RELIABILITY BASED DESIGN OF GEOTECHNICAL SYSTEMS BASED ON A DECOUPLED RELIABILITY ANALYSIS AND OPTIMIZATION APPROACH

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Reliability-based design intends to design an engineering system subject to probabilistic design constraints. This paper presents a reliability design optimization approach of geotechnical engineering problems based on a decoupled reliability analysis and optimization concept. Geotechnical and foundation systems involve considerable uncertainties in soil properties, loading conditions, and inadequate site characterization. A traditional reliability design optimization method, such as the reliability index approach, requires a double-loop optimization structure, since reliability analysis is implemented for constraint function evaluation using a numerical optimization algorithm. The decoupled approach separates the reliability analyses of the constraint functions from the design optimization loop. An adaptive metamodeling method is proposed to create approximate functions of the reliability indices, so that the reliability design optimization is performed using the approximate functions of the reliability constraints. Radial basis functions are implemented to build the metamodels. To progressively improve the accuracy of predicting the reliability indices, an adaptive sampling technique is employed, so that new samples are added using the optimal points found in previous design iterations. Numerical examples are solved and presented, and the proposed design approach works well. It is shown to be a useful alternative for solving reliability-based design of geotechnical and foundation systems.

Keywords: Engineering optimization, Geotechnical and foundation systems, Radial basis function (RBF), First-order reliability method (FORM).

1 INTRODUCTION

Design optimization of geotechnical and foundation engineering problems under uncertainties has attracted considerable attention in recent years. This is often referred to as reliability-based design optimization (RBDO) or reliability-based design (RBD). For geotechnical systems, randomness exists in design parameters, soil characterization, as well as loading conditions (Ang and Tang 2006). Direct probability-based design methods can explicitly incorporate uncertainties of random variables in the design optimization loop, since reliability analysis is directly performed to calculate failure probabilities or reliability indices (Wang et al. 2016, Fenton et al. 2016, Gao et al. 2019). For simple design problems involving single design variable, the RBD can be conducted in a trial-and-error manner (Low 2005, Liu and Low 2018). For more complicated design problems involving multiple design variables and reliability constraints, a numerical optimization procedure is typically required to find the optimal design satisfying all probabilistic constraints.
Various RBD formulations have been studied in other engineering areas and a conventional formulation has a nested double-loop structure (Aoues and Chateauneuf 2010, Valdebenito and Schuëller 2010). The design problem starts with optimization/minimization of the objective function as the outer loop, which is typically performed using a nonlinear programming algorithm or an evolutionary algorithm. The evaluation of the reliability constraints is carried out in the inner loop, which involves a numerical reliability analysis or inverse reliability analysis method (Babu and Basha 2008, Lü et al. 2017, Fang et al. 2019). To alleviate the numerical difficulties of the double-loop methods, other RBD formulations have also been studied (Li et al. 2019). A decoupled RBD approach based on a reliability index mapping concept has been applied to geotechnical problems (Zhang et al. 2020). Multi-objective robust design problems have also been studied in geotechnical engineering (Juang et al. 2013, Khoshnevisan et al. 2014, Zhong et al. 2020).

In this work, a decoupled RBD framework is studied which separates the reliability analyses of the constraint functions from the numerical design optimization. Metamodels are created to approximate the reliability indices, so that the RBD is performed using the approximate functions of the reliability constraints. In the literature, metamodeling methods using augmented radial basis functions (RBFs) have been successfully applied to engineering optimization and probabilistic analysis problems (Yin et al. 2016, Wang and Fang 2018, Wang et al. 2020). In this study, adaptive RBFs are implemented to build the metamodels of the reliability constraints. To progressively improve the accuracy of predicting the reliability indices, new samples are added using the optimal points found in the previous design iterations. The convergence criterion is checked and a new RBD iteration is initiated. Two numerical examples are solved and their solutions are obtained.

2 THE RELIABILITY DESIGN OPTIMIZATION PROBLEM

A general RBD problem is to minimize an objective function, as shown in Eq. (1 - 5),

\[
C(\mathbf{d}, \mathbf{\mu}_x)
\]

subject to

\[
P_{f,i}[g_i(\mathbf{d}, \mathbf{x}, \mathbf{p}) \leq 0] \leq P_{f,i}^\text{Target}, \quad i = 1, ..., n_p
\]  

(2)  

\[
g_i(\mathbf{d}, \mathbf{\mu}_x) \leq 0, \quad i = n_p + 1, ..., n_c
\]  

(3)  

\[
\mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U
\]  

(4)  

\[
\mathbf{\mu}_x^L \leq \mathbf{\mu}_x \leq \mathbf{\mu}_x^U
\]  

