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YIELD STRESS BASED ON NATURAL STRAIN THEORY UNDER CYCLIC SHEAR LOAD AFTER LARGE SIMPLE SHEAR

YASUYUKI KATO

Dept of Mechanical Engineering, College of Science and Technology, Nihon University, Tokyo, Japan

This paper describes the variation of yield stress under cyclic loads after applying large pre-deformation. In this research, the Natural Strain is used as a strain representation for describing an elastoplastic behavior under large deformation. Using test pieces which is already given a pre-deformation of large simple shear, the yield stress under cyclic shear loads in the forward direction and the reverse direction is examined. As for the method for estimating yield stress in each cycle, the slope of a tangent in the principal deviatoric stress and the principal deviatoric strain curve is adopted. The experiments are conducted with the different sizes of pre-deformation and strain amplitude in order to reveal the effect of pre-deformation on the yield phenomenon. Then, these experimental results are compared with the results of yield stress in the previous study, which is obtained under cyclic load for tension and compression after applying the large uniaxial tension.

Keywords: Elastoplastic behavior, Finite strain, Cyclic loading test, Slope of tangent, Ductile materials, Anisotropy, Strain amplitude, Proof stress.

1 INTRODUCTION

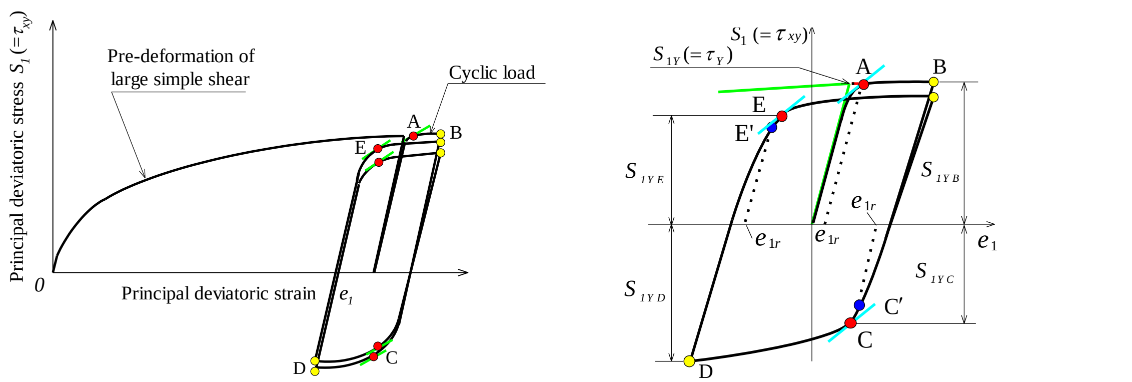
The purpose of this research is to reveal the yield phenomenon under large deformation on the basis of the Natural Strain theory. The Natural Strain is obtained by integrating an infinitesimal strain increment on an identical line element over the whole process of deformation path. Consequently, the shearing strain component is represented by a pure angular strain and it has the merit that can be accurately removed the rigid body rotation from the rotating angle of a line element. Moreover, since the additive law of strain on an identical line element can be satisfied, the strain rate can be clearly decomposed into the elastic component and the plastic component in the same manner as conventional infinitesimal deformation theory (Kato and Nishimura 1996). Therefore, the Natural Strain is effective strain representation which can systematically treat from infinitesimal deformation to the large deformation. In a series of our previous studies (Kato and Moriguchi 2003, Kato and Kazama 2013, Kato 2014, Kato 2016, Kato 2017), the anisotropy of yield surface generated under large deformation has been investigated. In general, when the structure undergoes intense repeated loads due to earthquake etc., a structural plastic failure arises even if the number of cycles is small. Therefore, it is an important issue for structural engineering to clarify the actual mechanism of low-cycle fatigue failure and the yield behavior under cyclic loads. Although the researches on the yield behavior under cyclic loads have been conducted for many years, many of these studies have been done within the range of an infinitesimal deformation. Hence, the detailed studies on the yield behavior under cyclic loads after applying a large pre-deformation have not been fully elucidated. In the case of ductile materials, the yield stress generated after applying large pre-deformation considerably increases as compared with the value of initial yield stress. Moreover, since the anisotropy in material is formed with an increase of deformation, the value of yield stress at pre-deformation side is different from the value of yield stress at opposite side because of the Bauschinger effect. In this research, the variations of yield stress with an increase of cycles when the cyclic load is applied with constant strain amplitude after giving a large pre-deformation is examined. The values of yield stress at pre-deformation side and opposite side gradually decrease and approach the constant value with an increase of cycles because of the effect of material anisotropy reduces with an increase of cycles.

