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ON THE MODULUS OF SUBGRADE REACTION FOR SHALLOW FOUNDATIONS ON HOMOGENOUS OR STRATIFIED MEDIUMS

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As known, the modulus of subgrade reaction of soil, k_s, is an essential parameter in designing flexible, shallow foundations based on the Winkler spring hypothesis, where, the foundation soil is modeled as a series of independent (elastic) springs having constant k_s. In this paper the various methods for calculating the k_s value are discussed, indicating that the more suitable one is Vesic's $k_s=0.65[EB^4/(E_bI_b)]^{1/12}E/[B(1–\text{v}^2)]$, where, E and v are the elastic constants of soil and E_b, I_b and B are the modulus of elasticity of the foundation material, the moment of inertial of the cross-section of the foundation and the foundation width respectively. In addition, it is recommended that, the proper soil modulus value is the one corresponding to v=0 for consistency with the deformation pattern of Winkler's springs (compression with no lateral deformation). In this respect, the author offers an effective method for calculating the equivalent elastic constants (E_{eq}, v_{eq}) for horizontally stratified soil mediums supporting shallow foundations. The same method can also be applied to reducing any homogenous (E, v) soil medium to an equivalent one having v_{eq}=0 and modulus E_{eq}.

Keywords: Winkler spring model, Mat foundations, Strip foundations, Flexible foundations, Equivalent elastic constants, Hetényi's model, Pasternak's model, Kerr's model, Elastic settlement analysis, Steinbrenner's elastic settlement solution.

1 INTRODUCTION

The modulus of subgrade reaction, k_s, is an essential parameter in a Winkler's spring type of analysis of shallow foundations, mainly flexible strip and mat foundations. In this type of analysis, an array of springs replaces the soil medium below the foundation; in this respect, k_s plays the role of the constant of springs. Apparently, in either homogenous or heterogeneous mediums, this value should effectively reflect the deformability of the medium examined. When plate-bearing test data are used, the basic equation for k_s is:

$$k_s = q/\delta \tag{1}$$

where, q is the load and δ is the respective displacement of the bearing plate (Eq. (1) is also the definition of k_s). It is apparent that, this value better represents homogenous soil mediums, supposing of course that the proper corrections have been applied for both the size and the shape of the footing (see Terzaghi *et al.* 1996). The influence depth of real-size footings, in practice, is often extended to soil strata with considerably different soil moduli as well as different groundwater conditions; thus, the use of Eq. (1) should be used with great caution. According to

Terzaghi *et al.* (1996) the influence depth of a BxL footing is z_I=2B(1+log$L$/B), meaning that z_I for a square loading plate of edge 0.3 m is only 0.6 m.

In addition to that the fact that k_s should effectively consider the stiffness of any soil layer being within the influence depth of footing, the k_s value should consider the stiffness of footing itself, the presence of groundwater or a possible groundwater rise in the influence zone (Pantelidis 2020a) and the fact that the modulus of soil is affected by both the shape of footing and the location on the plan-view of footing (Pantelidis 2019b). An overview of the most popular methods for calculating k_s is given in the section below.

2 SELECTING THE PROPER METHOD FOR CALCULATING k_s

A first alternative to the method relying on the plate-bearing test (recall Eq. (1)) is the use of the elastic theory (i.e. Steinbrenner's 1934 or Harr's 1966 solution). However, the latter, although suitable for the calculation of settlement of footings on multilayer soil systems applying the principle of superposition, does not take into account the rigidity of footings. The basic equation adopting Steinbrenner's formulations has as follows in Eq. (2) (Bowles 1996):

$$k_s = q/\delta = E\Big/\left[a'B'(1-\nu^2)I_s I_E\right] \tag{2}$$

where, α'=4 and B'=B/2 for the center of foundation (α'=1 and B'=B for the corner), I_s is the soil stratum thickness factor while I_E the embedment depth factor. This approach can also take into account the spatial variation of k_s over the plan-view of footing.

