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ON MODELING ACTIVITY CRASHING AND OVERLAPPING: A FIRST ALGORITHM

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Accelerating construction projects is a commonly used method for meeting the project deadlines and/or compensating for current delays. There are several approaches to speed-up a project, such as activity crashing and overlapping. Activity crashing means reducing activity durations through adding more resources. Activity overlapping means executing certain activities in parallel when they were supposed to be sequential in the original plan. Construction management literature is mostly focused on studying each acceleration mode separately, while the focal point of this paper is to develop a joint model and a first algorithmic implementation that involves both acceleration methods. Particularly, on reviewing the project management and scheduling literature, a mathematical model combining activity crashing and overlapping is reformulated. Also, a Genetic Algorithm is implemented on a fictitious case study. Preliminary findings of the model and algorithmic implementations identify that activity crashing alone and a mixed approach are preferred when significant compression is required, whereas activity overlapping is recommended for minimizing compression costs when only a small compression is required.

Keywords: Compression, Acceleration, Delay, Schedule, Stochastic, Productivity, Risk.

1 RESEARCH METHODOLOGY

Many construction projects suffer delays (Ballesteros-Pérez *et al.* 2015, 2016). One of the most common approaches used by project managers to deal with those delays and meet the deadlines is project acceleration (expedition) (Baker 1991). Haga (1998) defined project acceleration (schedule compression) by “shortening the normal duration of the project schedule without reducing the original scope of work.”

There are three methods for project acceleration: activity crashing, activity overlapping (fast-tracking) and activity substitution (Eduardo *et al.* 2008). Crashing is a method for reducing the project duration for the least possible cost through increasing the number of resources. The most common approaches to crash an activity are to add more laborers, use multiple shifts, offer overtime and add more resources; which will incur more cost (Baker 1991, Eduardo *et al.* 2008, Sahu and Sahu 2014). Activity overlapping involves starting the activity earlier than its planned start date so that it would commence before its predecessor(s) has finished; that is, converting some sequential activities into parallel ones (Meier *et al.* 2015). Similarly, this would require additional costs due to inevitable rework. This paper will deal simultaneously with both methods.

Based on the previous models found in the literature, a new merged model will be created and a Genetic Algorithm (GA) adopted due to the ability of this evolutionary technique to provide near-optimal and quick solutions. Furthermore, a case study of fictitious construction project will also be examined.

Crashing was the first approach to be studied and discussed in the context of project management (Eduardo *et al.* 2008). The most popular method for crashing PERT/CPM networks is based on the average duration of the activities overlooking any uncertainties that may lead to higher durations (Haga 1998, Sahu and Sahu 2014). One common approach to solve crashing problems is to use linear programming which was introduced by Fulkerson (1961). Also, goal programming was found to be quite efficient too when dealing with secondary objectives (Moore *et al.* 1978). The crashing part of the model proposed here will follow a deterministic approach with a linear relationship between duration decrement and cost increment similar to the overwhelming majority of models found in the literature.

Activity overlapping has been a subject for many researches since the early 1990s. Krishnan *et al*. (1997) were the first to introduce a model that explains how activity overlapping works. Terwiesch and Loch (1999) studied the effectiveness of activity overlapping on product development. Xiao and Si (2003) introduced an approach to calculate the total execution times of the upstream and downstream activities. Wang and Lin (2009) pointed out the importance of the process structure for more efficient overlapping using simulation algorithm similar to the approach followed in this paper. Following the research done by Bogus *et al*. (2011) and Ballesteros-Pérez (2017), this paper employed Monto Carlo simulations to calculate the average compression in the overlapping operations.

Although there is a need to study the two acceleration modes simultaneously, there are very few papers that have already handled this topic. Roemer and Ahmadi (2004) studied both crashing and overlapping concurrently in the modeling framework for the first time in the literature. Their model was based on an upstream and downstream approach that was suggested by Krishnan *et al*. (1997). Moreover, they studied the impact of crashing and overlapping solely and jointly on a multi-stage example. Eduardo *et al.* (2008) conducted a significant set of research that is relevant to this paper, yet their scope is much wider; they focused on combining the three modes of project acceleration. However, the two significant models neglected the stochastic nature of the activity duration in their study which is taken care of in this paper. In other words, the overlapping part was stochastically calculated while the deterministic approach was used to calculate the crashing part of the model.

