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DIRECT METHOD TO MODEL SEEPAGE FOR AN EMBANKMENT WITH A DRAIN

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Earth dams and embankments are used around the world to protect structures and facilities in low-elevation areas. If the phreatic surface intersects the downstream slope, the embankment is likely to fail, potentially resulting in enormous losses to life, infrastructure, and the environment. Drainage blankets are typically placed downstream to prevent the phreatic surface from intercepting the downstream slope. In the United States, the US Department of Interior requires a minimum of 10 feet clearance between the phreatic surface and embankment's downstream slope. The length of the drainage blanket to ensure this condition is typically estimated using a cumbersome and inefficient trial-and-error approach, assuming various blanket lengths until a satisfactory clearance is achieved. This paper presents a closed-form solution to the problem using a newly-developed formula to determine the blanket length for any specified clearance, which represents a significant advancement over the conventional trial-and-error graphical approach. Using the model presented in this paper, the quantity of seepage can also be estimated without the need to develop a flow net.

Keywords: Earth dams, Embankments, Blanket-length, Phreatic surface.

1 INTRODUCTION

Earth dams and embankments play an important role in protecting cities and installations from flooding conditions in low-lying areas. Since a significant hydraulic gradient is expected between the upstream and downstream sections of the dam or embankment, it is inevitable for seepage to occur across the barrier. Seepage inherently weakens the soil shear strength and adversely impacts other mechanical properties of the soil. Failures of embankments and earth dams are well documented in the literature (Foster et al. 2000, Sachpazis 2013). It is essential to route the seepage efficiently without compromising the structural stability of the barrier. Drainage blankets are used to maintain the structural stability of these barriers. The critical parameter in the design of the drainage blanket is its length. The length should be sufficiently long to guarantee a minimum clearance between the phreatic surface and the downstream slope (Al-Khafaji and Andersland, 1992).

The US Department of Interior and the Bureau of Reclamation requires a 10-foot minimum clearance and has also published guidelines for upstream and downstream slopes depending on construction materials, soil types, and types of dams (Chahar, 2004). Drains are typically constructed from coarse rock or a perforated pipe surrounded by crushed stone, and overlain by successively finer layers of sand and a filter to prevent fine soil material from being washed into it. Although properly-designed drains redirect the phreatic surface from the downstream slope, they can also serve to shorten the length of flow of seepage, causing greater quantities of seepage. Thus, consideration must be taken to design a drainage blanket long enough to keep the phreatic surface within the body of the dam, yet short enough to prevent excess volume of seepage (Creager and Barbour 1939).

Additionally, drains and filters are expensive to construct (Chahar 2004). Although transverse drains and filters carried into the downstream third of the embankment have been used since before 1910, the customary method for determining the correct length has been a trial-and-error approach (Creager and Barbour 1939). This method is cumbersome and error-prone. It is desirable to have a direct method to determine the required blanket length for a given clearance distance. Al-Khafaji (1991) developed an innovative direct determination method to determine the drainage blanket length, which is presented in this paper.

2 MATHEMATICAL FRAMEWORK

The definitional sketch of the embankment-drain problem is presented in Figure 1. The length of the drainage blanket is "L". The parabolic curve extends from point "0" at the upstream end to point "2" on the downstream end.

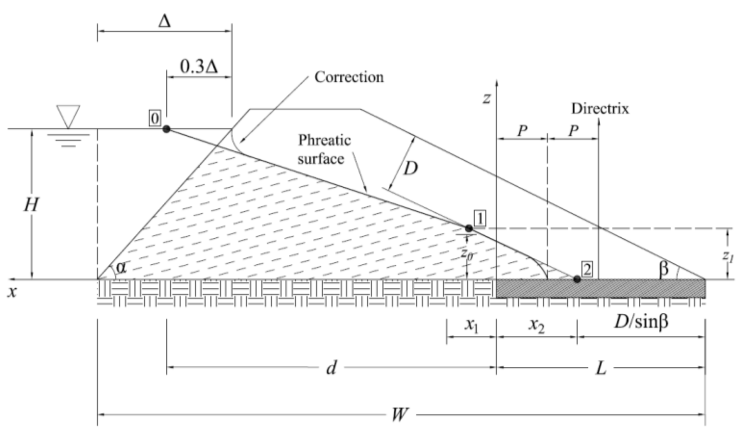

Figure 1. Definitional sketch of an earth dam with a drainage blanket (developed from Al-Khafaji and Andersland 1992).