A second alternative is the k_s value to be calculated from the allowable bearing stress over the corresponding settlement, however, this is essentially the same in concept with the procedure mentioned immediately above. As any widely acceptable elastic settlement analysis method can be adopted (EN 1997-1 2004), probably one of the most attractive choices to practitioners is the Mayne and Poulos' (1999) method, as it considers a number of factors, including the rigidity of footing. However, Mayne and Poulos' method returns overpredicted settlement values. Indicatively, assuming a footing with B=1 m and L=10 m over a medium with constant E with depth and ν=0.1, the Mayne and Poulos' method gives $\delta E/Bq\approx3.5$ for the center of the footing, if the latter is flexible, and approximately equal to 2.7 if the same footing is rigid; both values are much greater compared to those predicted from the theory of elasticity. Relatively, $\delta E/Bq$=2.5 using Harr's (1966) solution and 2.0 using the "characteristic point" concept (Kany 1974) (the settlement at the so-called "characteristic point" is considered to be the same as the settlement of the footing if the latter is assumed rigid). The inconsistency with the theory of elasticity (i.e. Harr's 1966 solution) increases as the L/B ratio increases and it is already quite visible for L/B ratios as low as 3 (see comparison offered by Mayne and Poulos' 1999).

A third alternative is the k_s expression derived from Vesic (1961). Vesic showed that for any beam of infinite length on elastic semi-space, the Winkler's hypothesis is valid, where, any such problem can be treated with reasonable accuracy using the following expression for the modulus of subgrade reaction per unit width of beam:

$$k_\infty = 0.65\sqrt[12]{\frac{EB^4}{E_b I_b}}\frac{E}{B(1-\nu^2)} \tag{3}$$

where, E_bI_b and B are the stiffness and the width of the beam and E and ν are the two elastic constants of soil. Eq. (3) can also be used for calculating the k_s value of beams with λL>2.25, where, λ=[k_sB/(4E_bI_b)]$^{1/4}$ (Vesic 1961). An important observation is that, Eq. (3) considers the rigidity of footing but, as it has been offered, it stands only for homogenous mediums. Thus, for stratified soils, the original soil system must be replaced by an equivalent homogenous medium having elastic constants E_{eq} and v_{eq}. The behavior of this equivalent medium must be identical to the original stratified one under the same loading conditions. Indeed, for compatibility reasons with the Winkler type of analysis (vertical displacement with no lateral deflection), the E_{eq} value should correspond to a Poisson's ratio value for soil equal to zero, i.e. v_{eq}=0.

Kerr (1965), in turn, considered two rows of springs, one over the other, combining Winkler's (or Hetényi's 1950) model with Pasternak's (1954) model (see Figure 1). Both Pasternak's and Hetényi's models aim at considering the shear interaction between the individual springs. To this effect, additional mechanical elements was introduced in these models so that the springs to be interconnected. More specifically, for considering the shear interaction between the springs, Pasternak used, in addition to the spring constant (k_s), the shear modulus of soil (G). In this respect, a beam or plate which h deforms only by transverse shear is used for connecting the ends of the springs (see Figure 1). Regarding Hetényi's model, a bending beam or plate having stiffness D is assumed over the springs. Despite of the increased complexity of Kerr's model, however, there is not convincing evidences in the literature that it simulates in an effective manner the behavior of a footing over a two-layer soil system. In addition, Kerr's model ignores the thickness of the two soil layers and thus, great error may be introduced in the analysis as the actual magnitude of loads reaching the lower soil layer is greatly affected by the thickness of the upper layer. Thus, the author sees no advantage of Kerr's model over Winkler's (or Hetényi's) model.