2 MODEL FORMULATION

2.1 Overlapping Model

Assume two activities: a predecessor activity (g) whose duration is d_g and its cost is C_g; and a successor activity (a) whose duration is d_a and its cost is C_a. To initiate the successor activity, two types of costs may be incurred: the upfront cost C_{ai} and the operational cost C_{aj}. The overlap that would occur between the two activities can be denoted by o. The delay can be represented by k and the float is denoted by f. By its very definition, fast-tracking requires the successor activity to be brought forward by a certain number of time units u so that it would start earlier than originally planned. The actual overlap would start when the activity's float vanishes, such that ($u > f$) and the difference between them is the overlap, such that ($o = u - f$). The occurrence of fast-tracking may incur some costs. Eq. (1) represents the average incremental cost Δc_a.

$$\Delta c_a = Risk(R) \times IMPACT(I) = \left(\frac{u-f}{d_g+k}\right)^{\alpha} \cdot \left(C_{ai} + \left(\frac{u-f}{d_a}\right)^{\beta} C_a\right) = \left(\frac{o}{d_g+k}\right)^{\alpha} \cdot \left(C_{ai} + \left(\frac{o}{d_a}\right)^{\beta} C_a\right) \quad (1)$$

when ($\Delta c_a \leq C_a$), ($0 \leq \alpha$), ($\beta \leq +\infty$) and $0 \leq o \leq MIN\ (d_a, d_g + k)$.

The predecessor's sensitivity is represented by the variable α, while variable β represents how quickly C_a is being spent while the successor is being executed. The model developed here assumes a case of probability which can be measured by multiplying the risk by the impact, as per Eq. (1). The proportional relationship between the risk (R), and overlap o suggests that the risk would be 0% when the overlap is zero, and the risk would be 100% when the overlap is maximal. The impact (I) is proportional to that amount of money (cost) allocated to the portion of the activity that would be overlapped. Also, it can be assumed for many construction projects on average conditions that there would be no upfront costs and both variables α and β are linear (i.e., $\alpha = \beta = 1$). This scenario is described by Eq. (2).

$$\Delta c_a = (R) \times (I) = \left(\frac{u-f}{d_g+k}\right) \cdot \left(\frac{u-f}{d_a} \cdot C_a\right) = \frac{C_a \cdot (u-f)^2}{d_a \cdot (d_g+k)} = \frac{C_a \cdot o^2}{d_a \cdot (d_g+k)} \quad (2)$$

when there is more than one predecessor for a certain successor (a), this scenario can be calculated using Eq. (3).

$$\Delta c_a = (R) \times (I) = \left(1 - \prod_{i=1}^{n} \left(1 - \left(\frac{o_i}{d_{g_i} + k_i}\right)^{\alpha}\right)\right) \cdot \left(C_{ai} + \left(\frac{o_{\max}}{d_a}\right)^{\beta} C_a\right) \quad (3)$$

with ($\Delta c_s \leq C_a$), ($0 \leq \alpha$), ($\beta \leq +\infty$) and $0 \leq o_{max} = \text{MAX}\ o_i \leq MIN\ (d_s, d_g + k)$

The number of days that a successor can be brought forward u can be calculated such that ($u = o + f$). The main concern now is to calculate the average effective amount of time u_{av} by which a successor activity can be brought forward using Eq. (4).

$$u_{av} = f \cdot Risk + (o + f) \cdot (1 - Risk) = f + o \cdot (1 - Risk) = f + o \cdot \left(1 - \left(\frac{o}{d_g + k}\right)^{\alpha}\right); u \geq f \quad (4)$$

The overall effective (expected) overlapping can be calculated for the project through the summation of the effective overlaps u_e of critical activities which have zero float (lies on the critical path CP). This can be summarized by equation (5).

$$u_e = \sum_{i \in CP} o_i \times (1 - Risk_i) \quad (5)$$

2.2 Crashing Model

As noted earlier, when it comes to crashing, the relationship between time and cost is deemed to be linear following previous models like Moore *et al*. (1978). Let us assume there is an activity (a); its normal duration, which is a subjective input by the user, is d_a^N, crashed duration is d_a^C, normal cost is C_a^N and the crashed cost is C_a^C. Having calculated the number of crashing days Cr, the crashed duration can be calculated using Eq. (6).

