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DYNAMIC RESPONSE OF RIGID ROADWAY PAVEMENT UNDER MOVING TRAFFIC LOADS

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The study of the dynamic response of rigid roadway pavements subjected to dynamic loads such as vehicle loads has received significant attention in recent years, because of the relevance to the design of pavements. This paper presents an analytical solution based on the Modified Bolotin Method to analyze rigid pavements under moving traffic loads. The concrete pavement is modelled as an orthotropic damped plate resting on a continuous elastic foundation, whereby at its edges it is partially fixed. The natural frequencies of the system are computed from a system of two transcendental equations, obtained from the solution of two auxiliary Levy's type problems. The dynamic vehicle load is expressed as a concentrated load of harmonically varying magnitude, travelling with a variable speed along the rigid pavement. A numerical example is given, demonstrating the applicability of the theory to rigid roadway pavements under actual loading conditions. Therefore, it may be expected that this dynamic load approach may lead to more economic solutions as compared to those obtained from the conventional static load approach.

Keywords: Rigid roadway pavement, Modified Bolotin Method, Moving traffic loads, Continuous elastic foundation, Auxiliary Levy's type problem.

1 INTRODUCTION

The dynamic response of rigid roadway pavements to moving vehicle and aircraft loads had been the subject of numerous studies in recent years. Although the importance of more accurate dynamic analysis of rigid pavements had been realized, analytical solutions were available for simple cases only, partly due to the mathematical complexity involved. The simplest representation of a continuous elastic foundation had been provided by Winkler (Kerr 1964) who assumed the base consisting of closely spaced independent linear springs. The Winkler model neglected the interconnection among the soil layers and as a result had imposed some serious limitations in the physical modeling of the sub-soil system. This limitation could be improved by modelling the sub-grade as a two-parameter medium, which provided shear interaction between individual spring elements. This type of continuous elastic foundation is called a Pasternak foundation. Zaman *et al.* (1993) presented an analysis of rigid pavements resting on a two-parameter elastic medium subjected to a moving load. In his work, the dynamic moving load was modelled as a spring-dashpot unit and the foundation system was modelled as a thin plate. Later in 2010, Patil *et al.* (2010)

studied the dynamic analysis of a rigid pavement resting on a two-parameter soil medium.

2 THE GOVERNING DIFFERENTIAL EQUATION OF MOTION

In this research work, a rigid roadway pavement is modelled as an orthotropic homogenous elastic rectangular plate resting on an elastic Pasternak foundation. According to the classic theory of thin plates, the transverse deflection *w(x,y,t)* of the plate satisfies the partial differential equation:

$$\begin{aligned} &D_x \frac{\partial^4 w(x,y,t)}{\partial x^4} + 2B\frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w(x,y,t)}{\partial y^4} + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} + \gamma h \frac{\partial w(x,y,t)}{\partial t} \\ &+ k_f w(x,y,t) - G_s \nabla^2 w(x,y,t) = p(x,y,t) \end{aligned} \tag{1}$$

where D_x is the plate flexural rigidity in the x direction, B is the plate torsional rigidity, D_y is the plate flexural rigidity in the y direction, ρ is the mass density, h is the plate thickness, γ is the damping ratio, k_f is the foundation stiffness coefficient, G_s is the shear modulus of the Pasternak foundation and *p(x,y,t)* is the dynamic load on the plate. The traffic load *p(x,y,t)* is modelled as an equivalent concentrated load of harmonically varying magnitude moving in the x directional axis of the plate that can be expressed as follows:

$$p(x,y,t) = p[x(t), y(t), t] = P(t)\delta[x - x(t)]\delta[y - y(t)] = P_0(1 + \frac{1}{2}\cos\omega_v t)\delta[x - v_0 t]\delta[y - \frac{1}{2}b] \tag{2}$$

where P_0 is the maximum amplitude of the vehicle, ω_v is the angular frequency of the vehicle, v_0 is the constant speed of the vehicle, b is the length of the rigid pavement in the y direction. Due to the use of dowels and tie bars to join the rigid roadway pavement, all four sides of the orthotropic plate have elastic vertical translational support as well as elastic rotational restraint along the sides. Thus, the boundary conditions for each side of the plate are as follows:

Elastic vertical support along x=0:

$$V_{x=0} = D_x\left(\left(\frac{\partial^3 w(x,y,t)}{\partial x^3}\right) + \left(\frac{B + 2G_{xy}}{D_x}\right)\left(\frac{\partial^3 w(x,y,t)}{\partial x \partial y^2}\right)\right) = ks_x w(x,y,t) \tag{3}$$

