

INTERACTION OF TWO SEMI-CYLINDRICAL CANYONS EXCITED BY SH WAVES

ABDUL HAYIR and SINAN EMRE CANKAYA

Dept of Civil Engineering, Istanbul Technical University, Istanbul, Turkey

The spectral analysis of strong earthquake ground motion needs detailed understanding of transfer function properties and source radiation along the wave propagation path. The main goal of this study is to evaluate the interaction of two semi-cylindrical canyons, which are subjected to the horizontally polarized shear-wave (SH-wave) and to find the transfer function properties of two canyons. In this study, the interaction of two semi-cylindrical canyons subjected to SH waves are considered and evaluated for a general angle of wave incidence. The method of Wave Function Expansion is derived, and the infinite series solution is obtained. Due to the complexity and convergence of infinite series including Bessel functions, the numerical results are limited. The convergence of the solution for the high frequencies requires the high order term. Moreover, the difficulties of this study come from convergence of the solution owing to interaction of two canyons having various dimensions and material properties. The closed-form solution of the problem shows that the surface topography can have prominent effects on incident waves when the wavelengths of incident motion are short compared to the radius of a canyon. The parameters, dimensions of the canyons, distance between two canyons, and the amplifications of the displacement amplitudes are obtained with respect to the incident angles of the waves and dimensionless frequency.

Keywords: Displacement amplitudes, Wavelengths, Analytical solution method, Wave expansion method, Surface amplifications, Local soil condition.

1 INTRODUCTION

The fundamental phenomenon of soil-structure interaction was examined by several analytical models accompanied by experimental observations. Among the physical phenomena investigated: the effects caused by local topography, the interaction -with other structures, and the dissipation of dynamic energy through the soil medium were described by exact series solutions (Wong 1975, Wong and Trifunac 1975). The soil-structure interaction (SSI) problems including a single structure and soil have been studied for many years (Hayir *et al.* 2001, Todorovska *et al.* 2002, Todorovska 1993). The effect of traveling seismic waves has been considered for several different types of single canyons such as cylindrical, semi-parabolic canyon in literature (Lee 1984, Lee *et al.* 2018, Lin *et al.* 2017). Structure-soil-structure interaction is investigated using a model of two shear walls supported by rigid foundations embedded in a soft layer over elastic bedrock.

In this article, the anti-plane response for incident plane SH wave is studied for two alluvial canyons as the local soil conditions. This article serves as an extension of the analytic work of wave expansion method and additional theorem.

2 THE MODEL OF TWO SEMI-CYLINDRICAL CANYONS

The model consists of a half-space in which two semi-cylindrical canyons settled on the free surface as in Figure 1. The half-space and canyons are assumed to be elastic, isotropic and homogeneous, with their material properties characterized by the rigidity μ , μ_v^1 and μ_v^2 , and the shear wave velocities β , β_v^1 and β_v^2 . γ is incident wave angle, c_x and c_y are phase velocity in x and y direction.

Two coordinate systems: the Cartesian and the cylindrical coordinate systems are employed to solve this boundary-valued problem.

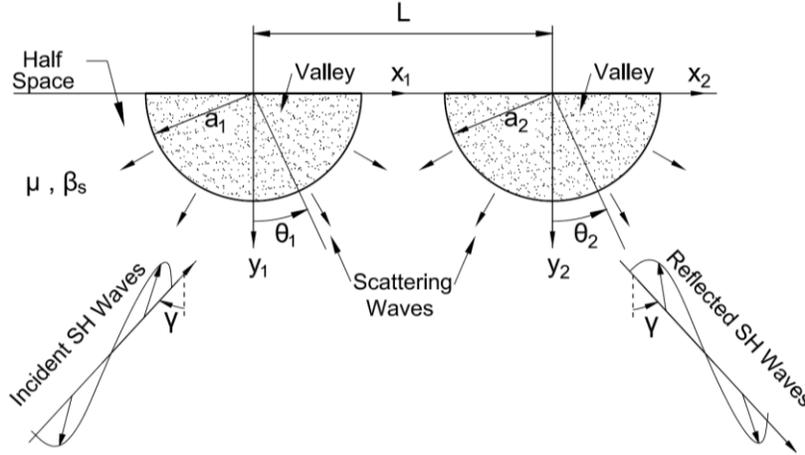


Figure 1. The model of the problem.