In our recent study (Kato 2016), using the test pieces made from high-purity tough pitch copper, the yield behavior under cyclic loads of tension and compression with the constant strain amplitude after applying a large pre-deformation of uniaxial tension has been examined. As for the estimation method for determining the yield stress, the slope of tangent in the deviatoric stress and strain curves has been used instead of the estimation method by conventional proof stress.

Moreover, in this study, focusing on the pre-deformation of large simple shear that the direction of the principal axis rotate and the line element of it also replace, the yield stress under cyclic shear loads in the forward direction and the reverse direction is examined. Then, in order to reveal the effect of pre-deformation on the yield phenomenon under shear deformation, the experiments are conducted with the different sizes of pre-deformation and strain amplitude.

2 METHOD FOR ESTIMATION OF YIELD STRESS UNDER CYCLIC SHEAR LOAD

Figure 1 (a) shows a deviatoric stress and deviatoric strain diagram when the simple shear in the forward direction and the reverse direction are subjected alternately to the test pieces after applying a pre-deformation of large simple shear. Here, this figure shows the principal deviatoric stress and principal deviatoric strain diagram, however, in the case of simple shear, the principal deviatotic stress S_1 coincide with the shear stress τ_{xy}. Moreover, Figure 1 (b) shows an enlarged view of the first cycle for the principal deviatoric stress and the principal deviatoric strain diagram. In this figure, as shown in point A, the value of yield stress when the shear load is applied again is already determined from the final yield state of pre-deformation of simple shear. In the neighborhood of the yield point A, the principal deviatoric stress and principal deviatoric strain curve becomes a shallow curve, and in this study, it is formulated as follows.



(a) Cyclic loads of simple share after large pre-deformation (b) Method for determination of yield stress

Figure 1. Method for determination of yield stress in the cyclic shear loads after large simple shear.

$$S_1 = a\left(1 - exp\left(b e_1\right)\right) + c e_1 + d \tag{1}$$

However, *a*, *b*, *c* and *d* are coefficients determined by using the *Levenberg-Marquardt Method*, which is one of the non-linear least-squares methods. Furthermore, the slope of tangent is also obtained by differentiating Eq. (1), and it is represented by Eq. (2).

$$\frac{d S_1}{d e_1} = -a b \, exp\left(b e_1\right) + c \tag{2}$$

Hence, the slope of tangent at yield point A can be derived by Eq. (2). Therefore, the slope of tangent at yielding can be specified in advance and it is indicated by strait blue line in Figure1 (b).

As for the yield stress in a cycle, firstly, the yield stress obtained after applying the shear deformation in the forward direction with specified constant strain amplitude is represented by a point B just before unloading of shear. Secondarily, the yield stress during reverse shear load is obtained by representing the gentle curve of stress by using the experimental equation (1). Then, the stress, which has the same value as the slope of tangent measured at the point A, i.e., the stress at point C, is assumed to be a yield stress. Thirdly, the yield stress obtained after applying the reverse shear strain with specified constant strain amplitude is represented by a point D. Finally, the yield stress at reverse shear side, which is derived by applying a shear load in forward direction again after unloading, is obtained by representing the gentle curve of principal deviatoric-stress by using Eq. (1) once again. And, the stress, which has the same value as the slope of tangent measured at the point A, i.e., the stress at point E, is assumed to be a yield stress.

On the other hand, the yield stress by proof stress is determined by using the value of the residual strain e_{1r} that is obtained by unloading from a point A. Hence, as shown in point C' and E', the proof stress is estimated smaller as compared with the estimated yield stress in this study.

3 EXPERIMENTAL METHOD

In the experiments, the cylindrical specimens, i.e. outer diameter 22[mm], inner diameter 16[mm] and gauge length 30[mm], are used. In order to apply the per-deformation of large simple shear, as for the material quality of specimens, the tough pitch copper, i.e., purity 99.99%, are adopted.

Next, as for the experimental condition, this experiment is composed of two stages. Namely, one is the experiments for applying the pre-deformation of large simple shear. Another one is the cyclic loading tests for the shear loading in the forward direction and the reverse direction.