Elastic vertical support along x=a:

$$V_{x=a} = D_x\left(\left(\frac{\partial^3 w(x,y,t)}{\partial x^3}\right) + \left(\frac{B + 2G_{xy}}{D_x}\right)\left(\frac{\partial^3 w(x,y,t)}{\partial x \partial y^2}\right)\right) = ks_x w(x,y,t) \tag{4}$$

Elastic vertical support along y=0:

$$V_{y=0} = D_y\left(\left(\frac{\partial^3 w(x,y,t)}{\partial y^3}\right) + \left(\frac{B + 2G_{xy}}{D_y}\right)\left(\frac{\partial^3 w(x,y,t)}{\partial x^2 \partial y}\right)\right) = ks_y w(x,y,t) \tag{5}$$

Elastic vertical support along $y=b$:

$$V_{y=b} = D_x\left(\left(\frac{\partial^3 w(x,y,t)}{\partial y^3}\right) + \left(\frac{B+2G_{xy}}{D_y}\right)\left(\frac{\partial^3 w(x,y,t)}{\partial x^2 \partial y}\right)\right) = ks_y w(x,y,t) \tag{6}$$

where G_{xy} is the shear modulus of the plate, ks_x is the elastic vertical translation stiffness in x direction, ks_y is the elastic vertical translation stiffness in y direction.

3 NATURAL FREQUENCIES OF THE SYSTEM

In order to solve the free vibration response of the problem described above, it is assumed that the principal elastic axes of the material are parallel to the plate edges and the free vibration solution of the problem is set as:

$$w(x,y,t) = W(x,y)\sin\omega t \tag{7}$$

where ω is the circular frequency of the system and $W(x,y)$ is a function of the position coordinates only. Then substituting Eq. (7) into the undamped homogeneous form of Eq. (1), the Eigen frequencies of the orthotropic plate with elastically restrained along its edges are obtained. By postulating the following Eigen frequency, which is analogous to the case of a plate simply supported at all edges (Alisjahbana and Wangsadinata 2012), natural frequencies of the system can be expressed as:

$$\omega_{mn}^2 = \left(\frac{\pi^4}{\rho h}\right)\left(D_x\left(\frac{p}{a}\right)^4 + 2B\left(\frac{pq}{ab}\right)^2 + D_y\left(\frac{q}{b}\right)^4\right) + \frac{k_f}{\rho h} + \frac{G_s}{\rho h}\left(\left(\frac{p\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2\right) \tag{8}$$

where p and q are real numbers to be solved from a system of two transcendental equations, obtained from the solution of two auxiliary Levy's type problems, also known as the Modified Bolotin Method (Pevzner 2000).

4 DYNAMIC RESPONSE OF THE RIGID ROADWAY PAVEMENT

The dynamic response of the plate can be found by using the method of separation of variables, which can be written in the following form:

$$w_{mn}(x,y,t) = \sum_{m=1}^{m}\sum_{n=1}^{n} X_{mn}(x)Y_{mn}(y)T_{mn}(t) \tag{9}$$

where $X_{mn}(x)$, $Y_{mn}(y)$ are Eigen functions, $T_{mn}(t)$ is a function of time which must be determined through further analysis. The differential equation for the coefficient functions $T_{mn}(t)$ can be expressed as:

$$\ddot{T}_{mn}(t) + 2\gamma\omega_{mn}\dot{T}(t) + \omega_{mn}^2 T_{mn}(t) = \int_0^a X_{mn}(x)dx\int_0^b Y_{mn}(y)dy\,\frac{p(x,y,t)}{\rho h Q_{mn}} \tag{10}$$

where Q_{mn} in Eq. (10) is a normalization factor that can be expressed as:

$$Q_{mn} = \int_0^a \int_0^b X_{mn}^2(x) Y_{mn}^2(y)\,dx\,dy \tag{11}$$

The particular solution of the temporal function $T_{mn}(t)$ can be represented in a form of the Duhamel convolution integral as follows:

$$T_{mn}^*(t) = \int_0^t \left(\frac{p(x,y,\tau)}{\rho h Q_{mn}} \int_0^a X_{mn}(x)dx \int_0^b Y_{mn}(y)dy \right) \left(\frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{1-\gamma^2}} \sin\left(\omega_{mn}\sqrt{1-\gamma^2}(t-\tau)\right)\right) d\tau \tag{12}$$

The general solution for the force response deflection of the plate to the traffic load *p(x,y,t)* expressed by Eq. (2) is given in integral form as follows:

