

# **DYNAMIC ANALYSIS OF FULLY ANCHORED CIRCULAR CYLINDRICAL LIQUIDS' STORAGE STEEL TANKS USING FINITE ELEMENT METHOD**

HANADI ABDULRIDHA LATEEF and ABDULAMIR ATALLA

*Civil Engineering, University of Basrah, Basrah, Iraq*

A vibration analysis of circular cylindrical steel liquid storage tanks anchored to rigid base is conducted. Empty, partially and completely liquid filled tanks are considered as well as tanks composed of two courses using ANSYS 11.0 finite element package. The tank wall is modeled using linear elastic shell finite element and a new method, based on the added mass approach, is developed to model the effect of the contained liquid. In this method the properties of the shell element is modified to include the effect of the contained liquid. The analysis includes four tank case studies which are empty, fully filled with water, and filled with changeable liquid level in addition to study the effect of the variable thickness of tank on the natural frequencies and mode shapes. The results show that the natural frequency of completely filled tall tank may be less by 70.7% than the natural frequency of empty tank. It is also found that a maximum value of natural frequency can be obtained when the lower thick course consists 0.75 of tank height and its thickness is four times that of the upper one. The natural frequencies decrease with the increasing in liquid level for tall tank. The natural frequency of completely filled tank is less by 70.7% than the natural frequency of empty tank.

*Keywords:* Added mass approach, Modified shell density, Vibration analysis, Variable shell thickness, Tall tank, Broad tank.

## **1 INTRODUCTION**

Early uses for liquid containers were found in the petroleum industry and in municipal water supply systems. As their numbers and sizes began to grow, their tendency to vibrate under seismic loading became a matter of concern (Haroun 1980).

## **2 MODELING OF THE WALL**

The wall of the tank is modeled as linear elastic shell element with four nodes and six degrees of freedom per node, SHELL63 in ANSYS 11.0 is used for this purpose (ANSYS 2007). This element is selected for its simplicity and adequacy in modeling thin shells.

## **3 MODELING OF THE CONTAINED LIQUID**

The dynamic of liquid storage tanks involves the interaction between the shell structure and the contained fluid. The liquid mass is divided into two parts: the impulsive and the

convective or sloshing mass, as shown in Figure 1. The impulsive mass experiences high accelerations; therefore, it controls the seismic loads (base shear and overturning moment) in the tank. The convective mass experiences very low accelerations; therefore, it contributes negligibly to the seismic loads in the tank (Malhotra 2006). Hence the sloshing effect is usually ignored in the dynamic and seismic analysis of liquid storage tanks (Malhotra 2006, Malhotra 1997).

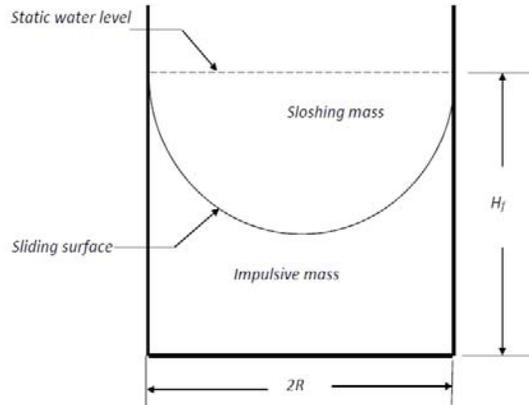


Figure 1. Spherical path Representation (Atalla 2008).

#### 4 MODIFIED SPHERICAL PATH REPRESENTATION

The Spherical Path Representation suggested by Atalla (2008) requires a long run time to calculate the results because it requires an additional type of element, i.e. mass element, to model the liquid in the tank. MASS21 element in ANSYS (2007) was selected because it is a single node element but has three dimensions capabilities. This element is preferred compared to the three dimensional block elements such as FLUID80 or SOLID45 because they have eight nodes, hence require larger storage capacity, and longer run time. The adequacy of MASS21 in modelling the liquid was verified in previous work by Atalla (2008).

In the present work, the added mass of the impulsive liquid at each ordinate is distributed on the whole volume of the shell element at that ordinate instead of representing it by a lumped mass element. In this modified method, only the shell element is used to represent both of the wall and the liquid as an added mass density to the wall of tank. The modified density  $\rho_{\text{modified}}$  of the shell at ordinate  $y$  is:

$$\rho_{\text{modified}} = \rho_s + \rho_{\text{added}} \quad (1)$$

where  $\rho_s$  is the mass density of steel, and  $\rho_{\text{add}}$  is the added mass density of the impulsive part of the contained liquid at ordinate  $y$ .

#### 5 APPLICATIONS

A free vibration analysis of two types of tanks, broad and tall is investigated. The tanks are fixed at the base and free at upper end without top cover. The properties of the tanks are summarized in Table 1.

Table 1. Dimensions of broad and tall tanks.