$$d_a^c = d_a^N - Cr \tag{6}$$

when $Cr_{min} \leq Cr \leq Cr_{max}$

Having calculated the crashed duration of activity (a), the actual crashed cost $C_a{}^C$ can be calculated using Eq. (7). Similar to the normal duration, the normal cost of each activity as well as the maximum crashed cost are subjective inputs by the user.

$$C_a^C = v_a \cdot Cr = v_a \cdot (d_a^N - d_a^C) \rightarrow \quad v_a = \frac{C_a^C - C_a^N}{d_a^N - d_a^C} \tag{7}$$

2.3 Concurrent Crashing and Overlapping

Having calculated the number of crashed days Cr, and the average effective amount of time u_{av}, the total project acceleration Ac can be calculated using Eq. (8).

$$Ac = u_{av} + Cr = f \cdot Risk + (o + f) \cdot (1 - Risk) + (d_a^N - d_a^C) \tag{8}$$

The additional total cost C_T incurred by an activity due to both crashing and fast-tracking can be calculated using equation (9).

$$C_T = \left(1 - \prod_{i=1}^{n} \left(1 - \left(\frac{o_i}{d_g + k_i} \right)^a \right) \right) \cdot \left(C_{ai} + \left(\frac{o_{\max}}{d_a} \right)^b C_a \right) + \left(v_a \cdot (d_a^N - d_a^C) \right) \tag{9}$$

2.4 Case Study

The two most important parameters that are being measured by the proposed model are the maximum acceleration that can be achieved and the additional cost associated with this compression. Thus, the mathematical model was applied to a case study consisting of 12 activities, a total duration of 100 days, and a total cost of $1000 as shown in Figure 1.

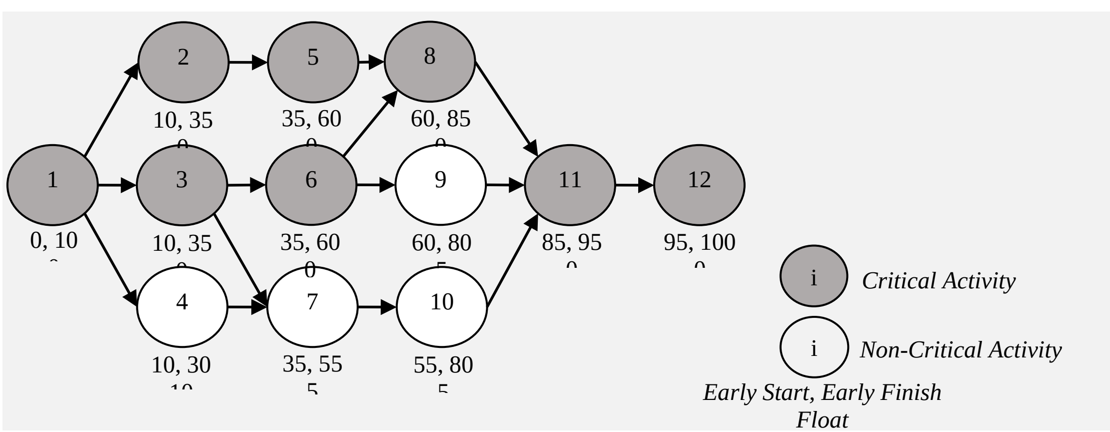

Figure 1. Network diagram for the case study.

A Genetic Algorithm (GA) model was developed using Excel Solver, as based on the mathematical models proposed earlier in the methodology section. The inputs for this model is shown in Table 1. The user shall decide on the crashed cost per day and the crashing limits which are assumed here to be 25% of the normal duration. Also, the amount of overlap allowed would be limited to 25%, which corresponds to a maximum risk of 25%.

Table 1. Input parameters for the Genetic Algorithm model.