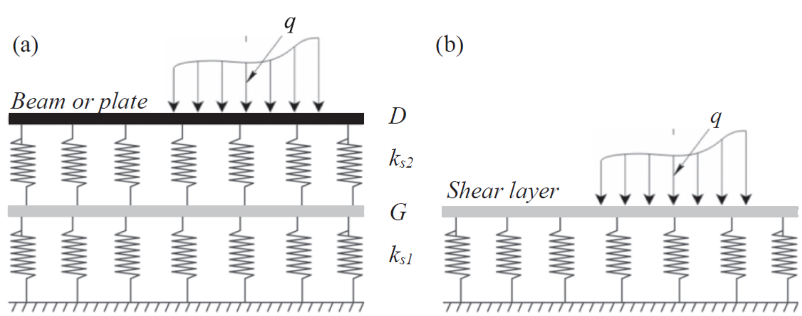

Figure 1. (a) Kerr's (1965) and (b) Pasternak's (1954) model for elastic foundations.

Based on the above, the author suggests that Vesic's (1961) method be used –recall Eq. (3)– along with a Poisson's ratio value equal to zero and the respective E_{eq} value. This E_{eq} value should reflect the deformability of the stratified medium, adverse groundwater regime (or future groundwater rise), the embedment depth of footing as well as the effect of both the shape of footing and the location on the plan-view of footing on the elastic modulus of soil. An in-depth review of the available methods calculating the equivalent elastic constants for the case of transversely loaded horizontally stratified soil mediums can be found in Pantelidis (2019a). One of the main findings of this review paper is that, in the vast majority of the cases, the existing methods return unrealistic E_{eq} values. Indeed, for the cases examined by Pantelidis (2019a), the maximum relative error introduced in the analysis using Egorov and Nichiporovich's (1961) weighted average method (best known as Bowles' (1996) method):

$$E_{eq} = \sum_{i=1}^{n} \left(h_i E_i \right) \Big/ \sum_{i=1}^{n} h_i \tag{4}$$

is as high as 83% and 63% on the unsafe and safe side respectively (where, E_i and h_i are the modulus and the thickness of the i-th layer respectively; the number of soil layers is n. Moreover, the current methods neglect the Poisson's ratio of soil strata, thus, reducing the problem to the v_{eq}=0 case is not possible. Consequently, the use of the current methods may easily lead to either non-economic or unsafe designs.

3 THE EQUIVALENT ELASTIC CONSTANTS OF SOIL MEDIUM (E_{eq}, v_{eq})

Apparently, there is an equivalent homogenous medium having pair of elastic constant values (E_{eq}, v_{eq}) that under the same loading conditions produces the same settlement with the original stratified medium. Adopting Steinbrenner's (1934) solution for a general BxL footing and equating these settlements as in Eq. (5):

$$q\left(a'B'\right)\frac{1-\nu_{eq}^2}{E_{eq}}I_s(H_n,\nu_{eq})I_E(\nu_{eq})C_w(\nu_{eq})=q\left(a'B'\right)\sum_{i=1}^{n}\frac{1-\nu_i^2}{E_i}\left(I_s(H_i,\nu_i)-I_s(H_{i-1},\nu_i)\right)I_E(\nu_i)C_w(\nu_i) \tag{5}$$

Solving the latter as for E_{eq}, the following expression for E_{eq} is obtained in Eq. (6):

$$E_{eq}=I_s(H_n,\nu_{eq})\cdot I_E(\nu_{eq})\cdot C_w(\nu_{eq})\cdot\left(1-\nu_{eq}^2\right)\Bigg/\sum_{i=1}^{n}\left[\frac{I_s(H_i,\nu_i)-I_s(H_{i-1},\nu_i)}{E_i}I_E(\nu_i)\cdot C_w(\nu_i)\cdot\left(1-\nu_i^2\right)\right] \tag{6}$$

where, H_i is the vertical distance extending from the level of foundation to the bottom of the i-th layer. Also, because H_0=0 m, $I_s(H_0,v_1)$ also equals to zero. C_w is the water table correction factor (Das 2017; Pantelidis 2020a; b) incorporated in the classical elastic solution. Eq. (6) has two unknowns, namely, E_{eq} and v_{eq}; however, the only real unknown is the E_{eq}. Regarding v_{eq}, any logical v_{eq} value can be used, under the precondition that this value will be used along with the respective E_{eq} value derived from Eq. (6). Besides, as shown later, the same settlement profile is obtained for any (E_{eq}, v_{eq}) pair of values satisfying Eq. (6). In a Winkler analysis, it is mandatory however that the equivalent Poisson's ratio of soil be equal to zero, because the v_{eq}=0 value corresponds to the deformation pattern of springs (compression with no lateral deformation).