SH waves are the horizontally polarized shear-wave and its governing equation of the problem in polar coordinates is shown in Eq. (1):

$$\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} = \frac{1}{\beta^2} \frac{\partial^2 u_z}{\partial t^2} \quad (1)$$

Boundary conditions for free surface are in Eq. (2):

$$\sigma_{\theta z} = \frac{\mu}{r} \frac{\partial u_z}{\partial \theta} = 0 \quad \text{at } \theta_1 = \pm \frac{\pi}{2} \quad \text{and } \theta_2 = \pm \frac{\pi}{2} \quad (2)$$

Eq. (3) shows the displacement continuities on the contact surface:

$$u_z = u_z^{v1} ; u_z = u_z^{v2} \quad \text{at } r_1 = a_1 \quad \text{and } r_2 = a_2 \quad (3)$$

and Eq. (4) shows the shear stress continuities contact surface:

$$\mu \frac{\partial u_z}{\partial r} = \mu_{v1} \frac{\partial u_z^{v1}}{\partial r} ; \mu \frac{\partial u_z}{\partial r} = \mu_{v2} \frac{\partial u_z^{v2}}{\partial r} \quad \text{at } r_1 = a_1 \quad \text{and } r_2 = a_2 \quad (4)$$

It is assumed that the excitation consists of plane harmonic SH waves with an amplitude of 0.5 and the angle of incidence both coordinate systems (Hayir *et al.* 2001) as given in Eq. (5) and Eq. (6);

$$u_{z1}^{i+r}(r_1, \theta_1) = \sum_{n=0}^{\infty} \{ (-1)^n \varepsilon_n J_{2n}(k r_1) \cos(2n \gamma) \cos(2n \theta_1) \} \quad (5)$$

$$\begin{aligned}
 & -2i \sum_{n=0}^{\infty} \{(-1)^n J_{2n+1}(k r_1) \sin((2n+1) \gamma) \sin((2n+1) \theta_1)\} \\
 u_z^{i+r}(r_2, \theta_2) = & \sum_{n=0}^{\infty} \{(-1)^n \varepsilon_n J_{2n}(k r_2) \cos(2n \gamma) \cos(2n \theta_2)\} \\
 & -2i \sum_{n=0}^{\infty} \{(-1)^n J_{2n+1}(k r_2) \sin((2n+1) \gamma) \sin((2n+1) \theta_2)\} \quad (6)
 \end{aligned}$$

where $\varepsilon_0 = 1$ and $\varepsilon_n = 2$ for $m \neq 0$

Scattering waves of the canyons are shown in Eq. (7) and Eq. (8) as follows:

$$u_z^{s1}(r_1, \theta_1) = \sum_{n=0}^{\infty} \{A_n H_{2n}^{(2)}(k r_1) \cos(2n \theta_1) + B_n H_{2n+1}^{(2)}(k r_1) \sin((2n+1) \theta_1)\} \quad (7)$$

$$u_z^{s2}(r_2, \theta_2) = \sum_{n=0}^{\infty} \{A_n^* H_{2n}^{(2)}(k r_2) \cos(2n \theta_2) + B_n^* H_{2n+1}^{(2)}(k r_2) \sin((2n+1) \theta_2)\} \quad (8)$$

The motion in the valley may be written as in Eq. (9) and Eq. (10):

$$u_z^{v1}(r_1, \theta_1) = \sum_{n=0}^{\infty} \{C_n \varepsilon_n J_{2n}(k_{v1} r_1) \cos(2n \theta_1) + D_n J_{2n+1}(k_{v1} r_1) \sin((2n+1) \theta_1)\} \quad (9)$$

$$u_z^{v2}(r_2, \theta_2) = \sum_{n=0}^{\infty} \{C_n^* \varepsilon_n J_{2n}(k_{v2} r_2) \cos(2n \theta_2) + D_n^* J_{2n+1}(k_{v2} r_2) \sin((2n+1) \theta_2)\} \quad (10)$$