(1) Experimental condition of pre-deformation of large simple shear

Four different types of shear deformation, namely, values of slip k = 0.37, 0.53, 0.98, 1.37 [-], i.e., values of the principal stretch λ = 1.2, 1.3, 1.6, 1.9 [-], are applied to the test pieces.

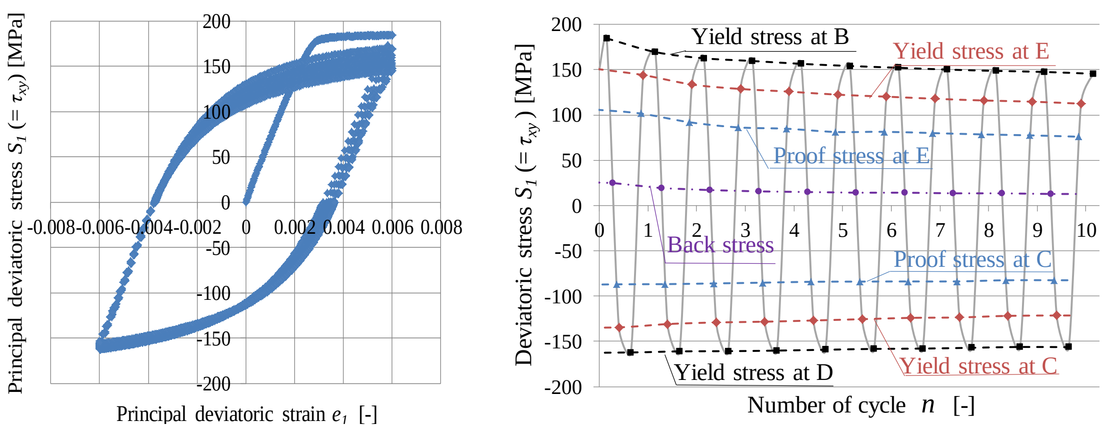
(2) Experimental condition of cyclic shear loads

After attaching the strain gauges to the test pieces, the experiments of the cyclic shear loading tests in the forward and the reverse direction are conducted with the constant strain amplitude, i.e., $\triangle e_1 = \pm 0.006$ [-], and in these experiments, the numbers of cycles n are all ten times. Then, the values of yield stress at points B, C, D and E in Figure.1 are determined, and the changes of yield stress with an increase of the number of cycle n are investigated. Moreover, as for the values of the principal stretch λ =1.2 and λ =1.3 [-], low-strain amplitude, i.e., $\triangle e_1 = \pm 0.003$ [-], is also examined in this research.

4 EXPERIMENTAL RESULTS

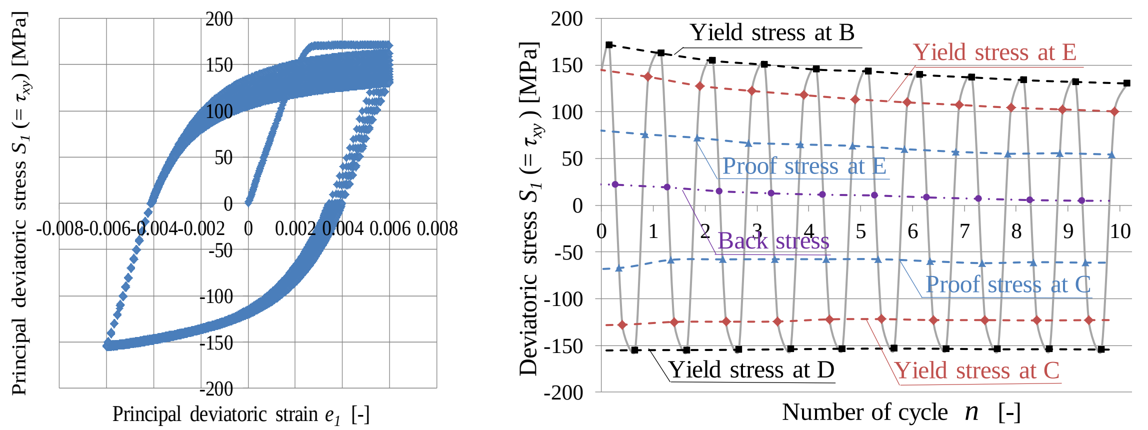
Figure 2, 3 and 4 show the experimental results obtained under cyclic shear loading tests after applying the pre-deformation of large simple shear with respect to the constant strain amplitude (the principal strain ⊿e_1 = ±0.006 [-]). In these figures, (a) is the deviatoric stress and deviatoric strain diagrams and (b) shows the variation of yield stress with an increase of the number of cycles, respectively. In these figures, Figure 2 shows the experimental results of the largest principal stretch among them, i.e., λ=1.9[-]. As obviously from this figure, the yield stress at the reverse shear side and the forward shear side, i.e., yield stress at C and D (see $S_{1\mathrm{YC}}$ and $S_{1\mathrm{YD}}$ in Figure 1 (b)) and yield stress at E and B (see $S_{1\mathrm{YE}}$ and $S_{1\mathrm{YB}}$ in Figure 1 (b)) have a decreasing tendency as number of cycles increase (see red or black broken lines). It can be seen from this figure that the decreasing tendency of yield stresses at reverse shear side, namely yield stress at point C and D, is small as compared with the forward shear side. Therefore, the back stress also decreases and approaches zero with an increase of number of cycle. On the other hand, results by conventional proof stress, namely, yield stresses at point C' and E', are represented by blue lines in Figure 2 (b). Hence, the estimated yield stresses at point C and E, namely, red curves, become large as compared with the results by proof stress. Furthermore, in these figures, Figure 4 shows the result of the smallest principal stretch, i.e., λ=1.2[-]. It is found from this figure that the decreasing tendency of yield stresses at E and B is small as compared with the results in Figure 2. Next, Figure 5 represents the experimental result of small strain amplitude (the principal stretch λ =1.2 [-], principal strain ⊿e_1=±0.003 [-]). In the case of small strain amplitude, the decreasing tendency of yield stress at the reverse shear side**,** namely yield stress at point C and D**,** slightly increases and approaches to the constant value as the number of cycle increase.

