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A SEQUENTIAL AUGMENTED METAMODELING METHOD FOR ENGINEERING RELIABILITY ANALYSIS

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Metamodeling methods provide useful tools to replace expensive numerical simulations in engineering reliability analysis and design optimization. The radial basis functions (RBFs) and augmented RBFs can be used to create accurate metamodels; therefore they can be integrated with a reliability analysis method such as the Monte Carlo simulations (MCS). However the model accuracy of RBFs depends on the sample size, and the accuracy generally increases as the sample size increases. Since the optimal sample size used to create RBF metamodels is not known before the creation of the models, a sequential RBF metamodeling method was studied. In each iteration of reliability analysis, augmented RBFs were used to generate metamodels of a limit state or performance function, and the failure probability was calculated using MCS. Additional samples were generated in subsequent analysis iterations in order to improve the metamodel accuracy. Numerical examples from literature were solved, and the failure probabilities based on the RBF metamodels were found to have a good accuracy. In addition, only small numbers of iterations were required for the reliability analysis to converge. The proposed method based on sequential RBF metamodels is useful for probabilistic analysis of practical engineering systems.

Keywords: Augmented radial basis function (RBF), Sequential metamodels, Monte Carlo Simulation (MCS), Failure probability.

1 INTRODUCTION

Due to the implicit and nonlinear performance functions involved, it is a challenging task to perform reliability analysis of practical engineering problems using traditional approaches, such as the first/second-order reliability methods (FORM/SORM) (Hasofer and Lind 1974, Kiureghian *et al.* 1987) and direct sampling-based methods (Rubinstein 1981, Au and Wang 2014). The FORM/SORM methods are used to find the most probable point or the design point (Hohenbichler *et al.* 1987, Lü *et al.* 2011). Since derivatives of system responses are required, the integration of FORM/SORM with a response analysis code, such as a finite element (FE) program, is not straightforward, especially for engineering applications that require expensive response evaluations. MCS or other sampling methods can be integrated with a commercial FE program in a rather straightforward manner, since derivatives of the performance function are not required. The direct implementation of MCS is computationally prohibitive for complex problems requiring expensive simulations. An efficient and simple metamodeling method is the response surface method (RSM) using a least-square polynomial regression model (Faravelli 1989, Kim and Na 1997, Gavin and Yau 2008, Kang *et al.* 2010, Bi *et al.* 2010). However, a global RSM has difficulty creating accurate metamodels for highly nonlinear response functions. To improve the accuracy of metamodels used in reliability analysis, other methods and techniques have also been developed to approximate implicit performance functions (Jin *et al.* 2001, Fang *et al.* 2005, Fang and Wang 2008, Chowdhury and Rao 2009, Zhao *et al.* 2014).

In this study, a sequential metamodeling approach based on augmented RBFs was studied and applied to engineering reliability analysis. After an augmented RBF metamodel was generated, MCS was used to calculate the probability of failure. The convergence criterion was checked and, if needed, additional sample points were created and more analysis iteration were applied. As the sample size increased, the RBF models became more accurate. The computational cost of the method depended on the number of implicit function evaluations, i.e., the total number of sample points used to generate a metamodel. To study the performance of the method, two examples from literature were adopted and numerical results were obtained.

2 THE RELIABILITY ANALYSIS PROBLEM

The failure probability, P_F, is written in Eq. (1) as (Madsen *et al.* 1986):

$$P_F \equiv P(\mathrm{g}(\mathbf{x}) \leq 0) = \int_{g(\mathbf{x})\leq 0} p_X(\mathbf{x}) d\mathbf{x} \tag{1}$$

where $\mathrm{g}(\mathbf{x})$ is a performance function and $\mathrm{g}(\mathbf{x}) \leq 0$ means the failure of an engineering system or component.

