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GEOMETRICALLY NONLINEAR FINITE ELEMENT ANALYSIS OF ARBITRARY THIN PLATES

SALEEMA PANDA and MANORANJAN BARIK

Dept of Civil Engineering, NIT, Rourkela, India

Many plate structures used especially in naval and aerospace applications undergo deflections that are not small in comparison to the thickness of the plate, but they are still small compared to other dimensions of the plate, so analysis must include the effects of the large displacements on the structures. This report developed an elegant finite element formulation for analyzing the geometrically nonlinear static behavior of arbitrary-shaped thin plates. An arbitrary planform of a whole plate was mapped into a square domain where a cubic serendipity shape function represented the arbitrary geometry. An ACM plate-bending element along with the inplane deformations was considered for the displacement function. The nonlinear formulation was done in the total Lagrangian coordinate system using [N]-notation, and the nonlinear governing equations were solved by Newton Raphson iterative method. It was found that the element was capable of accommodating different geometries just like isoparametric element. The efficacy of the element was shown by comparing the deflections and stresses at critical points of the plates of square, skewed, and circular geometries with previously published results.

Keywords: Nonlinear static analysis, Mapping, Newton Raphson Method, Arbitrary plates, Finite Element Method.

1 INTRODUCTION

Plated structures especially in naval and aerospace engineering may undergo large deflections under transverse loads, hence their additional effects must be accounted for in analysis. In a linear analysis, the geometry of the elements remain practically unchanged during loading process, for which the first order infinitesimal linear strain approximation suffices. But in a nonlinear analysis, because of the modification of the geometry based on the current displacements, applied forces are constantly adjusted for in the current state of equilibrium until the residual force is reduced to an acceptable level.

In this report, the geometrically nonlinear static behavior of arbitrary-shaped thin plates is studied. An ACM plate-bending element considered by Adini and Clough (1961) and Melosh (1963), along with the in-plane deformations, was considered for the displacement function. This element considered only thin plates and hence did not consider the shear deformation. This eliminated the shear-locking problem and generation of spurious mechanisms. In this formulation, the arbitrary planform of the whole plate was mapped into a square domain, and the nonlinear formulation used Von-Karman's nonlinear equation in the total Lagrangian coordinate system using [N]-notation. Further, the nonlinear governing equations were solved by the Newton Raphson iterative method.

2 MAPPING OF THE PLATE

The arbitrary shape of the plate was mapped (Barik and Mukhopadhyay 1998) approximately into a $[-1, +1]$ region in the $\xi - \eta$ plane, with the help of the cubic serendipity shape function (Zienkiewich and Taylor 1989). The mapped square plate was now discretized into a number of elements, and each element was mapped with the same cubic serendipity shape function to a natural coordinate element of domain $[-1, +1]$.

3 DISPLACEMENT INTERPOLATION FUNCTION

For the proposed element, the 4-noded rectangular ACM plate-bending element with five degrees of freedom (u, v, w, θx, θy) at each node was considered. The interpolation functions for the inplane and bending were the usual ones presented in detail in Barik and Mukhopadhyay (2002).

4 FORMULATION OF GEOMETRIC NON-LINEARITY

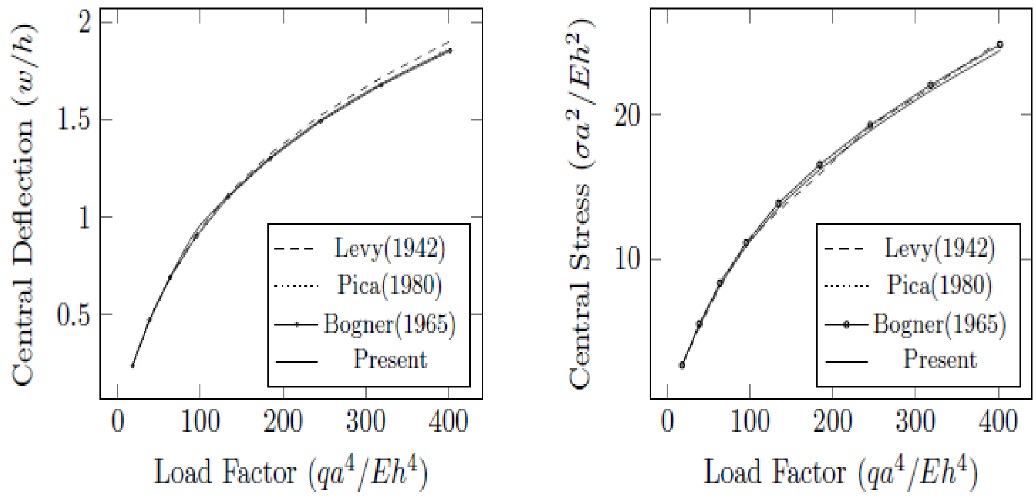
The geometrically nonlinear formulation (Mukhopadhyay et al. 2004) was done using the [N]-notation formulated by Mallet and Marcal (1968), and the displacements referred to the original configuration following the Lagrangian method.