It is noted that, Eq. (6) stands for any I_s factor which is part of an elastic settlement analysis equation of the form:

$$\rho=q\left(\text{Width or Diameter of foundation}\right)\left(1-\nu^2\right)I_sI_E/E \tag{7}$$

Regarding the embedment depth of footing, the author suggests the I_E factor given by Díaz and Tomás (2014) which is the product of an extensive parametric analysis based on 1,800 three-dimensional finite element models.

4 APPLICATION EXAMPLES

Schmertmann's (1970; "Fig. 6") application example is used as basis for setting up the examples presented herein. In this respect, the soil medium consists of 11 horizontal soil layers over bedrock (see Figure 2a). Three circular footings are considered, a flexible, a smooth rigid and a rough rigid footing resting on the surface of the above mentioned stratified medium; in all cases, the loading was uniform and equal to 200 kPa, while the diameter of the footing was 2.6 m. For all soil layers, a Poisson's ratio value equal to 0.4 was assumed (value considered by Schmertmann). The equivalent soil mediums were compared against the original stratified one through finite element analysis using Rocscience's RS2 with the criterion being that, two mediums are equivalent if they yield the same settlement under the same loading conditions. Considering that v_{eq}=0.4, Eq. (6) gave E_{eq}=9132 kPa while for v_{eq}=0, E_{eq}=11108 kPa. As shown in Figure 2b, the equivalent soil mediums gave almost identical settlement profiles, indicating the validity of the proposed method.

A second example is given. Let the long footing of Figure 3a founded also on the surface of the stratified soil system of Figure 2a; let also assume that the correction for footing shape has already been applied to these E values (Pantelidis 2019b). For convenience it is supposed that the elastic modulus below any point of the footing is equal to the respective one at the center of it. Using Eq. (6), E_{eq}=9743 KPa for v_{eq}=0.4 and E_{eq}=11599 KPa for v_{eq}=0 (for the corner of footing it stands that E_{eq}==10726 KPa and 12770 kPa respectively), whilst according to Bowles (Eq. (4)) E_{eq}=17765 KPa (the latter is independent of the location on the plan-view of footing and the Poisson's ratio value). Bowles' suggestion for the influence depth of footing (i.e. z_I=5B) has been applied to all cases. Applying Eq. (3), k_s was found equal to 7250 kN/m^3 for both (E_{eq}, v_{eq}) pair of values, whilst applying Eq. (4), k_s was equal to 13218 kN/m^3. It is confirmed that the foundation is flexible, as λL= 3.7>π (see Section 2). The analysis with jwinkler for strip footings (open source educational program; http://users.auth.gr/fkar/jWinkler/jWinkler.html) gave no noticeable difference in moments and shear forces of footing (diagrams not shown here). Small difference was observed in the soil reaction diagram and significant difference was found in the derived settlements (see Figure 3b). Favoring reproduction of author's example, the density of springs was 10 springs/m, whilst the elastic constants of concrete was E_b=21 GPa and v_b=0.15.