The directrix of the parabola is located at a distance of 2P from the left edge of the drainage blanket. At the upstream and downstream ends of the phreatic surface, the deviation from the parabolic shape is shown as the correction. At the downstream end, the phreatic surface ends at a distance of "P" from the left edge of the drainage blanket. The clearance between the phreatic surface and the top of the downstream embankment surface is "D". The length between point "0" and the left edge of the drainage blanket is "d". Casagrande suggested that a parabola could be used to approximate the phreatic surface for unconfined flow, except at the upstream and downstream slope boundaries (Al-Khafaji and Andersland 1992). The phreatic parabolic surface passes through the apex located at a distance of "P" from the left edge of the drain and point "0", which is located at a distance of 0.3L from the intersection of the upstream water level with the surface of the embankment. For a parabola, the distance from the focus to any point on the parabola is equal to the horizontal distance from that point to the directrix. So, for any point away from the boundaries, the phreatic surface is described by:

$$z^2 = 4P^2 + 4Px \tag{1}$$

where x and z are the coordinates of any point on the parabola and "P" is the parabola parameter ($P=\left(\frac{1}{2}\right)\left(\sqrt{d^2+H^2}-d\right)$). The slope of the parabola from Eq. (1) is:

$$\left(\frac{dz}{dx}\right)=2\left(\frac{P}{z}\right) \tag{2}$$

The slope of the parabola at point 1 is very nearly equal to the downstream slope $\tan\beta$. The vertical distance to point 1 "z_1" at point "x_1" can be expressed as:

$$z_1=2\left(\frac{P}{\tan\beta}\right) \tag{3}$$

Substituting into Eq. (1) and solving for x_1, we get:

$$x_1=\left(\frac{P}{\tan^2\beta}\right)-P \tag{4}$$

Since the angle of internal friction for most soils is less than 45^o (i.e., $\tan\beta < 1$), x_1 is always positive. The tangent to the phreatic surface can be written as:

$$z=(x\tan\beta)+P\left(\frac{1}{\tan\beta}+\tan\beta\right) \tag{5}$$

At point 2 (where $x = x_2$) , the tangent to the phreatic surface intersects the blanket. At this point z = 0 and Eq. 8 can be simplified to yield:

$$x_2=-\left(P+\frac{P}{\tan^2\beta}\right) \tag{6}$$

Since the above equation is negative for any value of the downstream slope angle, the length of the blanket is calculated using the absolute value of x_2:

$$L=|x_2|+\frac{D}{\sin\beta} = \left(P+\frac{P}{\tan^2\beta}\right)+\frac{D}{\sin\beta} \tag{7}$$

The width of the embankment "W" can be used to develop an expression for "P":

$$W=0.7\Delta+d+L=0.7\left(\frac{H}{\tan\alpha}\right)+\left(\frac{H^2}{4P}-P\right)+\left(P+\frac{P}{\tan^2\beta}\right)+\left(\frac{D}{\sin\beta}\right) \quad (8)$$

Eq. (8) is simplified to yield a quadratic in terms of "P". The quadratic can be solved to yield the following solution for "P" since only the positive root is meaningful.

$$P=\left[W-0.7\left(\frac{H}{\tan\alpha}\right)-\left(\frac{D}{\sin\beta}\right)\right]-\sqrt{\left[W-0.7\left(\frac{H}{\tan\alpha}\right)-\left(\frac{D}{\sin\beta}\right)\right]^2-2\left(\frac{H^2}{8}\tan^2\beta\right)} \quad (9)$$

Once the parabola parameter "P" is estimated, the length of the drainage blanket "L" can be calculated using Eq. (7) for any specified clearance "D", downstream slope "β", total embankment width "W", total height of the embankment "H", and the soil friction angle. This is a direct and concise method to calculate the drainage length.

Huang (1986) offered a method for locating the unsteady state phreatic surface in temporary earth dams, such as those used for refuse disposal and sediment control where a steady state phreatic surface may never develop in their lifespan. A straight line was used in Huang's analysis to approximate the phreatic surface in unsteady state and was determined to still be conservative. While such an approximation might suffice for rough estimates, it is desirable to have a more reliable method based on the mathematical framework presented in this paper.