The motion in the canyons may be written as in Eq. (11) and Eq. (12):

$$u_z^{v1}(r_1, \theta_1) = u_z^{i+r}(r_1, \theta_1) + u_z^{s1}(r_1, \theta_1) + u_z^{s2}(r_1, \theta_1) \quad (11)$$

$$u_z^{v2}(r_2, \theta_2) = u_z^{i+r}(r_2, \theta_2) + u_z^{s1}(r_2, \theta_2) + u_z^{s2}(r_2, \theta_2) \quad (12)$$

To determine the solution, it is mandatory to determine the unknown coefficients $A_n, B_n, A_n^*, B_n^*, C_n, D_n, C_n^*$ and D_n^* . For this purpose, boundary conditions Eq. (2) and continuity conditions Eq. (3) and (4) are used. Since the solutions of canyons are determined in different coordinates system (r_1, θ_1) and (r_2, θ_2) , the boundary conditions cannot be applied to the solutions directly. Therefore, one coordinate systems are converted to the other using Graf's addition theorem (Hayir and Bakirtas 2004). After this possess, the solutions are determined with the same coordinate system either (r_1, θ_1) or (r_2, θ_2) .

3 NUMERICAL EXAMPLES

The amplitudes and phases of these quantities are given in Eq. (13) as follows:

$$|u_z| = \sqrt{(Re u)^2 + (Im u)^2} \quad (13)$$

Dimensionless quantities $\eta = 2a/\lambda$, $ka = \pi\eta$

Figure 2 shows absolute free surface displacements or amplifications in z direction for various dimensionless wave frequencies between twin canyons which are softer than the half space. It is seen that the amplifications of canyons on the free surfaces are higher than the half space. When the canyons get softer, their amplifications get higher than the free surface amplifications of the half space as shown in Figure 3 and 4. Also while the frequency is getting bigger, the amplification is getting higher.

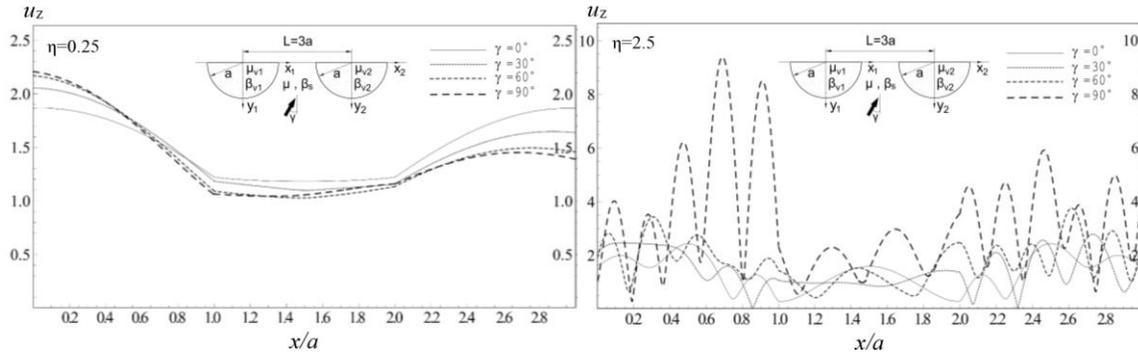


Figure 2. Absolute displacements u_z on the free surfaces for $\eta=0.25$ and $\eta=2.5$ in the case of $\rho_1/\rho=1/1.5$, $\beta_1/\beta=1/2$, $\rho_2/\rho=1/1.5$, $\beta_2/\beta=1/2$.