5 RESULTS

A square plate subjected to a uniformly-distributed load was analyzed for various load factors. Fig. 1(a) - 2(a) compares the present results with Levy (1942), Bogner et al. (1965) and Pica et al. (1980) for a clamped square plate, and Fig. 2(b) - 3(a) with Rushton KR. (1970) and Pica, A. et al. (1980) for a simply-supported square plate. A rectangular plate with different aspect ratios of 1.0, 1.5, and 2.0 were analyzed. Fig. 3(b) - 4(b) explains the present results for deflection, stress at the center, and stress at the center of an edge for clamped boundaries on all sides. A skew plate for all sides clamped was analyzed. The present results are shown in Fig. 5(a) - 6(a) which compare deflection, stress at the center, and stress at the center of an edge at different skew angles of 0°, 30°, and 45°. A clamped circular plate under a uniformly-distributed load was analyzed and compared with Schmit (1968) and Pica et al. (1980) in Fig. 6(b) - 7(b).

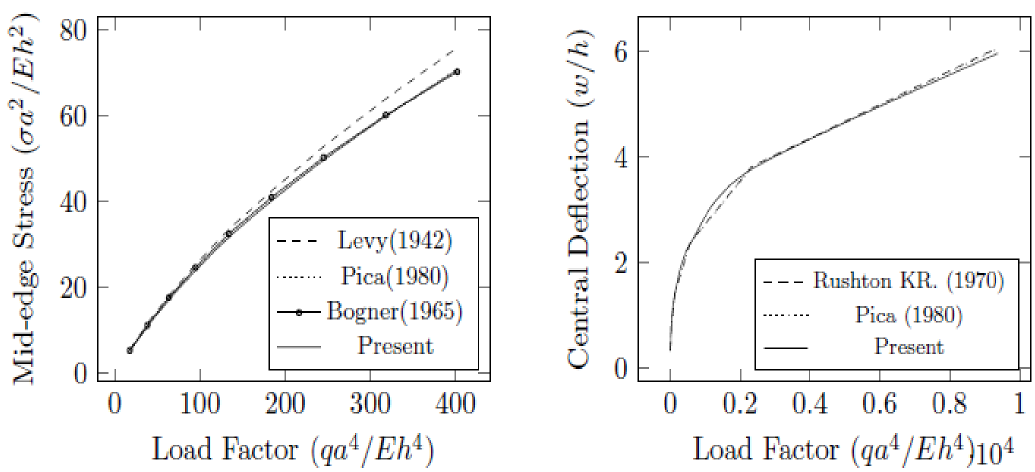
6 CONCLUSIONS

In order to have a geometrically-nonlinear static analysis of thin plates of arbitrary shapes, the formulation must be generalized by means of a mapping technique so that the analysis is performed in a square domain. In this report, cubic serendipity shape functions were used to represent the geometry of the plate, and four noded non-conforming ACM elements along with in-plane displacements were considered for displacement functions. In this context, the formulation was done in the total Lagrangian co-ordinate system, and the Newton-Raphson technique was used to solve nonlinear governing equations. A MATLAB code was written for the analysis, and results obtained for various geometries of the plate compared with available research were found to be in good agreement.



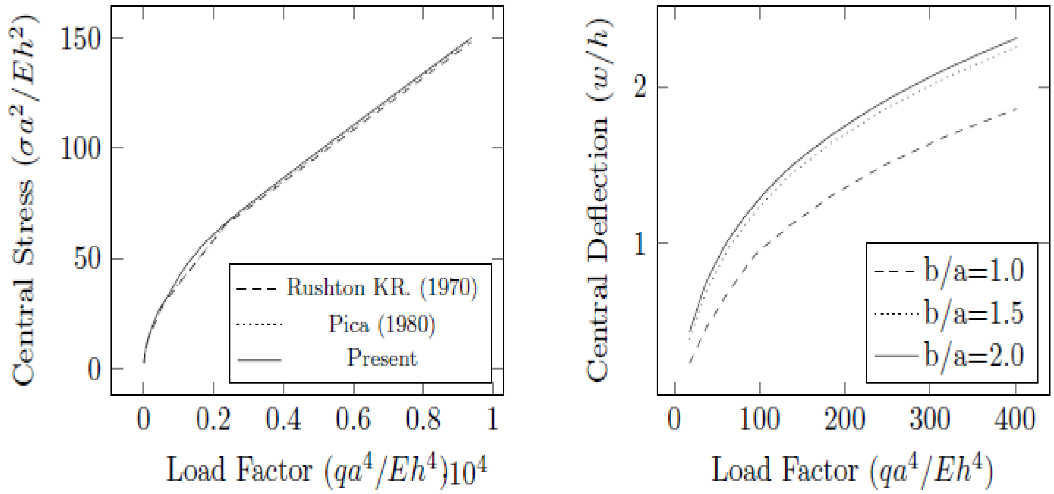
(a) Deflection at center of clamped square plate. (b) Stress at center of clamped square Plate.

