

# DESIGN OF ISOLATED BRIDGES USING POLYNOMIAL FRICTION PENDULUM ISOLATOR

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This paper is aimed to develop a design procedure of Polynomial Friction Pendulum Isolator (PFPI), a various-frequency sliding isolator, for decreasing the seismic responses of isolated bridges. Although sliding isolators have been widely used to mitigate seismic hazard, it may be not effective in decreasing the seismic responses of isolated structures subjected to near-field ground motions. The sliding surface of the PFPI is defined by a sixth-order polynomial function to avoid resonance under near-field ground motions. The restoring stiffness of the PFPI possesses softening section as well as hardening section. The structural acceleration response can be decreased by decreasing the restoring stiffness in softening section while the structural displacement response can be decreased by increasing the restoring stiffness in hardening section. However, it is difficult to determine the design parameters of PFPI in practical implementations. This study proposes a design procedure for the PFPI based on the bridge seismic design code in Taiwan. Designers can follow this procedure to easily design the bridge with PFPIs which satisfies the requirements of the code. The bridge with PFPIs designed by using this procedure is analyzed to realize the dynamic nonlinear responses of the bridge under artificial strong earthquake. The results show that the PFPIs effectively decrease the seismic responses of isolated bridges as compared with non-isolated bridges.

*Keywords:* Isolated bridge, Various-frequency sliding isolator, Seismic design, Dynamic response, Near-field ground motion.

## 1 INTRODUCTION

The seismic isolation with sliding bearings has been shown to be effective in protecting structures from earthquakes. The friction pendulum system (FPS) is currently one of widely-used sliding isolators. However, the fundamental vibration period of the structure with FPS isolators is elongated to a specific value due to the spherical sliding surface of FPS isolators. A resonant problem may occur when the structure with FPS isolators is subjected to a ground motion containing low-frequency components, such as a near-field earthquake (Lu *et al.* 2011). In order to avoid resonance and improve the performance of a sliding isolation system, a Polynomial Friction Pendulum Isolators (PFPI) is adopted in this study to design isolated bridges. The PFPI is similar to the traditional FPS, except that the geometry of its concaved sliding surface is defined by a sixth-order polynomial function rather than a spherical function. Therefore, the fundamental frequency of the structure with PFPIs continuously varies during vibration. In the past studies, PFPIs are proven to be capable of effectively mitigating the seismic

responses of isolated structures under near-field ground motions. Since the design parameters of PFPIs are more than those of traditional FPS isolators, it is difficult to determine the design parameters of PFPIs in practical implementations. This study proposes a design procedure for the PFPI based on the bridge seismic design code in Taiwan. Designers can follow this procedure to easily design the bridge with PFPIs which satisfies the requirements of the code.

## 2 POLYNOMIAL FRICTION PENDULUM ISOLATOR

Similar to a FPS isolator, a PFPI is mainly composed of a concaved sliding surface and a slider as shown in Figure 1. The slider slips on the sliding surface during earthquakes. The sliding surface is generally designed to be concaved and axially symmetric. As depicted in Figure 1, the cross-section of the sliding surface of the isolator can be expressed by a geometric function  $y(x)$  in an  $x$ - $y$  coordinate where  $y$  is the elevation of the sliding surface while  $x$  is the horizontal displacement of the slider. There are four forces acting on the slider including the vertical load applied on the slider,  $P$ , the normal contact force,  $N$ , the slider friction force,  $F_f$  and the horizontal shear force,  $U$ . The force  $U$  is induced by the relative horizontal motion between the superstructure and substructure. Lu *et al.* (2011) has derived the following equation to compute the horizontal shear force,  $U$ ,

$$U(x) = u_r(x) + u_f(x) \tag{1}$$

where  $u_r(x)$  and  $u_f(x)$  represent the restoring force and the friction force, respectively. Assuming the isolator is in its sliding state and the slope and frictional coefficient of the sliding surface is much smaller than one, these forces can be written as

$$u_r = P y'(x) \tag{2}$$

$$u_f = \text{sgn}(\dot{x})\mu P \tag{3}$$

where  $\mu$  denotes the friction coefficient between the slider and surface;  $\text{sgn}(\dot{x})$  means taking the sign of  $\dot{x}$ .

