PERFORMANCE COMPARISON OF DERIVATIVE-FREE OPTIMIZATION METHODS

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Structural optimization for parameters and non-parameters-based modelling are adopted in almost all area of science and engineering due to their inherent advantages in contrary to manual tuning. Typically, in the easiest case regression analysis is used to develop a data-based model for the future applications and forecasting. However, the aforementioned modelling approach may be suitable for well-defined data (i.e., elastic model) but might be difficult for complex data type where linear type model may show very poor performances. As a result, the adaptation of nonlinear type or complex models (e.g. a combination of linear and nonlinear parts) are unavoidable. For instance, modeling of a hysteresis type behavior would not be possible via any linear type models while a Bouc-Wen type nonlinear model would be a better choice. Hence, to avoid early mentioned issues and to achieve better performances, the derivative-free or search-algorithm optimization might be suitable alternatives. Herein, due to the underlying advantages, the derivative-free search algorithms have been adopted to optimize model performances and interpret both linear and nonlinear type data. To be precise, two different derivative-free algorithms namely, (i) Nelder-Mead Simplex Method, and (ii) Genetic Algorithm have been studied. The outcome of this study shows that in both former mentioned cases search-based algorithms have better performance in terms of capturing the behavior of the true data via optimization. The real-life applications of optimization algorithms are numerous as these tools are used in many branches of science and engineering.

Keywords: Derivative-free optimization algorithm, Parameters-based model, Nelder Mead Simplex Method, Non-parametric model, Genetic algorithm.

1 INTRODUCTION

The optimization algorithms are widely used in the various area of mathematics, economics, science, computing and engineering (Du et al. 2008, Martins and Ning 2021). The goal of any optimization algorithm is to minimize an objective function by finding a set of parameters those provides a best fit the true data (Hansen and Walster 1993, Lagarias et al. 1998, Kwok et al. 2006, Hamidi 2012, Miah 2020). The optimization algorithms can be separated based on their type such as linear, nonlinear, constrained, unconstrained. Among a wide range and type of optimization algorithms, the nonlinear unconstrained type optimization algorithms are gaining popularity due to their superior performance in terms of finding global optimization (where more than one local minima can found). In other words, the aforementioned optimization algorithm performs recursive iteration and the process is known-as heuristics. The most common type of heuristic type optimization algorithms is genetic algorithm (GA) (Holland 1992), particle swarm optimization (PSO) (Kennedy and Eberhart 1995), derivative-free type e.g. Nelder-Mead simplex method.
The GA algorithm has been employed to wide range of applications in optimization problems such as; aircraft landing systems (Mobaieen et al. 2012), modeling MR damper behavior (Kwok et al. 2007, Miah et al. 2015). While the PSO optimization algorithm is adopted to optimize the LQR controller parameters (Ghoreishi and Nekoui 2012), structural design optimization (Perez and Behdinan 2007), modeling MR damper behavior (Kwok et al. 2006), control system design (Hamidi 2012). However, the performance of individual optimization algorithm may vary significantly depending on the definition of the fitness function and given initial conditions. There are many researchers who have discussed and given guideline for setting optimization parameters and their optimizations criterions (Wolkowicz 1981, Nemirovskii and Yudin 1983, Vanderplaats 1984, Hansen and Walster 1993, Pinter 1995, Nocedal and Wright 1999, Todd 2001). Due to many inherent advantages of the nonlinear unconstrained optimization, herein, two nonlinear unconstrained optimization algorithms have been studied and their performances are compared. Last but not least, the applications of optimization algorithms are not limited to any specific field of science and engineering. The real-life applications can be found in modeling, parameters optimization, performance enhancement, and so on.

2 THE OPTIMIZATION ALGORITHMS

The optimization problem simply deals with minimization of a cost function by selecting a set of parameters that provides the best outcome of the desired problem. Optimization problems can be separated into two main categories such as discrete and continuous optimization. And the application of the optimization is extensive, the applications can be found in mathematics, medical science, finance and economics, computing, and engineering (Du et al. 2008, Martins and Ning 2021). Additionally, optimization problem can linear or nonlinear depending on the definition of the individual problem. Herein, two nonlinear unconstrained type optimizations are employed to investigate their performances.

2.1 Nelder-Mead Simplex Method

The Nelder-Mead simplex algorithm (NM) was introduced by Nelder and Mead in 1965. In short, the NM consist of the following steps: (i) order, (ii) reflect, (iii) expand, (iv) contract (outside and inside), (v) perform a shirk step. The detail formulation and more info about the method can be obtained through their original work (Nelder and Mead 1965). In MATLAB, a function called “fminsearch” can be used to perform the nonlinear and unconstrained optimization via NM. This algorithm is also known as derivative-free as a result the optimization process runs quite faster in contrast to it alternatives.