For $0 \le t \le t_0$:

$$\begin{aligned} w(x,y,t) = {} & \sum_{m=1}^{m}\sum_{n=1}^{n} X_{mn}(x)Y_{mn}(y)e^{-\gamma\omega_{mn}t}\left(a_{mn}\cos\left(\omega_{mn}\sqrt{1-\gamma^2}\,t\right) + b_{mn}\sin\left(\omega_{mn}\sqrt{1-\gamma^2}\,t\right)\right) + \\ & \sum_{m=1}^{m}\sum_{n=1}^{n} X_{mn}(x)Y_{mn}(y)\int_0^t \left(\frac{p(x,y,\tau}{\rho h Q_{mn}} \int_0^a X_{mn}(x)dx \int_0^b Y_{mn}(y)dy \left(\frac{e^{-\gamma\omega_{mn}(t-\tau)}}{\omega_{mn}\sqrt{1-\gamma^2}} \sin\left(\omega_{mn}\sqrt{1-\gamma^2}(t-\tau)\right)\right)\right) d\tau \end{aligned} \tag{13}$$

For $t > t_0$:

$$\begin{aligned} w(x,y,t) = {} & \sum_{m=1}^{m}\sum_{n=1}^{n} X_{mn}(x)Y_{mn}(y)\left(e^{-\gamma\omega_{mn}(t-t_0)}\left(w_{0mn}\cos\left(\sqrt{1-\gamma^2}\,\omega_{mn}(t-t_0)\right)\right)\right) + \\ & \sum_{m=1}^{m}\sum_{n=1}^{n} X_{mn}(x)Y_{mn}(y)\left(e^{-\gamma\omega_{mn}(t-t_0)}\left(\frac{v_{0mn} + \gamma\omega_{mn}w_{0mn}}{\omega_{mn}\sqrt{1-\gamma^2}} \sin\left(\sqrt{1-\gamma^2}\,\omega_{mn}(t-t_0)\right)\right)\right) \end{aligned} \tag{14}$$

in which w_{0mn}, v_{0mn} are the initial deflection and initial velocity at $t=t_0$.

Bending moments and vertical shear forces in the plate can be computed in terms of the deflection and its derivatives obtained from Eqs. (13) and (14) as expressed by the following equations:

Bending moments:

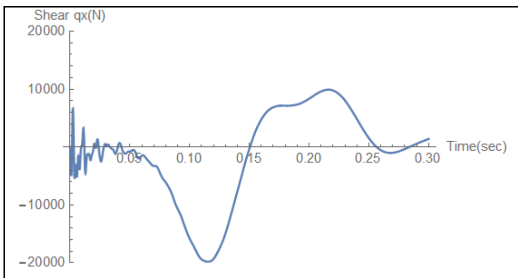
$$M_x = -D_x\left(\frac{\partial^2 w(x,y,t)}{\partial x^2} + \nu_y \frac{\partial^2 w(x,y,t)}{\partial y^2}\right) \qquad M_y = -D_y\left(\frac{\partial^2 w(x,y,t)}{\partial y^2} + \nu_x \frac{\partial^2 w(x,y,t)}{\partial x^2}\right) \tag{15}$$

Vertical shear forces:

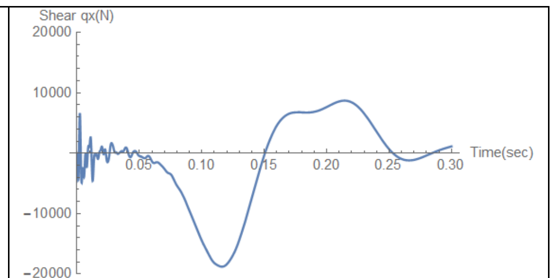
$$Q_x = -\frac{\partial}{\partial x}\left(D_x \frac{\partial^2 w(x,y,t)}{\partial x^2} + B\frac{\partial^2 w(x,y,t)}{\partial y^2}\right) \qquad Q_y = -\frac{\partial}{\partial y}\left(D_y \frac{\partial^2 w(x,y,t)}{\partial y^2} + B\frac{\partial^2 w(x,y,t)}{\partial x^2}\right) \tag{16}$$