Geometric Properties (m)	Tank Type		Material Properties		
	Broad	Tall	Steel		Water
Height	12	21.6	Modulus of Elasticity (GPa)	207	-
Diameter	36	14.4	Poisson's Ratio	0.3	-
Wall Thickness	0.0254	0.0254	Mass Density (kg/m <sup>3</sup> )	7850	1000

### 5.1 Case Study (1): Empty Tank

A broad tank of properties as listed in Table 1 is considered. The tank wall is divided in the longitudinal direction into 12 divisions, and in the circumferential direction into 100 divisions. This case is selected to verify the accuracy of the method, since the same dimensions was previously investigated by Saadi (2008). The resulting mode shapes and natural frequencies together with those obtained by Saadi are listed in Table 2. The comparison of the results shows that the natural frequency values obtained in the present work are in a good agreement with those obtained in the previous work.

Table 2. Values of Natural Frequency for Various Values of Circumferential Wave Number.

<i>N</i>	Natural Frequency (Hz)		
	Present work	Saadi [16]	Percentage difference
1	34.561	34.262	0.865
2	23.857	23.774	0.348
3	16.925	16.901	0.142
4	12.413	12.41	0.016

### 5.2 Case Study (2): Tanks Filled with Water

The effect of the contained water on mode shapes and natural frequencies is studied, by analyzing fully and partially liquid filled tanks. It is to examine the accuracy of the modified added mass method, by comparing results with those from previous works. The broad liquid filled tank investigated by Ramasamy and Ganesan (2006), Krishna and Ganesan (2006), and Atalla (2008) is reanalyzed. The tank has a length  $L = 12.2$  m, a radius  $R = 18.29$  m and wall thickness  $h = 0.0254$  m. The Values of the added mass calculated using the equations developed by Atalla (2008) and also by using the equations of the modified method developed in the present work are listed in Table 3. The variation of natural frequency with the circumferential wave number compared with those obtained in the previous works is shown in Figure 2. It is clear that the suggested method gives results which are in general agreement with the results of pervious works. It can also be noted that the relation between the natural frequency and mode shape of tank filled with water is similar to that of the empty one, i.e. the natural frequency decrease with increasing the circumferential wave number  $n$  in Figure 2.

Table 3. Values of the added mass at elements for broad tank.

y (m)	Added Mass (kg) at nodes	Modified Density Method		
		Course number	y (m)	Density (kg/m <sup>3</sup> )
0	12821.5	1	0-1.22	342392
1.22	11025.9	2	1.22-2.44	293572
2.44	9344.4	3	2.44-3.66	247957
3.66	7777.0	4	3.66-4.88	205545
4.88	6323.8	5	4.88-6.1	166338
6.1	4984.6	6	6.1-7.32	130334
7.32	3759.5	7	7.32-8.54	97534
8.54	2648.5	8	8.54-9.76	67937
9.76	1651.6	9	9.76-10.98	41545
10.98	768.73	10	10.98-12.2	18356
12.2	0	–	–	–

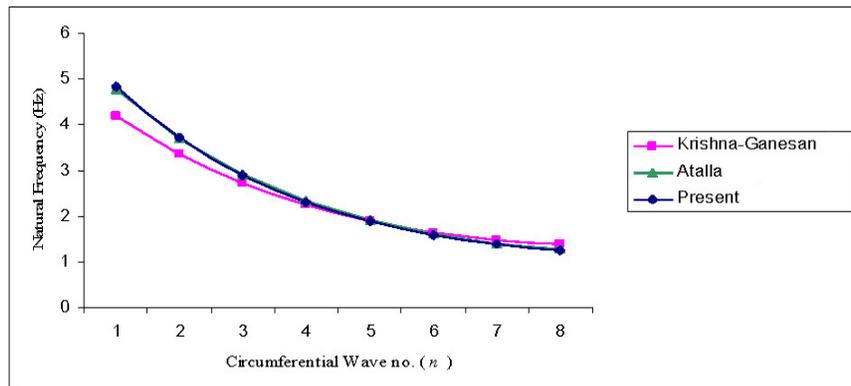


Figure 2. Values of natural frequency compared with previous works.

**5.3 Case Study (3): Effect of Liquid Level on Natural Frequency for Tall Tank**

The influence of the filling ratio ( $H_f/H$ ) on the dynamic characteristics is investigated by computing the natural frequencies and modes of the tall tank specified in Table 1. The effect of selected filling ratios (0, 0.25, 0.5, 0.75, and 1) on the coupled natural frequency of liquid - tank system for the first mode ( $m = 1$  and  $n = 1$ ) of tall tank is shown in Figure 3. This figure shows decrease in natural frequencies with increase in liquid level. The natural frequency of fully filled tank 5.72 Hz decreased by 70.7% than the natural frequency of empty tank 19.526 Hz. The results are in general agreement with the results of Saadi (2008).

