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ANALYSIS OF EMPIRICAL COMPRESSION INDEX EQUATIONS USING THE LIQUID LIMIT

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This paper proposes a new method to evaluate the reliability of published empirical formulas in terms of accuracy and applicability to different soil types. Different empirical models are proposed to properly approximate the compression index for a wide range of liquid limits and soil types. They were developed using a unique Soil Property Line (SPL) developed using a substantial number of published regression equations and compression data. Familiar empirical equations were examined for their reliability in predicting the compression index of clay for any liquid limit. A comparison was made between available and newly-proposed empirical formulas using combined regression data sets compiled independently by several authors. The newly proposed empirical compression index equations are applicable to wide ranges of clay soils, validating other published relationships. The degree of scatter and variations in the computed compression index values are minimized for any liquid limit.

Keywords: Compression data, Regression equations, Empirical equations, Compression index of clay, Soil Property Line, Formula.

1 INTRODUCTION

Several empirical equations have been developed to relate compression index (C_c) to soil index properties. Some equations are supposed to reflect C_c of all soils while others are limited to specific soil types and/or geography. Most authors used the correlation coefficient (R^2) as a lone measure to justify their applicability to a wide range of soils. Little or no information was provided relative to the number of data points used and/or the standard error. Further, the lack of uniformity in data collection and data interpretation makes it difficult to verify the accuracy of derived empirical equations. However, a large number of publications are now available to warrant a closer look at the validity, accuracy, and usefulness of many available empirical formulas for C_c estimation of fine-grained soils to their liquid limit (LL). The more widely used equations to estimate C_c are those developed by Skempton (1944), Terzaghi and Peck (1948), and Hough (1957). Other less well-known equations include Tsuchida (1991), Azzouz et al. (1976), and Koppula (1981). Besides statistical measures, these equations seem to lack a logical and/or theoretical basis. The applicability of many of these equations to organic soils has not been established. 3-D models clearly show that consolidation pressures cannot be ignored in organic soils irrespective of the index property being used; C_c for clay sediments is actually related to consolidation pressure. Al-Khafaji and Andersland (1981) showed that the use of C_c in settlement calculations for organic soils is not justified. For a majority of practical problems, combining mineral and organic soils data is not suitable. This paper undertakes an exhaustive comparative study of available empirical equations, comparing their applicability to available published and independently-collected data. Also, more insight is provided into the nature of future development of empirical equations.

2 AVAILABLE EMPIRICAL COMPRESSION INDEX EQUATIONS

Empirical equations to estimate the C_c are valuable because they are generally viewed as substitutes for consolidation tests. Approximate C_c values are important in preliminary settlement studies and indicate the magnitude of C_c for conducting consolidation tests. The soil index property used to estimate C_c should be easily measured in the laboratory. The majority of empirical formulas linearly relate C_c to LL, natural water content, and *in situ* void ratio, as shown in Table 1.

Table 1. Empirical equations for the compression index approximation using the liquid limit.

Equation	**Applicability**	**Reference**
C_c = 0.007((LL – 7)	Remolded clays	Skempton, 1944
C_c = 0.0186(LL – 30)	Motley clays from Sao Paulo city	Cozzolino, 1961
C_c = 0.006(LL – 9)		Azzouz et al., 1976
C_c = 0.014LL – 0.168		Park and Lee, 2011
C_c = 0.0046(LL – 9)		Bowles, 1989
C_c = 0.007(LL – 10)	Remolded clays	Skempton 1944
C_{c10} = 0.009(LL – 8)	Osaka Bay clay	Tsuchida 1991
C_c = 0.009(LL – 10)	Normally consolidated clays	Terzaghi and Peck, 1967
C_c = 0.006(LL – 9)	All clays with LL V 100%	Azzouz et al., 1976
C_c = 0.009(LL – 8)	Osaka Bay clay	Tsuchida, 1991

Skempton conducted consolidation tests on remolded specimens for many types of clay where the initial moisture content of the materials were at the LL, and developed a relationship between C_c and LL. Terzaghi and Peck suggested that Skempton's equation could be modified to reflect a C_c of normal consolidation by simply multiplying it by a factor of approximately 1.3. The empirical expressions in Table 1 share one commonality – all are based on regression analysis of laboratory test data. Hough (1957) was the first to recognize that important differences exist between organic and mineral clay soils and suggested two different empirical equations to estimate C_c for the two types of soils. He also introduced several formulas to estimate C_c for cohesionless soils. Lambe and Whitman (1969) suggested that empirical expressions were not reliable, based in part on a graphical correlation between ratio $C_c/(1 + e_0)$ versus natural water content for a number of soil samples.