Lastly, Figure 6 shows the experimental result of cyclic load for tension and compression after applying the pre-deformation of uniaxial tension. The yield stress in each position in the cycle gradually decreases with an increase of the number of cycle. However, the decreasing tendency of the yield stresses at tension side, namely yield stress at point B and E, are small as compared with the yield stress in the case of shear deformation.



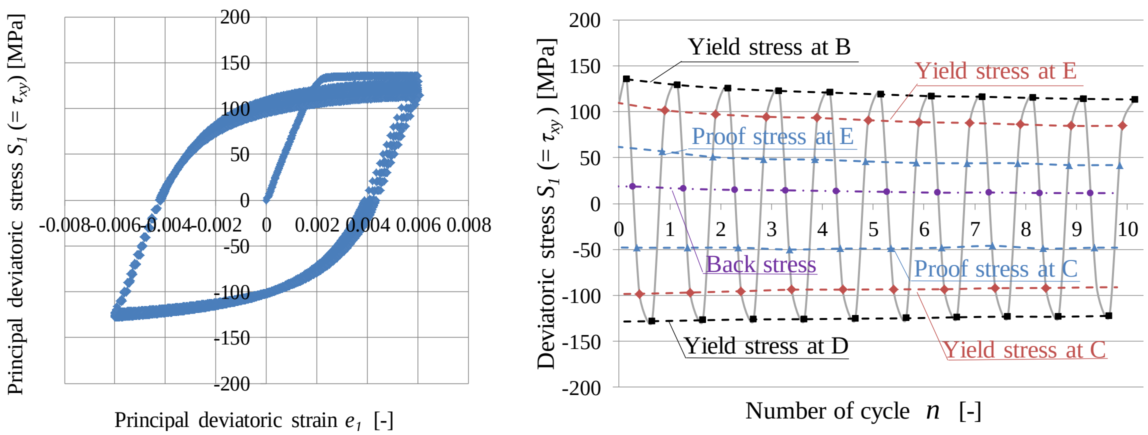
(a) Deviatoric stress and deviatoric strain (b) Variation in yield stress with the number of cycles

Figure 2. Variation of yield stress in cyclic loading for simple shear (λ=1.9 , ⊿e_1 = 0.006).