Ahmed (2005) modeled the location of the phreatic surface as a first-order partial differential equation with analytical and finite-element techniques. His analysis was conducted with the perspective that the phreatic surface is a constantly fluctuating boundary between the saturated and unsaturated zones of an unconfined aquifer, and is at atmospheric pressure. The errors between the numerical and analytical solutions were found to be insignificant.

Yussuf *et al.* (1994) developed a finite-difference solution to the Boussinesq equation using a Du Fort-Frankel method to predict the location of the phreatic surface. In the analysis, the numerical solution predicted a higher phreatic surface than the analytical solution and it was suggested that the numerical solution using the finite-difference approach might be more accurate. Although these authors have used finite-element and finite difference methods to estimate the shape of the phreatic surface, the parabolic equation presents a simpler and effective alternative. An accurate estimate of the length of the drainage blanket is critical because a short drainage blanket length can lead to inadequate clearance distance; an excessively large blanket length can lead to excessive seepage, which results in additional slope stability problems.

Chahar (2004) developed an equation to determine the maximum effective length of a horizontal drainage blanket corresponding to the maximum distance from the downstream slope cover to the phreatic line. The author concluded that if the drainage blanket length is increased beyond this length, it does not aid in increasing the downstream slope cover, and may even lead to increased volume of seepage due to a shorter path of flow. Chahar (2004) suggested that the distance from the downstream slope cover to the phreatic line is independent of the filter length and only dependent on the properties of the dam's geometry, such as the free board, top width, and upstream and downstream slopes. However, this is not the case because the parabola parameter "p" is clearly a function of "D", which is the distance between the phreatic surface and the downstream slope. Messaid and Boudoukha (2008), assumed the free surface follows a parabolic contour, and developed charts to estimate the length and position of the drainage blanket based on their study. Chahar (2004) also provided equations and charts to determine the necessary length for a given dam. The approach presented in this paper does not require use of charts to estimate the length of the drainage blanket.

3 ESTIMATION OF SEEPAGE

The Laplace equation can be used to characterize the flow through an embankment. However, the estimation of seepage analytically or using approximate numerical solutions is situation-specific. As a consequence, a general solution to solve for seepage flow through embankments is not readily available. This problem is solved using the approach presented in this paper. In fact, the calculation of the parameter "P" also allows for an estimation of the quantity of seepage for an embankment with a drain, upon application of Darcy's law. Figure 1 shows that at $x = 0$, $z = z_0 = 2P$. Therefore, the hydraulic gradient or the slope of the phreatic surface at $x = 0$ is equal to 2. Now, the seepage can be estimated for a unit width of embankment as:

$$q=kiA=k\left(\frac{dz}{dx}\right)\left(z_0\right)=2kP \tag{10}$$

From Eq. 10, it is clear that seepage can be estimated if the hydraulic conductivity and the parameter P for an earth embankment are known. Ghanbari and Zaryabi (2014) developed equations and charts to estimate seepage forces linked with drainage blanket length, and to estimate seepage flow. Ilyinksy *et al*. (1998) developed equations using an inverse method, to estimate the quantity of seepage. They suggested that understanding the flow of water along the phreatic surface was intricately linked to the drainage blanket length. Graber (2007) reported that the design of geosynthetic and aggregate subsurface drains requires open channel flow assumption rather than pressurized flow assumed by many authors. These results are more directly applicable to the design of subsurface drains. Based on the mathematical treatment presented in this paper, the quantity of seepage for an embankment with a drain can be estimated without the use of a flownet.

4 SUMMARY AND CONCLUSIONS

Embankments and earth dams are used widely across the globe as a cost-effective solution. Appropriate design of an embankment with a drain requires an accurate estimate of the blanket length and the seepage flow. In this paper, a method to solve both problems was presented using an innovative mathematical treatment to generate a closed-form solution. Using the equations presented in this paper, the length of the drainage blanket and the quantity of seepage can be estimated if the clearance between the downstream slope and the phreatic surface, downstream slope, total embankment width, total height of the embankment, the soil friction angle, and the hydraulic conductivity of the soil are known.

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