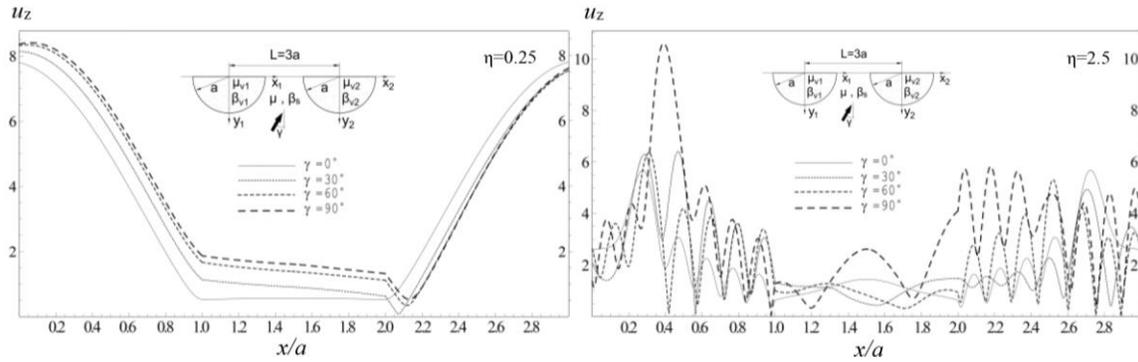


Figure 3. Absolute displacements u_z on the free surfaces for $\eta=0.25$ and $\eta=2.5$ in the case of $\rho_1/\rho=1/1.5$, $\beta_1/\beta=1/3$, $\rho_2/\rho=1/1.5$, $\beta_2/\beta=1/3$.

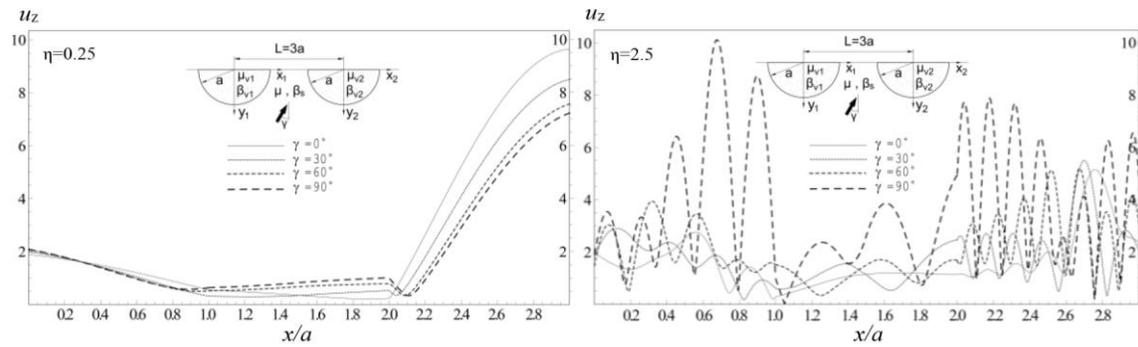


Figure 4. Absolute displacements u_z on the free surfaces for $\eta=0.25$ and $\eta=2.5$ in the case of $\rho_1/\rho=1/1.5$, $\beta_1/\beta=1/2$, $\rho_2/\rho=1/1.5$, $\beta_2/\beta=1/3$.

Figure 5 indicates that when the distance between the twin's canyons are getting bigger, the interaction of the canyons vanishes. Figure 6 shows that when canyons geometries are different, the amplifications are affected due to the scattering waves. In Figure 6, the diameter of the one canyon is half of the other. In this case, the incident waves hit the bigger canyons and scattered back and effected the other canyons amplifications. The same event occurs in Figure 7.

