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AN ADAPTIVE RBF METHOD FOR DESIGN OPTIMIZATION OF BUILDING STRUCTURES

QIAN WANG, LUCAS SCHMOTZER, and YONGWOOK KIM

Dept of Civil and Environmental Engineering, Manhattan College, Riverdale, USA

Design of building structures has long been based on a trial-and-error iterative approach. Structural optimization provides practicing engineers an effective and efficient approach to replace the traditional design method. A numerical optimization algorithm, such as a gradient-based method or genetic algorithm (GA), can be applied, in conjunction with a finite element (FE) analysis program. The FE program is used to compute the structural responses, such as forces and displacements, which represent the design constraint functions. In this method, reading and writing the input/output files of the FE program and interface programming are required. Another method to perform structural optimization is to create an approximate constraint function, which involves implicit structural responses. This is referred to as a surrogate or metamodeling method. The structural responses can be expressed as approximate functions, based on a number of preselected sample points. In this study, an adaptive metamodeling method was studied and applied to a building structure. The FE analyses were first performed at the sample points, and metamodels were constructed. A gradient-based optimization algorithm was applied. Additional samples were generated and additional FE analyses were conducted so that the model accuracy could be improved, close to the optimal design points. This adaptive scheme was continued, until the objective function values converged. The method worked well and optimal designs were found within a few iterations.

Keywords: Reinforced concrete (RC) buildings, Adaptive metamodels, Finite element (FE), Gradient-based optimization algorithm.

1 INTRODUCTION

When designing a building, structural engineers must strive to create the most efficient design possible. This design is often found by changing many variables within certain constraints set forth by applicable building and design codes. This process is known as structural design optimization (Kirsch 1993, Arora 2017). Large-scale building optimization typically requires the use of structural analyses and iterative procedures (Chan and Wang 2005, Zou *et al.* 2007). Most analyses are too complex to perform without the use of FE analyses. As a result, engineers had to integrate FE codes with an optimization algorithm (Arora and Wang 2005). This can be an expensive process, as it requires extensive computer coding. As a result, alternative methods have been explored. Some methods include the use of metamodels. Metamodels serve to create simple and accurate models without directly integrating complex FE models in optimization loops. The metamodel is an approximate function representing the actual response function of a structure (Jin *et al.* 2001, Bi *et al.* 2010, Yin *et al.* 2016).

In this work, an alternative metamodeling method was explored. It used augmented radial basis functions (RBFs) to approximate the results of the structural responses (Fang and Wang

2006). These structural responses were obtained through FE analysis of preselected sample points. After the initial results were obtained, FE software was no longer needed, nor required to be directly integrated in the numerical optimization loop. Once the explicit response functions became available, a traditional gradient-based algorithm was applied so that an optimal design could be found. Furthermore, an adaptive technique was developed so that new sample points could be added and the accuracy of the RBF models could be improved. The proposed optimization technique was exemplified on a hypothetical three-dimensional (3D) reinforced concrete building. The technique was used to determine the thicknesses of various shear walls in the structure for an optimized torsional-resistant design.

2 OPTIMIZATION PROBLEM FORMULATION AND SOLUTION

2.1 General Formulation

A general objective function $C(\mathbf{x})$ is seen below in Eq. (1). It is written in terms of the design variables, **x**. The objective function is minimized and subject to the constraint function g(**x**) given by Eq. (2). Eq. (3) specifies the lower and upper limits of design variables:

$$C(\boldsymbol{x}) \tag{1}$$

subject to

$$g(\boldsymbol{x}) \leq 0 \tag{2}$$

$$\boldsymbol{x}^L \leq \boldsymbol{x} \leq \boldsymbol{x}^U \tag{3}$$

2.2 Metamodels Based on RBFs

RBFs have been used for complex optimization problems. They seek to define a function with the creation of a new function or metamodel, $\tilde{g}(\mathbf{x})$, as seen in Eq. (4):

$$\tilde{g}(\boldsymbol{x}) = \sum_{i=1}^{n} \lambda_i \phi(\|x - x_i\|) \tag{4}$$

where

n = number of sample points;
λ_i = coefficient of the basis function;
ϕ = basis function;
$\mathbf{x}$ = vector of design variables;
$\mathbf{x}_i$ = vector of design variables at *i*th sample point;
$\|\mathbf{x} - \mathbf{x}_i\|$ = Euclidean norm.

The coefficients of the new model are dependent on the value of the unknown function at some sample points. The RBFs can be highly accurate with the addition of linear or quadratic functions. If an RBF implements with linear or quadratic functions to approximate the true function, it is considered augmented (Fang and Wang 2006).

2.3 An Adaptive RBF Approach

The efficiency of the optimization process was improved with the use of adaptive RBFs. The adaptive RBF optimization processes started with initial sample points. Once a metamodel function was created, based on the sample points, a gradient-based optimization algorithm was applied. Then, for adaptive RBF, additional sample points were generated around the newly found optimum point. The corresponding FE analyses were conducted at the additional sample points. Optimization was performed again with the additional sample points, increasing the accuracy of the function around the optimal design point. This adaptive scheme was continued, until the objective function values converged. The number of additional sample points at each iteration was chosen to be twice of the total number of design variables. An illustration of additional sample points is show in Figure 1.