The PFPI isolator stiffness can be defined in two different approaches. One is the tangent stiffness,  $k_t(x)$ . The other is the secant stiffness,  $k(x)$ . They are defined as

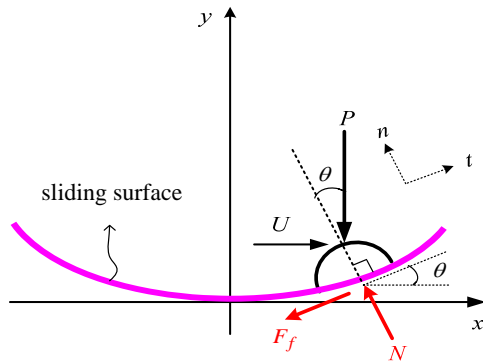


Figure 1. Force components applied on the slider of a PFPI isolator.

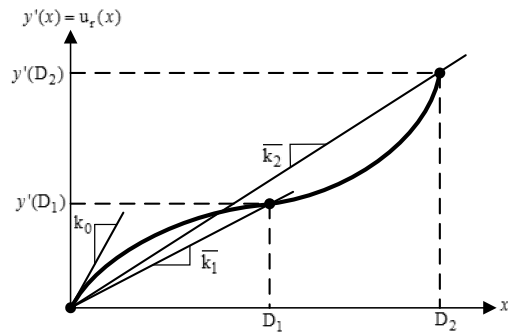


Figure 2. The restoring force vs. displacement of a PFPI.

$$k_r(x) = \frac{du_r}{dx} = P y''(x) \quad (4)$$

$$k(x) = \frac{u_r(x)}{x} = \frac{P y'(x)}{x} \quad (5)$$

The restoring force and stiffnesses of the PFPI system depend on the geometric function  $y(x)$  of the sliding surface as shown in Eqs. (2), (4), and (5). Therefore, the demanded mechanical property of the isolator can be realized by choosing the proper function  $y(x)$ . To satisfy multiple objectives in seismic design, this study defines the geometric function by the following sixth-order polynomial.

$$y(x) = \frac{1}{6}ax^6 + \frac{1}{4}cx^4 + \frac{1}{2}ex^2 \quad (6)$$

where  $a$ ,  $c$ ,  $e$  are three constant coefficients to be determined. Taking the first derivative of Eq. (6) and then substituting it into Eq. (2) yields the restoring force of the PFPI as

$$\bar{u}_r(x) = \frac{u_r(x)}{P} = y'(x) = ax^5 + cx^3 + ex \quad (7)$$

where the restoring force in Eq. (6) has been normalized with respect to the vertical load  $P$ .  $\bar{u}_r(x) = y'(x)$  is also called the normalized restoring force. Likewise, the normalized isolator stiffness  $\bar{k}(x)$  can be obtained as

$$\bar{k}(x) = \frac{k_r(x)}{P} = \frac{y'(x)}{P} = ax^4 + cx^2 + e \quad (8)$$

The normalized restoring force  $\bar{u}_r(x)$  of the PFPI defined by the fifth-order polynomial in Eq. (7) is an anti-symmetry curve as shown in Figure 2. The curve has three inflection points. One of the inflection points must be located at the origin; while the other two points are determined by the three polynomial coefficients  $a$ ,  $c$  and  $e$  in Eq. (7). By properly choosing the values of the three coefficients  $a$ ,  $c$  and  $e$  the restoring-force function  $y'(x)$  of the PFPI will possess a softening section followed by a hardening section as shown in Figure 2. In the softening section, the isolator stiffness is decreased while the stiffness is increased in the hardening section. The purpose of the softening section is to mitigate the acceleration response for an earthquake below the design level while the purpose of the hardening section is to reduce the isolator drift for an extreme earthquake beyond the design level.

Since the three coefficients  $a$ ,  $c$ ,  $e$  in Eq. (7) are purely mathematical, such coefficients are converted into other parameters having more engineering meaning (Lu *et al.* 2013). In Taiwan highway bridge seismic design code, two-level earthquakes shall be considered, including maximum design earthquake (MDE) and maximum considered earthquake (MCE). Assume  $D_1$  and  $D_2$  are the maximum displacements of

the isolator under MDE and MCE, respectively. Let  $\bar{k}_1$  and  $\bar{k}_2$  be the isolator's secant stiffnesses at two isolator displacements  $D_1$  and  $D_2$ , respectively; while  $\bar{k}_0$  be the tangent stiffness at the origin. Applying the three assumptions into Eq. (7), the three coefficients  $a$ ,  $c$ ,  $e$  can be written as

$$a = \frac{-(D_2/D_1)^2(\bar{k}_1 - \bar{k}_0) + (\bar{k}_2 - \bar{k}_0)}{D_1^2 D_2^2 [(D_2/D_1)^2 - 1]} ; c = \frac{(D_2/D_1)^4(\bar{k}_1 - \bar{k}_0) - (\bar{k}_2 - \bar{k}_0)}{D_2^2 [(D_2/D_1)^2 - 1]} ; e = \bar{k}_0 \quad (9)$$