**6.4 Case Study (4): Effect of the Variable Thickness of Empty Tank**

In this case the broad tank given in Table 1 is considered to be made of steel sheets with two values of thickness. The thickness of the lower and upper courses are  $h_1 = 0.0254$  m and  $h_2 = 0.0127$  m, respectively. The natural frequencies and circumferential wave number for the first three modes determined for different heights of the lower course are listed in Table 4. The results show that when the height of lower course increases, the

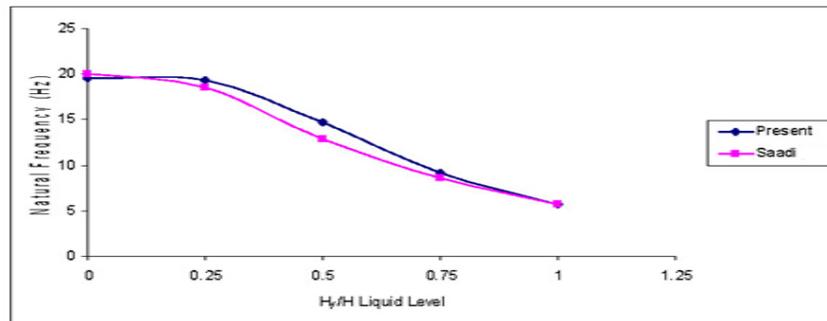


Figure 3. Coupled natural frequency for various filling ratios for tall tank.

natural frequency increases. When the height of lower course is 0.75 of tank height, the natural frequency is 20% greater than when the lower course is 0.25 of tank height. For the values of the ratio of ( $h_2/h_1$ ) equal (0.25, 0.5, and 0.75), the relation between the height of the lower course  $y$  and the natural frequency for the broad empty tank is shown in Figure 4. This figure shows that the natural frequency increases with the increase in height of lower course for all ( $h_2/h_1$ ) ratios and maximum natural frequency is obtained for  $h_2 = 0.25 h_1$ .

Table 4. Mode shape and natural frequency of variable wall thickness for  $h_2 = 0.5h_1$  for broad tank.

Height of Lower Course (m)	First Mode		Second Mode		Third Mode	
	$n$	f(Hz)	$n$	f(Hz)	$N$	f(Hz)
0	15	2.588	14	2.915	13	2.685
3	14	2.881	15	2.915	13	2.951
6	14	3.219	15	3.261	13	3.283
9	13	3.61	12	3.629	14	3.733
12	11	3.611	12	3.67	10	3.756

## 7 CONCLUSIONS

Dynamic analysis of fully anchored circular cylindrical steel water tanks is undertaken using the finite element method. A new method was developed to model the effect of the contained liquid on the dynamic response of the liquid storage tank. In this method no additional type of elements is used to model the liquid in the tank, only the shell element is used to model the wall as well as the liquid that is modeled as a mass density added to the density of the tank's wall. The natural frequencies are found to decrease with the increasing in liquid level for tall tank. The natural frequency of completely liquid filled tank is less by 70.7% than the natural frequency of empty tank. For empty broad and tall tanks, a maximum natural frequency found when the thickness of the upper quarter of the tank height is one fourth of thickness of the lower portion.

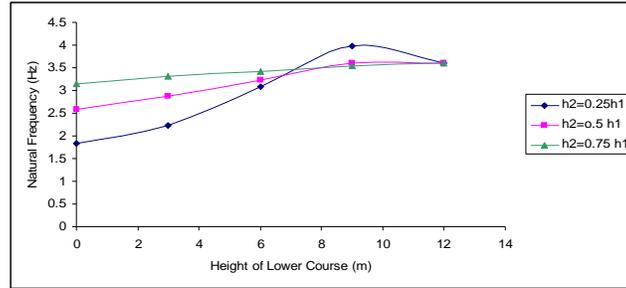


Figure 4. Relation between the natural frequency and height of lower course.

## References

- ANSYS, Element Reference Manual, Release 11, *Swanson Analysis Systems, Inc.*, 2007.
- Atalla, A., Vibration analysis of circular cylindrical liquid storage tanks using finite element technique, *PhD. Thesis in Civil Engineering*, University of Basrah, 2008.
- Malhotra, P. K., Earthquake induced sloshing in tanks with insufficient freeboard, Report, *Structural Engineering International 3/2006*, SEI Editorial Board.
- Malhotra, P. K., Method for seismic base isolation of liquid-storage tanks, *Journal of Structural Engineering*, ASCE, Vol.123, No.1, January 1997.
- Haroun, M. A., Dynamic analysis of liquid storage tanks, *Report EERL 80-04*, Pasadena, California, February 1980.
- Thomson, W. T, *Theory of Vibration with Applications*, 3<sup>rd</sup> edition, Prentice Hall, 1988.
- Krishna, B. and Ganesan, N., Polynomial approach for calculating added mass for fluid-filled cylindrical shells, *Journal of Sound and Vibration*, Vol. 291, 2006, PP. 1221-1228.
- Saadi N. O., Free vibration analysis of free-clamped circular cylindrical liquid storage tank, *M. Sc. Thesis*, University of Technology, 2008.