2.2 Genetic Algorithm

The genetic algorithm (GA) is widely used as an optimization tool that uses the concept of biological evolution. Though principles of evolution were already in the field of science and engineering but in the 1970s John Holland is the one who has introduced the detail of the application of GA (Holland 1992). Briefly, the GA contains the following three main steps: (i) selection of parents, (ii) crossover, and (iii) mutation. In those steps the parents are selected via a stochastic process, later child is form from the parents, and, finally, mutation process changes the individual parents to form children. This is a recursive process the simulation goes on and on until it reaches the global minima. In others words, the iteration process continuous until the best fit of the objective function by selecting a set of optimized parameters are found. A function called “ga” is given in MATLAB to perform the optimization via genetic algorithm. It needs to be noted that
the performance of the optimizer heavily depends on the definition of the problem and proper initialization. If the aforementioned conditions are not satisfied, each iteration may take quite long or may just stop.

3 RESULTS AND DISCUSSION

This study has performed the optimization numerically by using MATLAB. It is mentioned earlier that the built-in functions such as “fminsearch” and “ga” have been used to perform the optimization by utilizing NM and GA, respectively. In order to evaluate the performances two examples are used and both optimizations algorithms have been employed to get the best results. To perform the optimization an objective function has been defined as given below in Eq. (1),

\[ f^{\text{opt}} = \sqrt{\frac{1}{N} \sum_{n=1}^{N} |f_{\text{act}} - f_{\text{sim}}|^2} \]  

(1)

where \( f^{\text{opt}} \) is the function that needs to be optimized, \( f_{\text{act}} \) indicates the true function, \( f_{\text{sim}} \) is a representative function that can render the true data. As an example, a sample iteration plots of optimizers (e.g. GA and NM) have been depicted in Fig. 1.

![Fig. 1. Sample iteration results of GA (left) and NM (right).](image)

3.1 Simple Example

A nonlinear curve has been selected as a simple example. And the objective function is defined to minimize the error between the true data to optimized model data.

![Fig. 2. True data compared with the optimized results of GA and NM.](image)
All the optimization conditions such as initialization and objective function remain same for both algorithms. The optimized results by GA is depicted in Fig. 2 (left). It is mentioned earlier both optimization algorithms have been employed to optimized the same objective function. The iteration plot and the optimized results by NM is given in Fig. 2 (right). It can be easily seen that both algorithms have managed to render the true behavior quite accurately. Finally, an inter comparison between GA and NM have been conducted (see Fig. 3). The aforementioned figure clearly shows the efficacy of both optimization algorithms.

![Fig. 3. Inter comparison of optimized results of GA and NM.](image)

### 3.2 Complex Example

In addition to the aforementioned example a more complex hysteresis type example has been selected. Similar to the previous example, in this case also, for the comparison of the overall performances, all the optimization conditions remain same for both optimizers.

![Fig. 4. The true hysteresis data compared with the optimized results of GA.](image)

Fig. 4 shows the optimized results of GA and it is evident that the true hysteresis behavior has been rendered by the GA quite accurately. Later, the same optimization is performed by the NM optimizer and the optimized results and iteration plot is shown in Fig. 5. Again, the optimized
performance has confirmed that even in case of such complex behavior NM is capable of capturing the true phenomena quite effectively.

![Graph showing true data and optimized results via NM.](image)

**Fig. 5.** True data and the optimized results via NM.

![Graph showing optimized performance comparison between GA and NM.](image)

**Fig. 6.** Optimized performance comparison between GA and NM.

As a last check the inter comparison of the GA and NM is performed (see Fig. 6) and it is observed that both optimization algorithms are effective in rendering such complex hysteresis type behavior.

### 4 CONCLUSIONS

This study has investigated the performances of two nonlinear unconstrained optimizers. To evaluate and compare the performances two examples have been selected and both optimizers have been employed to optimize the objective functions. In short, for both examples it has been noticed that NM algorithm shows better performances in terms of best fit and the required iterations time. However, it has been observed that the performances of GA can be heavily affected on the initialization and the definition of the objective function. And also, it needs to be noted that the studied optimizers have quite different inherent optimization process. Hence, the performance of the individual algorithm may depend on the complexity of the optimization problem as well as initialization. Due to many real-life applications such as linear or nonlinear model’s parameters...
optimization, performance enhancement of existing model, this study provides very useful information that may help engineers in their decision-making process for optimization related problems.

Acknowledgments
The authors highly appreciate the support of the Graz University of Technology (TU Graz), Graz, Austria.

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