ID	**Inputs**													
	Duration			***Cost (c)***			***Relationships***				***Model Limits***			
	ND	CD	Slack	Cost (c)	CC	CC/day	IPs	IPs durations			Crashing		Overlapping	
	d_a^N	d_a^C	f	C_a^N	C_a^C	v_a	g_i	d_g			Cr_{min}	Cr_{max}	o_{min}	o_{max}
1	10	8.20	CA	50	63	5.00	-	-			0	2.5	-	-
2	25	18.76	CA	50	63	2.00	1	8.2			0	6.3	0	4.1
3	25	18.81	CA	100	125	4.00	1	8.2			0	6.3	0	4.1
4	20	16.51	5	50	63	2.50	1	8.2			0	5.0	0	4.1
5	25	18.84	CA	100	125	4.00	2	18.8			0	6.3	0	9.4
6	25	18.84	CA	100	125	4.00	3	18.8			0	6.3	0	9.4
7	20	15.11	5	50	63	2.50	3,4	18.8	16.5		0	5.0	0	5.1
8	25	18.80	CA	100	125	4.00	5,6	18.8	18.8		0	6.3	0	5.5
9	20	15.59	5	50	63	2.50	6	18.8			0	5.0	0	9.4
10	25	18.80	5	100	125	4.00	7	15.1			0	6.3	0	7.6
11	10	7.51	CA	100	125	10.00	8,9,10	18.8	15.6	18.8	0	2.5	0	3.6
12	5	3.76	CA	150	188	30.00	11	7.5			0	1.3	0	3.8

ND: Normal Duration CD: Crashed Duration CC: Crashed Cost IP: Intermediate Predecessor

Maximum crashing and maximum overlapping were then calculated as shown in Table 2. Also, Monte Carlo simulation was utilized; the values were simulated 10,000 times. The costs were calculated using the mathematical model explained earlier.

Table 2. Output parameters for the Genetic Algorithm model.

Activity ID	**Outputs**					
	Optimum Crashing	***Active overlaps***	***Active overlaps cost***	***Actual crashing cost***	***Overlap Risk***	***Effective overlap***
	(Solution)	***(Solution)***	Eq. (3)	Eq. (7)	Eq. (3)	Eq. (5)
	Cr	o	(ΔCa)	C_a^C	R	u_e
1	**1.80**	-	-		-	-
2	***6.24***	***3.76***	4.58	12.47	0.46	2.04
3	***6.19***	***3.87***	9.69	24.78	0.47	2.04
4	***3.49***	***2.46***	2.23	8.74	0.30	1.72
5	***6.16***	***5.89***	9.82	24.64	0.31	4.04
6	***6.16***	***5.48***	8.47	24.65	0.29	3.88
7	***4.89***	***3.41***	3.95	12.24	0.35	2.21
8	***6.20***	***4.07***	8.34	24.80	0.39	2.50
9	***4.41***	***5.32***	4.81	11.04	0.28	3.82
10	***6.20***	***5.13***	9.26	24.78	0.34	3.39
11	***2.49***	***3.42***	21.71	24.86	0.48	1.79
12	***1.24***	***3.76***	74.94	37.19	0.50	1.88

The results summarized in Table 3 show that at the same limits, activity fast-tracking led to lower compression than activity crashing yet cheaper. It can be said that fast-tracking is preferred for low-to-moderate delays while activity crashing and the concurrent approach is preferred for extreme delays. Also, a combination of the two approaches shall be considered.

Table 3. Outcome summary Table.

Initial project cost	$1,000.0	Effective crashing (***Cr***); Eq. (6)	24.1
Total overlap cost	$157.8	Effective overlapping $\boldsymbol{u_e}$;(Deterministic); Eq. (5)	12.1
Total crashing cost	$230.2	Effective acceleration $\boldsymbol{Ac_e}$ (Deterministic)	**36.2%**
Compression cost (***Ct***); Eq. (9)	$388	Average acceleration actually achieved ***Ac***; Eq. (8)	32.1
Percentage of cost increase	**39%**	**(Average from project duration simulations)**	**32.1%**

3 CONCLUSIONS

The model proposed in this paper followed previous crashing and fast-tracking models, and it combined them in one model that is based on both deterministic and stochastic approaches. One advantage of this model is its ability to calculate the total compression achieved, as well as extra costs that result from using the two scheduling acceleration modes either separately or jointly. Also, the model not only considered the case of the single predecessor, but also the multi-predecessor scenarios.

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