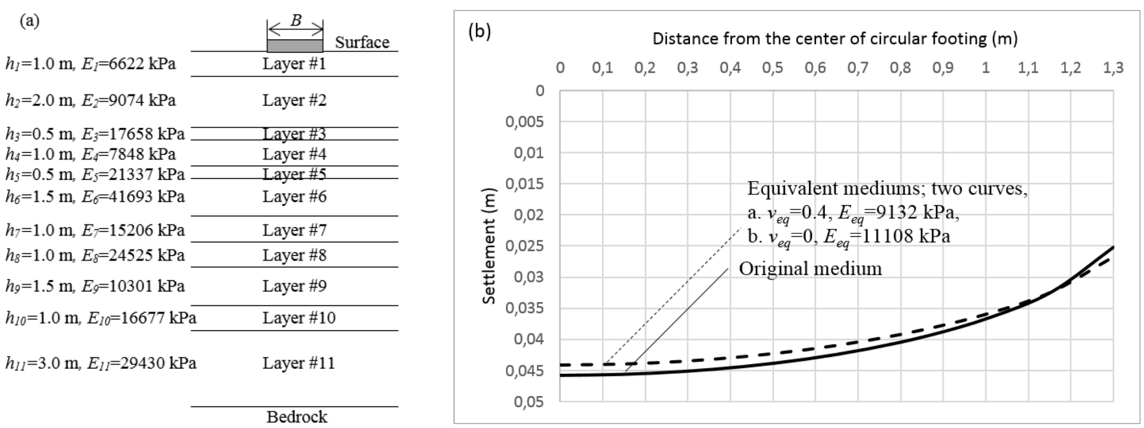

Figure 2. a) Soil strata thickness and soil elastic moduli values (from: Schmertmann 1970); v=0.4 for all soil strata, b) Settlement versus distance from the center of circular footing example chart.

5 SUMMARY AND CONCLUSIONS

As known, the modulus k_s, which is a measure of soil-structure rigidity, is widely used in designing flexible, shallow foundations using the Winkler spring hypothesis, where, the foundation soil is conveniently replaced by an array of springs having constant k_s. Among the available methods discussed in the present paper for calculating k_s, the author suggests the use of Vesic's (1961) approach. The latter requires both E and v of soil to be known. However, because the deformation pattern of Winkler's springs corresponds to an equivalent Poisson's ratio value for soil equal to zero (i.e. vertical displacement with no lateral deflection), Vesic's method should be used along with the (E_{eq}, v_{eq}=0) pair of values. In this respect a method for calculating the two equivalent elastic constants for the case of stratified mediums over a shallow foundation is proposed. The same method can also be used for reducing any homogenous (E, v) soil medium to an equivalent one having modulus E_{eq} and v_{eq} equal to zero. The effectiveness of the proposed method was illustrated through application examples.

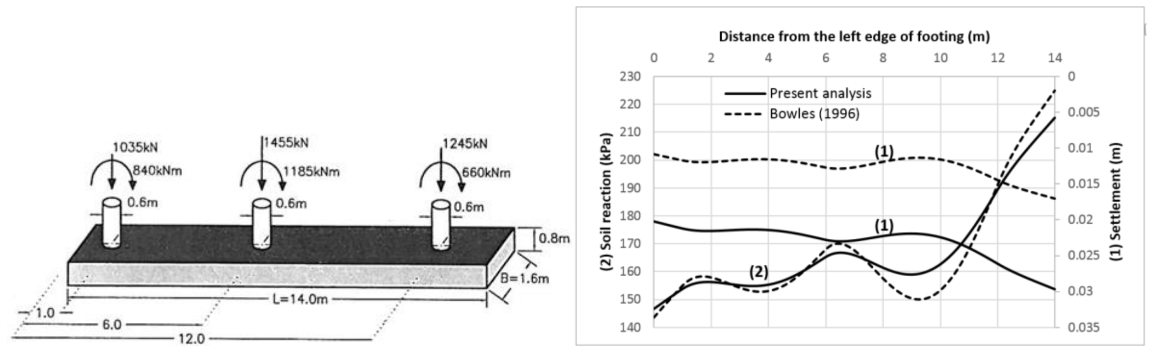

Figure 3. a) Geometry and loading of the example long footing, b) Soil reaction and settlement versus distance from the left edge of footing example chart.

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