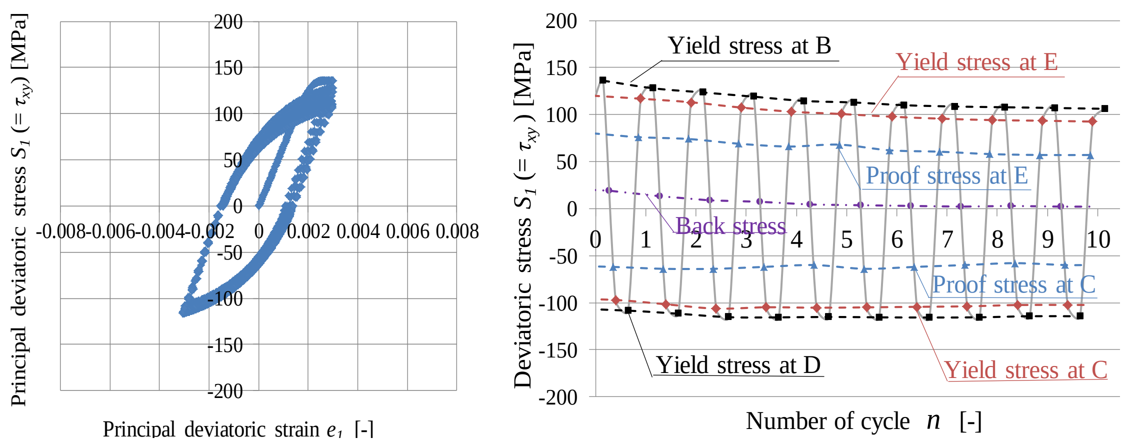
(a) Deviatoric stress and deviatoric strain (b) Variation in yield stress with the number of cycles

Figure 3. Variation of yield stress in cyclic loading for simple shear (λ=1.6 , $\triangle e_1$ = 0.006).



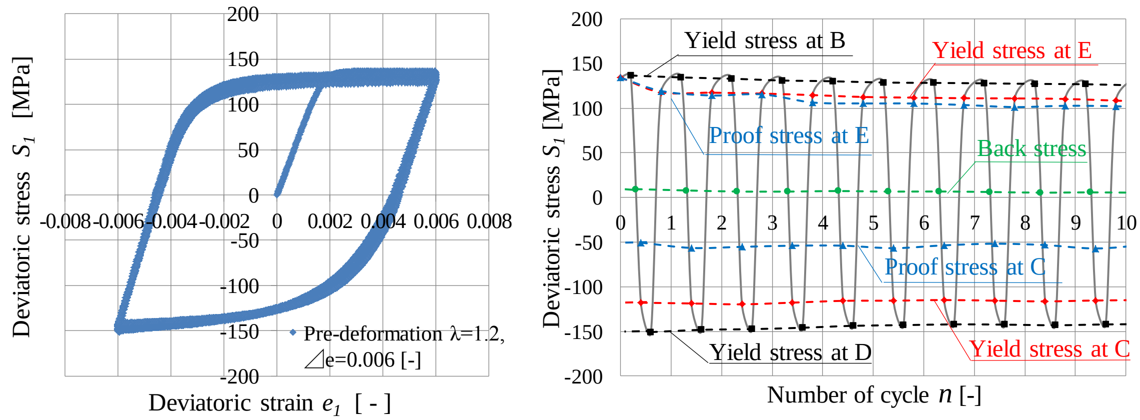
(a) Deviatoric stress and deviatoric strain (b) Varia tion in yield stress with the number of cycles

Figure 4. Variation of yield stress in cyclic loading for simple shear (λ=1.2 , $\triangle e_1$ = 0.006).



(a) Deviatoric stress and deviatoric strain (b) Variation in yield stress with the number of cycles

Figure 5. Variation of yield stress in cyclic loading for simple shear (λ=1.2 , $\triangle e_1$ = 0.003).



(a) Deviatoric stress and deviatoric strain (b) Variation in yield stress with the number of cycles

Figure 6. Variation of yield stress in cyclic loading for uniaxial tension (λ=1.2 , $⊿e_1$ = 0.006).

5 CONCLUSIONS

In this study, using the test pieces, which was already applied the large simple shear, the yield stress under cyclic shear load was estimated. As a result, the following conclusions are obtained:

(1) The yield stresses proposed in this study estimated by using the slope of a tangent have a decreasing tendency with number of cycle increase. However, the decreasing tendency reduces as the number of cycles increases, and they approach to the constant value.

(2) In the case of the large pre-deformation of simple shear, the decrease tendency of the yield stress at the forward shear side becomes larger as compared with the yield stress at the reverse shear side that is the opposite direction to the pre-deformation.

(3) In case of small strain amplitude, the yield stress at the reverse shear side slightly increases and approaches to the constant value as the number of cycle increase.

(4) The yield stress by proof stress is small as compared with the yield stress estimated in this study at both the forward shear side and the reverse shear side.

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