5 RESULTS AND DISCUSSION

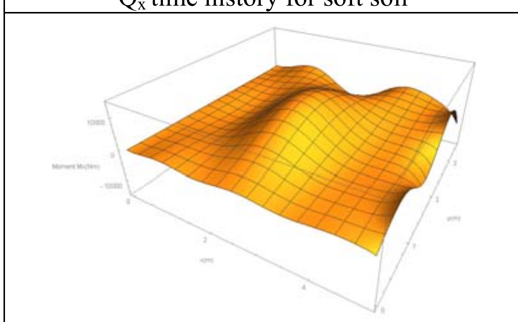
A rectangular orthotropic plate resting on an elastic Pasternak foundation subjected to a moving harmonic load is considered. The numerical values for the rigid roadway pavement and the moving traffic load are as follows: a = 5 m, b = 3.5 m, h = 0.25 m, E_x = 2.7x10^9 N/m^2, E_y = 2.25x10^9 N/m^2, υ_x = 0.18, υ_y = 0.15, ρ = 2.5x10^3 kg/m^3, $ks_{1x}=ks_{2x}=ks_{1y}=ks_{2y}$ = 2.5x10^6 N/m/m, $kr_{1x}=kr_{2x}=kr_{1y}=kr_{2y}$ = 1 N-m/rad/m, P_0 = 80 kN, v = 60 km/h and γ = 5%, 10%. Pasternak foundation parameters are as follows: case 1 (soft soil condition) G_s = 9.52x10^6 N/m, k_f =2.72x10^7 N/m^3; case 2 (medium soil condition) G_s = 1.904x10^7 N/m, k_f =5.44x10^7 N/m^3 and case 3 (hard soil condition) G_s = 3.808x10^7 N/m, k_f = 1.088x10^8 N/m^3. The traffic load moves along the centerline of the plate, parallel to the x-axis with constant velocity and the load frequency is ω_v =50 rad/s. It is assumed that the load moves from one end of the plate to the other and continues on to the adjacent plate. Many factors have effects on the dynamic response of the plate resting on a Pasternak foundation subjected to moving load. The main parameters are elastic foundation stiffness, load velocity and load frequency of the moving load. It can be observed from Fig. (1) that by increasing the stiffness of the soil, the dynamic response of the plate decreases so are the load amplitude and velocity. The absolute maximum dynamic deflection at the center of the plate has been plotted in Fig. (2). It can be seen from Fig. (2) that the dynamic deflections occur when the velocity of the vehicle is close to the critical velocity.



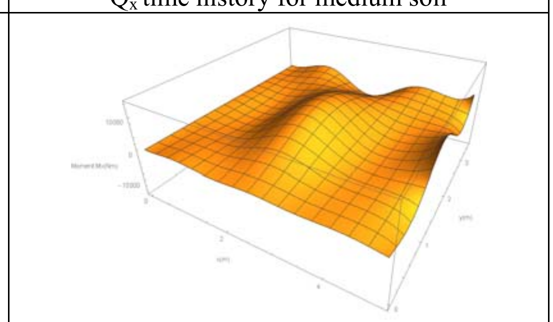
Q_x time history for soft soil



Q_x time history for medium soil

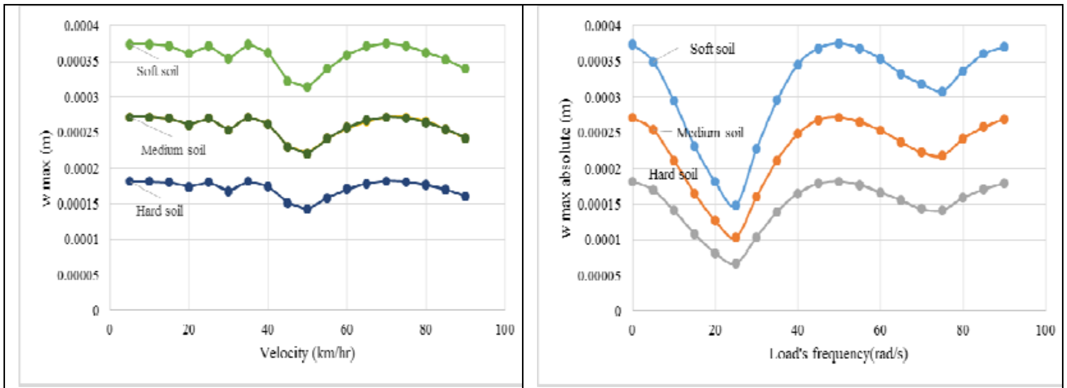


M_x distribution for soft soil



M_x distribution for medium soil

Figure 1. Various dynamic responses of the plate at near resonance condition for soft soil condition (left) and medium soil condition (right).



Response spectra as a function of velocity	Response spectra as a function of loads frequency

Figure 2. Maximum dynamic deflection as a function of velocity and load's frequency for case 1 (soft soil condition), case 2 (medium soil condition) and case 3 (hard soil condition).

6 CONCLUSIONS

A Modified Bolotin Method (MBM) for dynamic analysis of rigid pavements under moving vehicle loads is presented. The concrete pavement sits on top of soil medium which is modelled by Pasternak model. At any peak the critical velocity was observed to increase while the corresponding maximum deflection was found to decrease with the increase in the value of soil modulus. The effect of foundation stiffness and load velocity is an important parameter for deflection of the plate under dynamic condition. On increasing the load velocity, for a finite plate, there is a critical velocity. On increasing the load frequency, for a finite plate there is a critical frequency of the load. For the dynamic load, as the velocity increases, the absolute maximum dynamic deflection increases first until the velocity becomes close to the resonant velocity and decreases again after the resonant velocity.

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