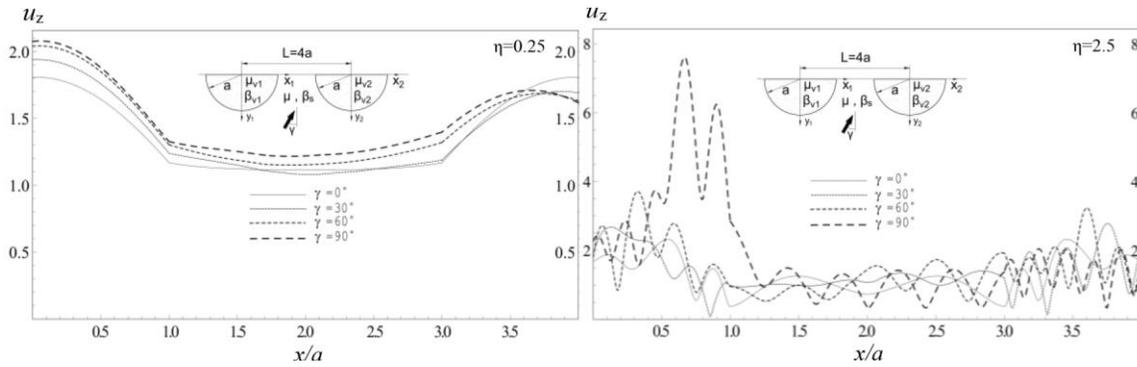


Figure 5. Absolute displacements u_z on the free surfaces for $\eta=0.25$ and $\eta=2.5$ in the case of $\rho_1/\rho=1/1.5$, $\beta_1/\beta=1/2$, $\rho_2/\rho=1/1.5$, $\beta_2/\beta=1/2$.

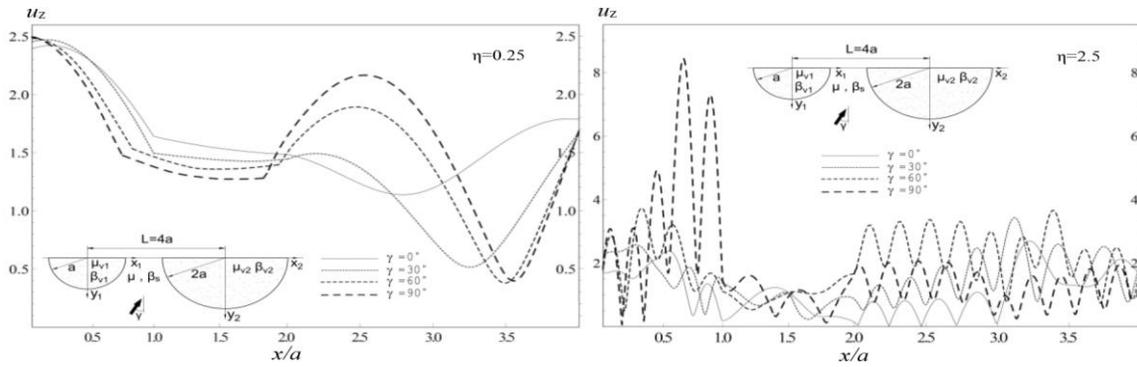


Figure 6. Absolute displacements u_z on the free surfaces for $\eta=0.25$ and $\eta=2.5$ in the case of $\rho_1/\rho=1/1.5$, $\beta_1/\beta=1/2$, $\rho_2/\rho=1/1.5$, $\beta_2/\beta=1/2$.