### 3 DESIGN PROCEDURE OF ISOLATED BRIDGE WITH PFPIS

Lu *et al.* (2013) has proposed a conceptual design for the PFPIS. In this paper, a design procedure of isolated bridges using PFPIS is proposed based on Taiwan highway bridge seismic design code. Firstly, bridge engineers determine the maximum deck displacement  ${}^D D_d$  and  ${}^M D_d$  under MDE and MCE, respectively, according to the expected performance of bridges and practical limitations. Then the following procedures are provided to design the isolators to meet the provisions in the code.

#### 3.1 Design in Maximum Design Earthquake

In general, isolators are installed between deck and substructure, effective periods of seismically isolated structure are used to accommodate both the stiffnesses of the substructure and isolators. Compared to the period of non-isolated bridges, the period of isolated bridges are much longer and in the region far from the constant design spectral acceleration plateau to reduce the seismic-induced force. Assume the period of the isolated bridge is  ${}^D T_{eff}$  as

$${}^D T_{eff} = r_D T_{0D} \quad (10)$$

where  $T_{0D}$  is the period at the end of constant design spectral acceleration plateau for MDE.  $r_D$  is a magnification factor and should be larger than one. Then the effective stiffness  ${}^D k_{eff}$  of all isolators and substructures at displacement  $D_1$  for the isolators and displacement of  ${}^D D_{sub}$  for the substructure can be calculated.

$${}^D k_{eff} = \frac{4\pi^2(m_d + m_{sub})}{{}^D T_{eff}^2} \quad (11)$$

where  $m_d$  is the mass of the deck and  $m_{sub}$  is the mass of the substructures.

If the stiffness of the substructures is  $k_{sub}$ , the stiffness and displacement of the isolators,  $\bar{k}_1$  and  $D_1$ , respectively, can be calculated as

$$\bar{k}_1 = \frac{{}^D k_{eff} k_{sub}}{(k_{sub} - {}^D k_{eff})}, \quad D_1 = {}^D D_d - \frac{\bar{k}_1 {}^D D_d}{k_{sub} + \bar{k}_1} \quad (12)$$

The maximum isolator shearing force  $V_1$  can be calculated by multiplying  $\bar{k}_1$  by  $D_1$ . If the  $V_1$  is larger than expected, go back to increase the period of the isolated bridge,  ${}^D T_{eff}$ , and then recalculate the  $\bar{k}_1$  and  $D_1$  of the isolators. Next step is to calculate the equivalent viscous damping ratio for the isolators,  $\xi_1$ .

$$\xi_1 = \frac{4Q_1 D_1}{2\pi k_1 D_1^2} \quad (13)$$

where  $Q_1$  is the characteristic strength of the isolator as shown in Figure 3. If  $\xi_1$  is larger than 5%, the equivalent viscous damping of the isolated bridge,  ${}^D\xi_{eff}$ , must be calculated, which is not shown herein. Consequently, the maximum deck displacement  ${}^D D_d$  is checked by using the following calculation.

$${}^D D_d = \frac{S_{ad} {}^D T_{eff}^2 g}{4\pi^2} \quad (14)$$

where  $S_{ad}$  is the spectral acceleration of the isolated bridge under MDE.  $g$  is the acceleration of gravity. If the difference between the assumed  ${}^D D_d$  and calculated  ${}^D D_d$  is larger than the tolerance, it is required to reprocess the design procedure.