Figure 1. Effects at center of clamped square plates.



(a) Mid-edge stress of clamped square plate. square (b) Deflection at center of clamped plate.

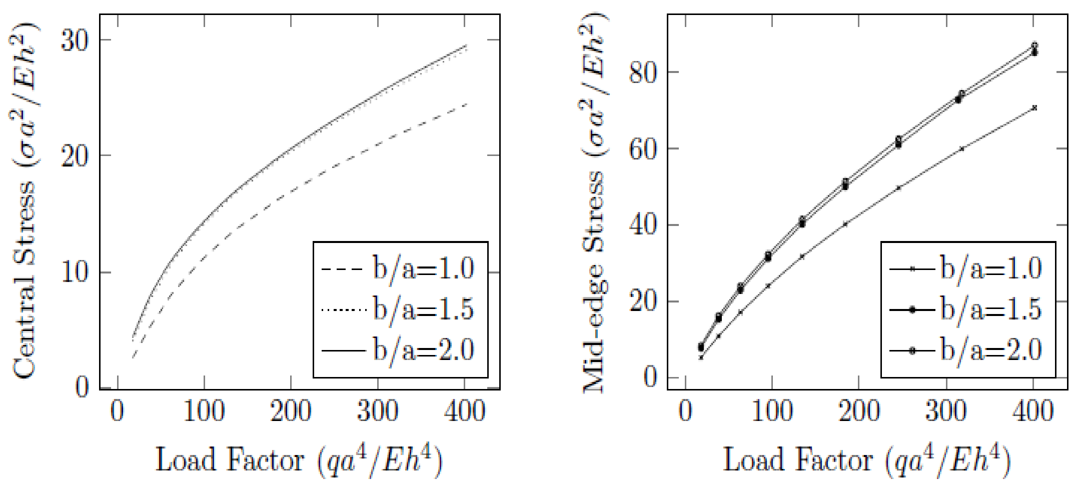
Figure 2. Effects at edge and center of clamped plates.



(a) Stress at center of SS square plate.

(b) Central deflection of clamped rectangular plate.

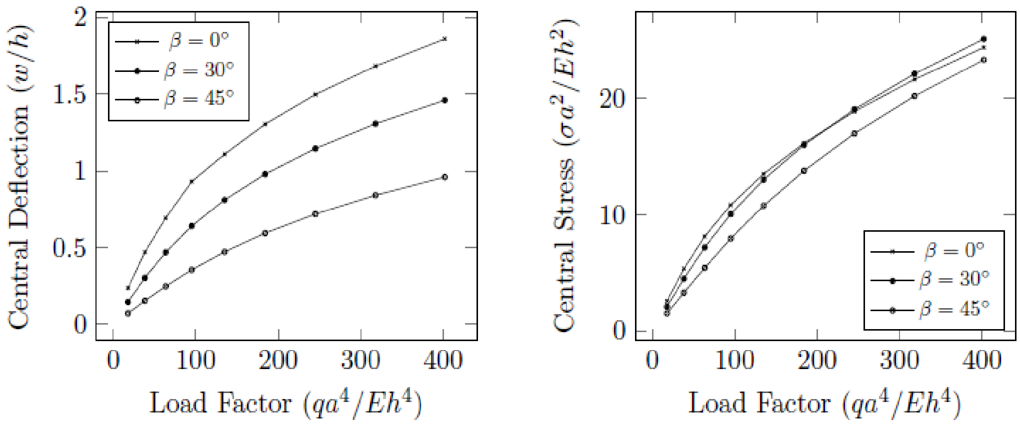
Figure 3. Effects on square and rectangular plates.



(a) Central stress of clamped rectangular plate.