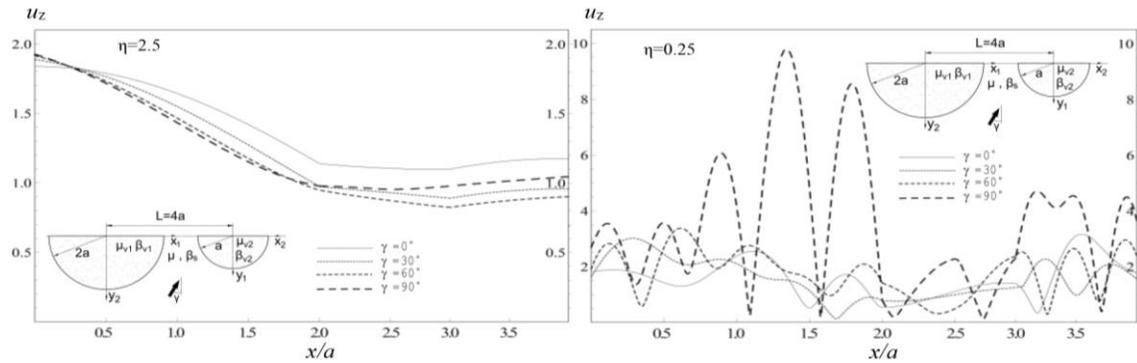


Figure 7. Absolute displacements u_z on the free surfaces for $\eta=0.25$ and $\eta=2.5$ in the case of $\rho_1/\rho=1/1.5$, $\beta_1/\beta=1/2$, $\rho_2/\rho=1/1.5$, $\beta_2/\beta=1/2$.

4 CONCLUSIONS

In this study, the effects of local soil conditions have been observed clearly for two local soils having semi-circular diameters. It is well known that one of the vital reasons of the earthquake damage is local soil conditions. The local soil conditions change the characteristics of surface

seismic response. Therefore, it is vital to figure out the amplifications of the soils during strong ground motion. Due to the characteristics of soils, the amplification might be big or small. Considering the geometrical and physical properties of the valleys and the surrounding environment, the degree of complexity of the amplitude results increases with increasing frequency of incoming waves. At a certain point on semi-cylindrical valley surfaces, spectral amplifications are largely dependent on the angle of incidence of SH waves.

The results show that if the shear wave velocity in the semi-cylindrical valleys decreases and other model variables remain constant, the overall amplification in the valleys will increase. In the alluvial valleys, where one is larger than the other, the small valley may be subject to large displacements due to the effect of the waves scattered from the large valley. As the distance between the valleys increases, the wave scattering effects decrease. Alongside this distance increase, only the effects of incoming and reflected waves could be seen on the soil. In some parts of the valleys, standing waves formed when SH waves interact with incoming, reflected and scattered waves.

References

- Hayir, A., Todorovska, M. I., and Trifunac, M. D., *Antiplane Response of a Dike with Flexible Soil-Structure Interface to Incident SH Waves*, Soil Dynamics and Earthquake Engineering, 21(7), 603-613, October, 2001.
- Hayir, A., and Bakirtas, I., *A Note On a Plate Having a Circular Cavity Excited by Plane Harmonic SH Waves*, Journal of Sound and Vibration, 271(1-2), 241-255, March, 2004.
- Lee, V. W., *Three-Dimensional Diffraction of Plane P, SV and SH Waves by A Hemispherical Alluvial Valley*, Soil Dynamics and Earthquake Engineering, 3(3), 133-44, July, 1984.
- Lee, V. W., Lin, C. H., and Teng, L., *Diffraction Around a Semi-Parabolic Canyon in an Elastic Half-Space by Out-of-Plane SH-Waves, II: Rocking and Resultant Rotations*, Journal of Earthquake Engineering, 22(1), 126-46, 2018.
- Lin, C. H., Teng, L., and Lee, V. W., *Surface Rotations on and Around a Semi-Parabolic Canyon in an Elastic Half-Space Induced by Out-of-Plane SH-Waves, I: Torsion*, Journal of Earthquake Engineering, 21(7), 1070-89, 2017.
- Todorovska, M. I., *Effects of the Wave Passage and The Embedment Depth for in-Plane Building-Soil Interaction*, Soil Dynamics and Earthquake Engineering, 12(6), 343-55, 1993.
- Todorovska, M. I., Hayir, A., and Trifunac, M. D., *A Note On Tsunami Amplitudes Above Submarine Slides and Slumps*, Soil Dyn. Earthq. Eng., 22(2), 129-141, February, 2002.
- Wong, H. L., *Dynamic Soil-Structure Interaction, Report EERL-75-01*, Pasadena, California: Earthquake Engineering Research Lab, California Institute of Technology, 1975.
- Wong, H. L., and Trifunac, M. D., *Two-Dimensional, Antiplane, Building-Soil-Building Interaction for Two or More Buildings and for Incident Plane SH Waves*, Bulletin of Seismological Society of America, 65(6), 1863-85, December, 1975.