P. D., and Papadakis, G. C. (2001). "Diagonal shear failure and size effect in RC beams without web reinforcement." Journal of Structural Engineering, 127(7), 733-742.