(5)  

where \(\mathbf{d}\) is the deterministic design variable vector, \(\mathbf{x}\) is the random design variable vector (mean vector \(\mathbf{\mu}_x\)), and \(\mathbf{p}\) is the random parameter vector (mean vector \(\mathbf{\mu}_p\)). The reliability constraint function in Eq. (2) can be expressed in terms of reliability index, as

\[
\beta_i(\mathbf{d}, \mathbf{\mu}_x) = \beta_i[g_i(\mathbf{d}, \mathbf{x}, \mathbf{p}) \leq 0] \geq \beta_i^\text{Target}, \quad i = 1, ..., n_p
\]  

(6)  

where \(\beta_i(\mathbf{d}, \mathbf{\mu}_x)\) and \(\beta_i^\text{Target}\) are the reliability index and target reliability index of the \(i\)th \((i = 1, ..., n_p)\) reliability constraint. Eq. (3) represents all deterministic design constraints.
3 THE RELIABILITY DESIGN OPTIMIZATION APPROACH

This section introduces the RBD approach using RBF metamodels and the overall RBD procedure.

3.1 RBF Metamodels

To decouple the numerical reliability analysis and design optimization, an approximate model, i.e., metamodel, of the reliability index in Eq. (6) is created, as shown in Eq. (7),

$$\tilde{\beta}_i(D_V) = \tilde{\beta}_i(d, \mu_d) \geq \beta_i^{\text{Target}}, \ i = 1, \ldots, n_p$$

where \(D_V = \{d, \mu_d\}\) is the design variable vector. An augmented RBF metamodel of the reliability index, \(\tilde{\beta}_i\), can be expressed as in Eq. (8),

$$\tilde{\beta}(D_V) = \sum_{j=1}^{n} \lambda_j \phi_9(D_V - D_{V_j}) + \sum_{k=1}^{p} r_k f_k(D_V)$$

in which the first part is the basic RBF model. An augmented RBF model is generally more accurate than the basic model (Wang and Fang 2018, Wang et al. 2020).

3.2 A Sequential Optimization Procedure

The following steps highlight the overall RBD procedure of this study:

(i) Generate initial sample points.
(ii) Perform reliability analyses of all performance functions at the initial sample points. Any reliability analysis method, such as Monte Carlo Simulations (Rubinstein 1981) or first-order/second-order reliability method (FORM/SORM) (Hohenbichler et al. 1987), can be applied. In this study, an alternative FORM is implemented and details of this first-order method can be found in the literature (Low and Tang 2007).
(iii) Construct RBF models of the reliability indices using all sample points.
(iv) Find a new optimal design point by performing numerical optimization based on RBF models of the reliability constraints.
(v) Check the stopping criterion and stop the design process. If the prescribed stopping criterion is not met, continue to the next step.
(vi) Perform reliability analyses of all performance functions at the current optimal design point (as an additional sample point). Go to step (iii).

4 ILLUSTRATIVE EXAMPLES

Two illustrative examples from the literature are solved using the proposed RBD approach: a mathematical example and a geotechnical system problem.

4.1 Example 1 – A Mathematical Problem

This is a mathematical example with two deterministic design variables, \(d = \{d_1, d_2\}\), and two independent random parameters, \(p = \{p_1, p_2\}\) (Aoues and Chateauneuf 2010). The two random parameters follow normal distributions with mean values of \(\{5.0, 3.0\}\) and a coefficient of variation of 0.3. The RBD problem is to minimize the following objective function in Eq. (9-12),

$$C(d) = d_1^2 + d_2^2$$

subject to
\[ \beta_1(d) \geq \beta_1^{\text{Target}} = 2.32 \quad (10) \]
\[ g_1(d,p) = \frac{1}{\xi} d_1 d_2 p_2^2 - p_1 \quad (11) \]
\[ 5.0 \leq d_1, d_2 \leq 15.0 \quad (12) \]

To create the RBF metamodel of the reliability constraint in Eq. (10), a total of nine initial sample points are generated. Reliability analyses are carried out using FORM (Low and Tang 2007), and the reliability indices are used to create an initial RBF metamodel of the reliability index. An optimal point \( d = \{5.241, 5.241\} \) is found in the first RBD iteration. The optimal objective of 54.945 represents an error of 12.92\%, compared to the final converged design objective (63.095). Reliability analysis is performed at this point and an updated RBF metamodel of the reliability index is created using ten samples, in order to find a new optimal point \( d = \{5.720, 5.720\} \). In the second RBD iteration, the optimal objective is 65.439 and the error is reduced to 3.71\%. This process continues, and a final converged optimal design point \( d = \{5.617, 5.617\} \) is found with a final optimal objective of 63.095. The sequential RBF metamodeling method and design optimization work well. To achieve convergence of the design, a total of four iterations and twelve sample points are required, which represent twelve reliability analyses of the probabilistic constraint in Eq. (10). The four RBD iterations are listed in Table 1.