3 A SEQUENTIAL APPROACH OF AUGMENTED RBF METAMODELS

3.1 Augmented Metamodels

An augmented RBF metamodel of a performance function $\mathrm{g}(\mathbf{x})$ consists of two parts, as (Fang *et al*. 2005, Fang and Wang 2008) given in Eq. (2):

$$\tilde{\mathrm{g}}(\mathbf{x}) = \sum_{i=1}^{n} \lambda_i \phi(\|\mathbf{x} - \mathbf{x}_i\|) + \sum_{j=1}^{p} c_j f_j(\mathbf{x}) \tag{2}$$

The first part in Eq. (2) is an RBF function, and the second part is the augmented polynomial function, respectively. An augmented RBF model generally provides a more accurate approximation than the basic RBF model (Fang *et al*. 2005, Fang and Wang 2008).

3.2 Monte Carlo Simulations

MCS can be applied in conjunction with the metamodel $\tilde{\mathrm{g}}(\mathbf{x})$ to estimate the failure probability, P_F, as written in Eq. (3):

$$P_F \equiv P(\mathrm{g}(\mathbf{x}) \leq 0) = \frac{1}{N} \sum_{i=1}^{N} \Gamma[\tilde{\mathrm{g}}(\mathbf{x}^i) \leq 0] \tag{3}$$

3.3 Overall Procedure of Reliability Analysis

The overall reliability analysis procedure using augmented RBF is as follows:

(i) Determine sample sizes used in the first and additional iterations.
(ii) Generate an initial sample pool.
(iii) Calculate performance function values at all initial sample points. This step require FE analysis or other numerical analysis methods for practical applications.

(iv) Construct augmented RBF metamodels of performance functions using all available sample points.
(v) Compute failure probability using MCS or another sampling method.
(vi) Check convergence between two successive iterations. If the failure probability convergence is achieved, stop the reliability analysis procedure; otherwise continue to the next step. In this study, a relative error of ε = 1.0% is used as the convergence criterion.
(vii) Generate additional sample points.
(viii) Evaluate performance functions at the additional sample points, then go to Step (iv). This step requires additional FE analyses for practical applications.

4 NUMERICAL EXAMPLES

To study the method, numerical examples form literature were solved. In this work, a mathematical problem and an engineering problem were investigated and numerical solutions were obtained.

4.1 Example 1 – A Mathematical Example

The first example is a mathematical problem with two independent random variables (Kim and Na 1997, Chowdhury and Rao 2009). The nonlinear performance function is written in Eq. (4) as:

$$g(\mathbf{x}) = e^{(0.2x_1+6.2)} - e^{(0.47x_2+5.0)} \quad (4)$$

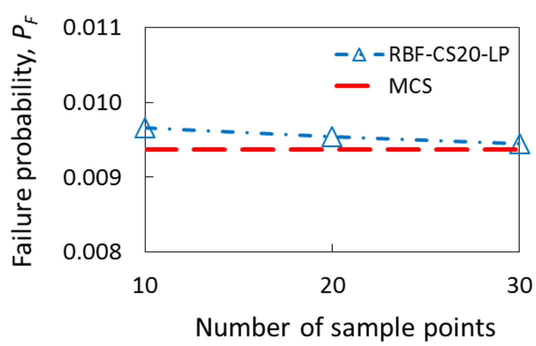

Figure 1. Failure probability vs. sample size.

Both x_1 and x_2 follow a standard normal distribution, i.e., zero mean and unit standard deviation. In order to compare results, MCS was applied using the original analytical performance function, and a failure probability 0.009372 was obtained. The sequential RBF metamodeling method was applied and the failure probability values were computed and compared with 0.009372. The RBF metamodel started with ten (10) sample points, and ten (10) more sample points were added in each subsequent iteration. In the first iteration, the estimated failure probability was 0.009654, representing an error of 3.0%. At convergence, the failure probability became 0.009443, with the error reduced to 0.8%. As more samples were employed, the RBF model accuracy was improved. The variation of failure probability versus the sample size for this example is plotted in Figure 1. It took three iterations for the reliability analysis steps to converge, corresponding to a total of thirty (10+10+10=30) sample points.