3 VALIDITY OF EMPIRICAL COMPRESSION INDEX EQUATIONS

Examination of consolidation data provided by Lambe and Whitman (1969), Rendon-Herrero (1980), and Mayne (1980) illustrates the need for an objective and rational method to validate empirical equations for compression index approximation. While nonlinear and multiple regression equations may be applicable in certain cases, these are not recommended due to inherently large fluctuations in approximated dependent parameters (C_c). Therefore, a new method is proposed to qualitatively and quantitatively determine the validity of linear regression equations used to estimate C_c. A number of regression equations were developed using one or more combinations of three independently compiled data sets and the linear empirical formulas listed in Table 1. A linear model relating the C_c to LL was assumed:

$$C_c = \alpha_L + (\beta_L)(LL) \quad (1)$$

α_L and β_L are the regression coefficients relating C_c to LL for a given data set. Regression analysis was then performed using the combined data set (182 data points). Objectivity and unbiased analysis require that one must not be selective in choosing data points used in regression analysis. For this reason, the range of C_c was arbitrarily limited in the ranges of 0-3, 0-2, 0-1, 0-0.5, and 0-0.25, and the corresponding regression equations were developed. The LL was limited to ranges of 0-200, 0-100, 0-75, and 0-50 and the corresponding regression equations determined. This process was applied to each of the three independent data sets reported by Lambe and Whitman, Herrero, and Mayne, using the same limits on C_c and LL. The resulting regression coefficients (α_L and β_L), correlation coefficients (R^2), standard errors σ_e, average liquid limit LL_{avg}, and average compression index C_{cavg} are shown in Table 2.

Table 2. Regression analysis results for compression index as a function of liquid limit (Based on data reported by Lambe and Whitman, Herrero, and Mayne).

Based on Combined data								
Eq. No.	C_{cavg}	LL_{avg}	Limit	# Points	R^2 (%)	σ_e	α_L	β_L
R1	0.469	80.56	Full Range	182	48.6	0.4682	0.077507	0.004859
R1-1	0.432	78.65	$0\le Cc \le 3$	181	61.6	0.2621	0.14321	0.003673
R1-2	0.41	72.72	$0\le Cc \le 2$	179	49.3	0.2629	0.141712	0.003691
R1-3	0.329	63.51	$0\le Cc \le 1$	165	31.8	0.195	0.178307	0.002378
R1-4	0.234	51.54	$0\le Cc \le 0.5$	134	31.4	0.096	0.04774	0.003614
R1-5	0.156	44.62	$0\le Cc \le 0.25$	81	8.7	0.0532	0.104064	0.001173
R1-6	0.37	61.24	$0\le LL \le 200$	172	64.9	0.1788	-0.161484	0.008671
R1-7	0.3	54.71	$0\le LL \le 100$	156	33.2	0.1683	-0.035164	0.00612
R1-8	0.262	47.59	$0\le LL \le 75$	128	27.1	0.1513	-0.08336	0.00726
R1-9	0.196	38.34	$0\le LL \le 50$	73	7.1	0.1364	-0.016861	0.00555

Careful examination reveals that reducing the data in Table 2 by only a few points has dramatic effects on R^2. This is true irrespective of the total number of data points analyzed. For example, the R^2 for the combined data increased from 48.6 to 61.6 after the number of data points was reduced from 182 to 181 points. This makes it difficult to decide which data points to include or exclude from the analysis. In general, one should use empirical formulas with high correlation and low standard error. On that basis, one may select a number of empirical equations for a given range of C_c or LL. The implication is that no regression equation can do the job of correctly predicting the C_c over the full range of LL values expected for soil. Hence, Eq. (R1-6) is likely the most reasonable empirical expression for soils with LL less than 200%. This is because it is based on 172 data points and has the highest R^2 with a relatively small corresponding standard error. Other empirical expressions may be selected for different LL ranges. The derived empirical expressions appear to be varied and dependent on the number of data points involved. At first glance, it seems impossible to derive any substantive conclusions. Fortunately, consideration of the regression coefficients α_L and β_L shows that they are related linearly irrespective of R^2. Although regression coefficients corresponding to small correlations indicate lack of trend, the relationship between the regression coefficients holds as shown in Figure 1.

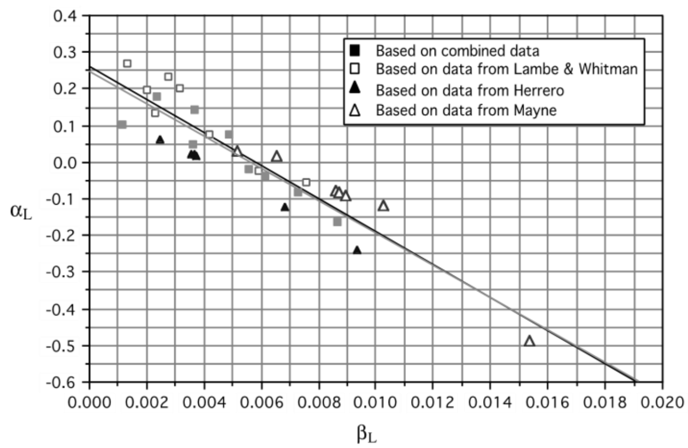

Figure1. Property line for the compression index and liquid limit.