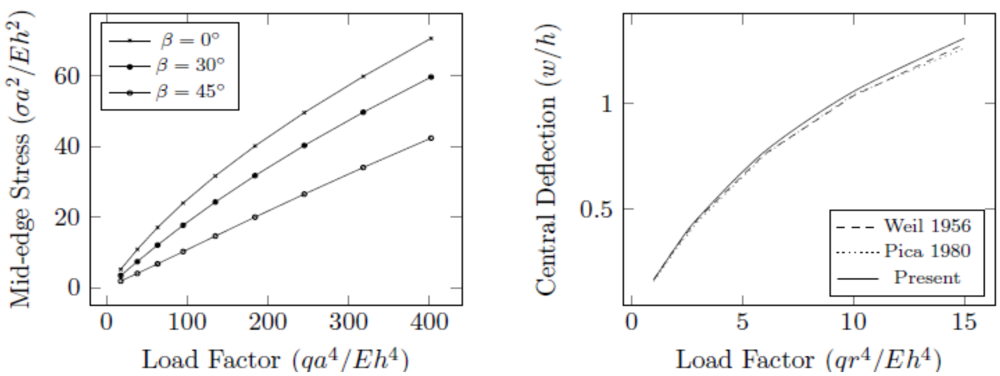
(b) Midedge stress of clamped rectangular plate.

Figure 4. Effects on clamped rectangular plates.



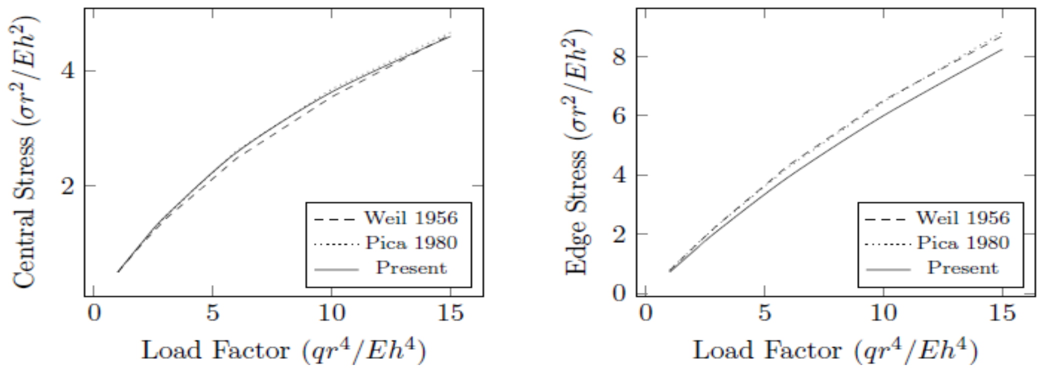
(a) Deflection at center of clamped skew plate. (b) Central stress of clamped skew plate.

Figure 5. Effects on clamped skew plates.



(a) Midedge stress of clamped skew plate. (b) Central deflection of clamped circular plate.

Figure 6. Effects on clamped skew and circular plates.



(a) Central stress of clamped circular plate. (b) Edge stress of clamped circular plate.

Figure 7. Stress on clamped circular plates.

References

Adini, A. and Clough, R., Analysis of Plate Bending by the Finite Element Method, Report submitted to the *National Science Foundation*, G7337, 1961.

Barik, M. and Mukhopadhyay, M., A new stiffened plate element for the analysis of arbitrary plates, *Thin-Walled Structures*, 40, 625–639, 2002.

Barik, M. and Mukhopadhyay, M., Finite element free flexural vibration analysis of arbitrary plates, *Finite Elements in Analysis and Design*, 29, 137– 151, 1998.

Bogner, F.K., Fox, R., and Schmidt, L., The generation of inter-element compatible stiffness and mass matrices by the use of interpolation formulae, *In Proceedings of the Conference on Matrix Methods in Structural Analysis*, Wright-Patterson A.F.B, 1965.

Levy, S., Square Plate with Clamped edges under normal pressure producing large deflections, *NASA Technical Note*, 847, 1942.

Mallet, R. and Marcal, P., Finite element analysis of nonlinear structures, *Journal of Structural Division*, 94, 2081–2105, 1968.

Melosh, R.J., Basis for derivation of matrices for the direct stiffness method, *AIAA*, J.1, 1631-7, 1963.

Mukhopadhyay, Madhujit and Sheikh, A. H., *Matrix and Finite Element Analysis of Structures*, chapter 17, Ane Books Pvt. Ltd., first edition, 2004.

Pica, A., Wood, R., and Hinton, E., Finite element analysis of geometrically nonlinear plate behavior using a Mindlin formulation, *Computers and Structures*, 11(3), 203–215, 1980.

Rushton KR., Large deflection of plates with initial curvature, *International Journal of Mechanical Sciences*, 12, 1037–51, 1970.

Schmit R. Large deflections of a clamped circular plate, *Journal of the Engineering Mechanics Division*, ASCE, 94(EM6), 1603–6, 1968.

Zienkiewich, O.C. and Taylor, R.L., *The Finite Element Method*, McGraw-Hill, 4^{th} edition, 1989.