4.2 Example 2 – A Soil Settlement Problem

This example is a soil settlement problem, as shown in Figure 2 (Ang and Tang 1975, Chowdhury and Rao 2009). An empirical equation for normally loaded clay is used to calculate the settlement of point A due to new construction. If the upper bound settlement is 2.5 in., the performance function is written in Eq. (5) as:

$$g(\mathbf{x}) = -\frac{C_c}{1+e_0} H \log\left(\frac{p_0+\Delta p}{p_0}\right) + 2.5 \tag{5}$$

Table 1 lists the the distribution types, mean values, and coefficients of variation (CV) of the five independent random variables. These variables include:

C_c = compression index of the clay layer;
H = thickness of the clay layer;
e_0 = void ratio of the clay layer before construction;
p_0 = original effective pressure at point B before construction; and
Δp = pressure increase at point B due to construction.

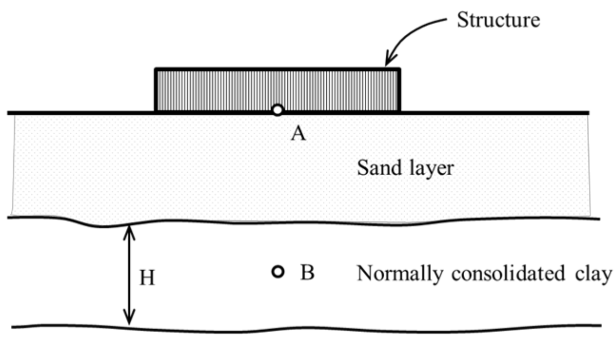

Figure 2. A soil settlement problem.

Table 1. Random variables.

Random variable		Distribution	Mean	CV
C_c	(N/A)	Gaussian	0.396	0.25
e_0	(N/A)	Gaussian	1.19	0.15
H	(in)	Gaussian	168	0.05
p_0	(ksf)	Gaussian	3.72	0.05
Δp	(ksf)	Gaussian	0.5	0.2

In this example, a total of thirty (30) sample points were initially generated. Ten (10) additional sample points were generated in each following iteration and added to the sample pool. It took four iterations, i.e., a total of sixty (30+10+10+10=60) sample points, for the failure probability to converge. The variation of failure probability versus the sample size is plotted in Figure 3. The failure probability was computed as 0.008096 based on MCS and the original analytical function. In the first iteration, the failure probability was estimated to be 0.08788. This represented an 8.6% error. At the 4th iteration, the error was decreased to 0.8%. The reliability analysis method worked well. To obtain a reasonable accuracy, around forty (40) to fifty (50) sample points were needed in this example.

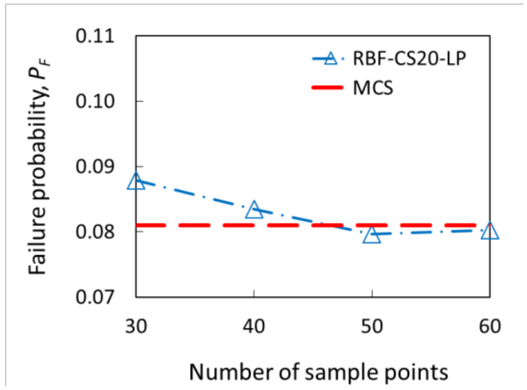

Figure 3. Failure probability vs. sample size.

5 SUMMARY AND CONCLUDING REMARKS

RBFs and augmented RBFs can be used to create accurate metamodels of linear or nonlinear performance functions. In a reliability analysis, MCS can be integrated with RBF metamodels, and applied to calculate the failure probability. To improve the standard approach using MCS and RBF metamodels, a sequential RBF metamodeling approach in reliability analysis was studied in this work. The RBF metamodels were applied in an iterative manner, so that the accuracy of metamodels were improved. Two numerical examples were presented. To evaluate the proposed method, numerical accuracy and computational efficiency were studied and the method worked well. More research work is needed to apply the method for practical engineering problems involving expensive response simulations.

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