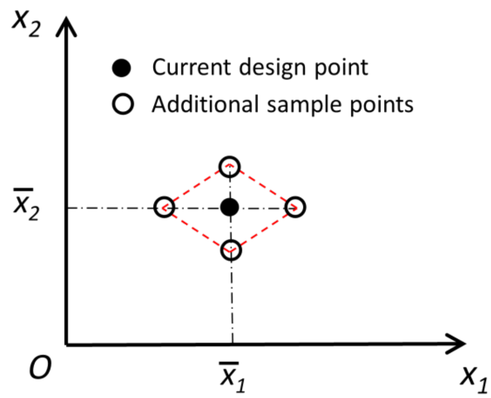

Figure 1. An illustration of additional sample points.

3 A REINFORCED CONCRETE BUILDING EXAMPLE

Figure 2 shows the typical floor plan at the base of a hypothetical seventeen-story reinforced concrete structure, optimized with the use of RBFs. At the top of the structure, the two towers are connected. This design was susceptible to a large amount of torsion. The shear walls of the structure are highlighted in Figure 2. For design optimization, the shear walls were grouped with three separate thicknesses: w_1, w_2, and w_3. Each wall's thickness was constrained to be between 0.4 and 0.8 meters. Two separate optimizations were run. The first sought to minimize the first torsional period (T_t) in the structure relative to the first translational period (T_1). The second sought to minimize the cross-sectional area of the shear walls. The two optimization processes are written below in mathematical terms. The first optimization formulation, optimization 1, seeks to minimize the ratio of T_t to T_1, as seen in Eq. (5):

$$C(w_1, w_2, w_3) = \frac{T_t}{T_1} \tag{5}$$

The second optimization formulation, optimization 2, seeks to minimize the total cross-sectional area of all shear walls. The second optimization formulation is subject to two separate constraints. Again, the shear wall thickness must maintain the same thickness range. In addition, the structure must remain resistant to torsion. Therefore, Eq. (5) is implemented as a constraint, as seen in Eq. (6):

$$g(w_1, w_2, w_3) = \frac{T_t}{T_1} - 0.75 \leq 0 \tag{6}$$

The torsional and translational periods of the building were obtained from FE analyses using SAP2000 program (Computer and Structures 2011). The adaptive metamodel method began with nine initial sample points. Six additional sample points were added in each optimization iteration. To compare the results, optimization with the global RBF method was also performed. Thirty-one sample points were generated and thirty-one separate FE analyses were performed. Every analysis was based on separate shear wall thicknesses randomly determined through the Latin hypercube sampling method.

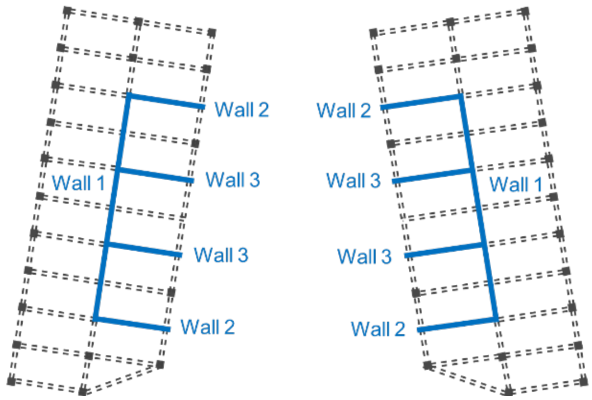

Figure 2. A typical floor plan.

Table 1 shows the optimal design results using the two methods for two optimization formulations. Both methods provided accurate functions to optimize the shear walls of the structure. For the adaptive RBF method, only fifteen and twenty-one FE analyses were required to find the optimal designs for the two formulations, respectively. The optimal designs were further verified, using the FE analysis results, compared with the approximate values from RBF models. Very small errors were observed for both the adaptive and global RBF methods. The adaptive RBF method was shown to find more accurate optimal design than the global RBF method. The adaptive method proved to be both efficient and effective.

Table 1. Optimal design results.

	Adaptive RBF (6 samples/iteration)		Global RBF		Lower bound	Upper bound
	1st optimization formulation	2nd optimization formulation	1st optimization formulation	2nd optimization formulation		
w_1 (m)	0.800	0.424	0.800	0.441	0.400	0.800
w_2 (m)	0.400	0.400	0.400	0.400	0.400	0.800
w_3 (m)	0.400	0.400	0.400	0.400	0.400	0.800
T_t/T_1 (RBF)	0.673	0.750	0.647	0.750		
T_t/T_1 (FE)	0.669	0.750	0.669	0.745		
T_t/T_1 (% error)	0.6%	0.0%	3.3%	0.7%		
No of samples	15	21	31	31		

4 CONCLUDING REMARKS

Structural optimization was performed with the use of adaptive RBFs. The RBFs were employed to develop metamodels, based on output from various FE analyses. The described process was demonstrated on a hypothetical reinforced concrete building. The objective was to use the researched techniques in order to minimize the torsional response of the structure while remaining within established constraints. In this study, it was proven that the adaptive method was very efficient: It required a relatively small number of sample points, and in turn, a relatively small number of FE analysis. This provides an efficient method for optimizing large complex structures, alternative to using large number of expensive FE analyses and/or complex computer coding for input/output integration with FE codes. This method should be further researched and applied to other structural engineering optimization problems.

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