### 3.2 Design in Maximum Considered Earthquake

Since it is intended to decrease the isolator displacement under maximum considered earthquake, the hardening section of the PFPI's restoring properties is utilized to design the isolated bridge. The effective stiffness of the isolator,  $\bar{k}_2$ , at displacement  $D_2$  under MCE should be larger than the stiffness  $\bar{k}_1$  under MDE. Assume the stiffness  $\bar{k}_2$  to be 1.0-sec. period under MDE and MCE, respectively.

$$\bar{k}_2 = \frac{S_{M1} \bar{k}_1}{S_{D1}} \quad (15)$$

The displacement  $D_2$  can be calculated by the second equation of Eq. (12). Referring to the first equation of Eq. (12), the effective stiffness  ${}^M k_{eff}$  of the isolated bridge can be determined. Then the period of the isolated bridge,  ${}^D T_{eff}$ , is calculated by the known masses and stiffnesses. Following the same design procedure in MDE by replacing  $S_{ad}$  by  $S_{aM}$ , one can finally determine the  $\bar{k}_2$  and  $D_2$  of the isolators.

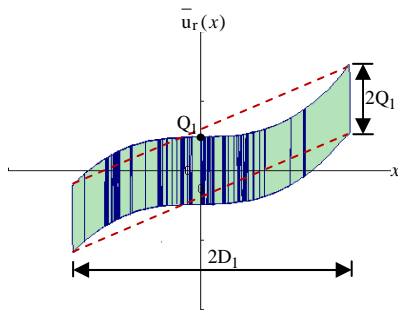


Figure 3. Enclosed area of one-cycle hysteresis loop of a PFPI.

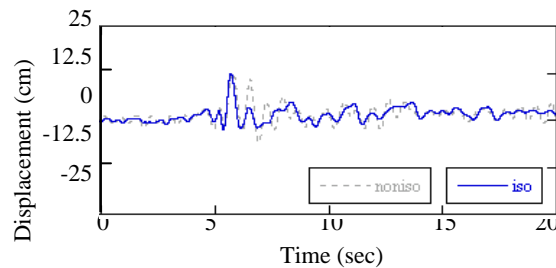


Figure 4. Deck displacement of non-isolated and isolated bridge.

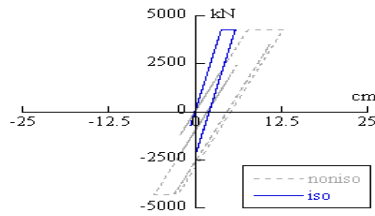


Figure 5. Hysteresis loop of column of non-isolated and isolated bridge.

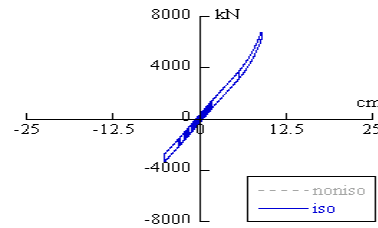


Figure 6. Hysteresis loop of the PFPI.

#### 4 DESIGN EXAMPLE AND RESULTS

A typical five-span continuous isolated bridge is designed by using PFPIs. A column with the effective deck mass on the top can be taken apart as a unit for design with its properties: deck mass 600 ton, column mass 243 ton, column lateral stiffness 112.7 MN/m, structure damping ratio 5%. Assume  ${}^D D_d$  and  ${}^M D_d$  are 0.15 m and 0.22 m, respectively. Choose  $r_d$  to be 2.0 and  $\bar{k}_0$  to be the same as  $\bar{k}_1$ . Based on the above design procedure, the displacements  $D_1$  and  $D_2$  of the PFPIs are 0.110 m and 0.147 m, respectively, while the corresponding effective stiffnesses  $\bar{k}_1$  and  $\bar{k}_2$  are 32.4 MN/m and 41.9 MN/m, respectively.

The seismic performance of the designed isolated bridge is checked by using structural dynamic analysis. The input ground motion is an artificial excitation whose response spectral accelerations are almost identical to the design spectrum in the code. Figures 4 and 5 presents the comparison of the deck displacements and the column hysteresis loops between non-isolated bridge and isolated bridge. Also the hysteresis loop of the PFPIs is shown in Figure 6. It can be observed that the seismic performance of the isolated bridge with the PFPIs has improved satisfactorily.

#### 5 CONCLUSIONS

A design procedure for the PFPI based on the bridge seismic design code in Taiwan is proposed in this study. Bridge engineers can follow this procedure to easily design the bridge with PFPIs which satisfies the requirements of the seismic code. A practical typical isolated bridge is designed by using PFPIs with application of this procedure. This isolated bridge is analyzed under artificial strong ground motion. The results show that the PFPIs effectively decrease the seismic responses of isolated bridges as compared with non-isolated bridges.

#### References

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