<table>
<thead>
<tr>
<th>Iteration number</th>
<th>Number of samples</th>
<th>Design variables</th>
<th>Reliability index (RBF) ( \beta_1 )</th>
<th>Reliability index (Original function) ( \beta_1 )</th>
<th>( C(d) )</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>( d_1 ) 5.241</td>
<td>2.320</td>
<td>2.247</td>
<td>54.945</td>
<td>12.92%</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>( d_1 ) 5.720</td>
<td>2.320</td>
<td>2.339</td>
<td>65.439</td>
<td>3.71%</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>( d_1 ) 5.620</td>
<td>2.320</td>
<td>2.321</td>
<td>63.164</td>
<td>0.11%</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>( d_1 ) 5.617</td>
<td>2.320</td>
<td>2.320</td>
<td>63.095</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

4.2 Example 2 –A Drilled Shaft Example

This is an example adapted from the literature (Juang et al. 2013, Khoshnevisan et al. 2014). An axially loaded drilled shaft in loose sand is considered. The diameter \( B \) and length \( D \) of the drilled shaft are treated as random design variables, i.e., \( x = \{B, D\} \), and their mean values are \( \mu_x = \{\mu_B, \mu_D\} \). The RBD is to minimize the total concrete volume, as

\[ C(\mu_x) = \frac{\mu_B^2 \mu_D}{4} \quad (13) \]

subject to

\[ \beta_1(\mu_x) \geq \beta_1^{\text{Target}} = 3.2 \quad (14) \]
\[ \beta_2(\mu_x) \geq \beta_2^{\text{Target}} = 2.6 \quad (15) \]
\[ g_1(x,p) = Q_{ULS} - F_a - F_t \quad (16) \]
\[ g_2(x,p) = Q_{SLS} - F_a - F_t \quad (17) \]
\[ 0.3 \leq \mu_B \leq 0.6 \quad m \quad (18) \]
\[ 10.0 \leq \mu_D \leq 25.0 \quad m \quad (19) \]
The ultimate limit state (ULS) and serviceability limit state (SLS) of a drilled shaft are defined in Eq. (16) and Eq. (17), and their target reliability indices are 3.2 and 2.6, respectively. In addition to the two random design variables, this example also includes seven random parameters. More details of the random design variables and parameters as well as the explicit forms of the ULS and SLS performance functions in Eq. (16) and Eq. (17) are available in the literature (Khoshnevisan et al. 2014), therefore they are not presented here.

In the first RBD iteration, a total of nine sample points are generated, and the RBF metamodels of the reliability indices in Eq. (14) and Eq. (15) are created. An optimal point $\mathbf{x} = \{0.406, 16.443\}$ is found and the optimal objective is 2.129 m$^3$. Reliability analyses are performed at this point and updated RBF metamodels of the reliability indices are created, before numerical optimization is applied again. This process continues and the final convergence is achieved in the fourth iteration using a total of twelve sample points. The final optimal design point is $\mathbf{x} = \{0.367, 20.009\}$ and the converged optimal objective is 2.112 m$^3$. The iteration history of RBD is listed in Table 2.

### Table 2. Example 2: RBD iterations.

<table>
<thead>
<tr>
<th>Iteration number</th>
<th>Number of samples</th>
<th>Design variables</th>
<th>Reliability index (RBF)</th>
<th>Reliability index (Original function)</th>
<th>$C(\mathbf{x})$ (m$^3$)</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>$\mu_B$ (m) 0.406</td>
<td>$\mu_B$ (m) 16.443</td>
<td>$\beta_1$ 3.200 $\beta_2$ 3.313</td>
<td>2.129</td>
<td>0.79%</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>$\mu_B$ (m) 0.319</td>
<td>$\mu_B$ (m) 25.000</td>
<td>$\beta_1$ 3.200 $\beta_2$ 3.275</td>
<td>1.995</td>
<td>5.53%</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>$\mu_B$ (m) 0.363</td>
<td>$\mu_B$ (m) 20.321</td>
<td>$\beta_1$ 3.200 $\beta_2$ 3.284</td>
<td>2.103</td>
<td>0.44%</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>$\mu_B$ (m) 0.367</td>
<td>$\mu_B$ (m) 20.009</td>
<td>$\beta_1$ 3.200 $\beta_2$ 3.283</td>
<td>2.112</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

### 5 SUMMARY AND CONCLUDING REMARKS

A decoupled RBD approach is studied in which the reliability analysis and numerical optimizations are performed in a sequence. In addition, a metamodeling technique is adopted and integrated with a sequential optimization procedure so that the accuracy of the reliability analysis is progressively improved in RBD iterations. The RBD approach is applied to two illustrative examples and it works well. The design convergence is achieved in only a few iterations. The proposed design approach is a general method and provides a useful alternative for solving RBD of geotechnical engineering problems. Further research is helpful to extend the proposed framework to RBD of other complex civil engineering problems.

### References