Two linear regression relationships were determined between the regression coefficients. The first relationship (Eq. (2) with R^2=0.808) pertains to regression coefficients for the combined analysis of all the data sets:

$$\alpha_L = 0.24608 - 43.949\beta_L \tag{2}$$

The second regression equation (Eq. (3) with R^2=0.863) was derived using the three separate analyses on data reported by Lambe and Whitman, Herrero, and Mayne:

$$\alpha_L = 0.2624 - 45.190\beta_L \tag{3}$$

Clearly the differences between Eqs. (2) and (3) are minor, and therefore an average relationship can be defined for regression coefficients of empirical formulas relating C_c to LL. This relationship is referred to as the Compression Liquid Limit (CLL) Line or Property line, and is given approximately as:

$$\alpha_L = 0.254 - 44.57\beta_L \tag{4}$$

Eq. (4) represents a soil property line relating to applicability of all linear relationships between C_c and LL. This is a significant finding in that it is now possible to examine available empirical formulas and judge whether such relationships are meaningful. More importantly, Eq. (4) applies irrespective of the R^2 associated with a given formula. It appears that positive α_L-values would result when large LL values are used in regression analysis, suggesting that organic soils are represented by points above the line α_L=0. More data is needed to examine this hypothesis. Equations developed by Bowles, Mayne, Terzhagi, and Skempton are compared with Eq. (4) in Figure 2.

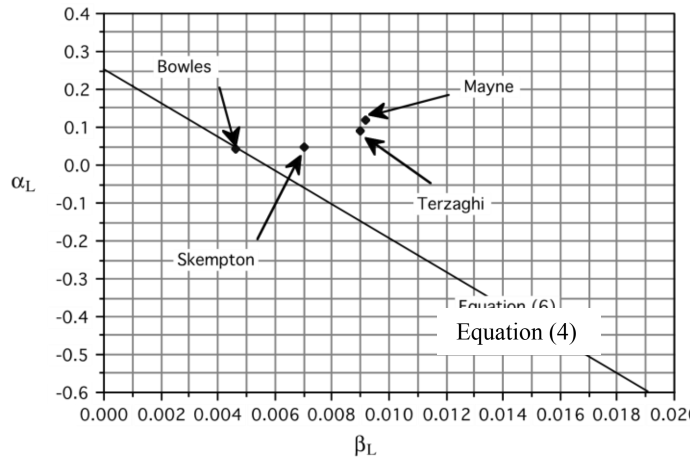

Figure 2. Relationship between regression coefficients for relationships between compression index and liquid limit.

Clearly, Mayne's empirical equation is farthest from the CLL line (Eq. 4). It is important to note that the equation published by Mayne (1980) was in error. The equation derived based on Mayne's own data plots close to the CLL line. Mayne's published equation was significantly influenced by a single data point corresponding to C_C=7.145 and LL=426. In fact, the Mayne empirical equation is closer to the CLL line when the two points corresponding to liquid limit greater than 200% were removed from the data set. All cited empirical equations overestimate the C_c, implying that these estimates are on the safe side. Eq. (4) shows that the Terzhagi and Peck equation based on multiplying Skempton's equation by 1.3 might be risky. Skempton's equation itself plots fairly close to the CLL line. Based on Eq. (4) and the analysis of the combined data, it is now possible to make a recommendation relative to C_c estimation using LL. Table 2 shows that Eq. R1-6 has the largest R^2 of equations corresponding to combined data, with a value of β_L, which is nearly 0.009.

4 CONCLUSIONS

Most empirical equations used to estimate compression index of soils in terms of soil index properties have been developed using data for disturbed as well as undisturbed soils. The variability of soil parameters, soil types, and machine- and operator- errors makes it impossible to suggest a unified approach to compression index estimation. Unlike mineral soils, organic soils are highly unstable and their properties change under constant effective consolidation pressure. Consequently, prediction of C_c should be limited to mineral soils. Most empirical formulas to estimate C_c are based on liquid limit, water content, and void ratio, and most are linear relationships restricted to one independent variable. While some of these empirical equations are restricted to specific soils, others are supposedly applicable to all soils. Use of these formulas is often legitimized based on the R^2 value but no attempt has been made to examine their applicability to independently-compiled data. Consideration of a number of widely-known empirical compression equations with data revealed interesting and useful possibilities. Examination of data scatter reveals that high values of LL are generally associated with organic and volcanic soils. The inclusion of such data points in derivations of empirical formulas could alter the applicability of many of these equations to mineral soils. The variability of C_c relating to organic soils is well documented. In fact, Al-Khafaji and Andersland (1981) have shown that the use of C_c in settlement calculations of organic soils is not justified. Based on work presented in this paper (Figure 2), it may become possible to define regions of applicability to a variety of soils. In fact, it may even be possible to suggest correlation coefficients based on the number of data points included in regression.

Dedication

This paper is dedicated to Dr. and Mrs. Orlando Andersland.

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