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RELATION BETWEEN NONLINEAR RESPONSE TO BI-DIRECTIONAL SEISMIC INPUT AND MECHANICAL PROPERTIES OF STRUCTURES

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The effect of horizontal bi-directional seismic input is not considered in current designing method with equivalent linearizing method, even though there have been cases where a nonlinear response to bi-directional input exceeds the one to one-directional input. The objective of this paper is to grasp the characteristics of nonlinear response of structures to bi-directional seismic input. With an analytical model of a single-mass two-degrees-of-freedom system, we conducted examinations with parameters of mechanical properties: shape of yield surface and yield resistance. It was verified that the response to bi-directional input was often much greater than the one-to-one directional input, and oblateness of the elastic response to bi-directional input was concerned with the characteristics of bi-directional elasto-plastic response.

Keywords: Yield surface, Yield resistance, Oblateness of elastic response, Equivalent linearizing method, Plastic theory, Flow rule.

1 INTRODUCTION

A design with an equivalent linearizing method and a seismic response spectrum is one of the ways to create earthquake resistance. However, the effect of horizontal bi-directional seismic input is generally not considered. Although the response to bi-directional input sometimes exceeds the one-to-one directional input (Zhou *et al.* 1989, Wada and Hirose 1989, Takahashi *et al.* 1993), characteristics of horizontal bi-directional nonlinear response with respect to the nonlinear response to horizontal bi-directional seismic input are still not understood. This study ascertains the adaptability of response evaluation method with equivalent linearizing method to bi-directional nonlinear response, examining the relationship between nonlinear response to bi-directional input and mechanical properties of structures.

We conducted parametric investigation for the relationship between mechanical properties of structures and bi-directional response displacement by single-mass system with two degrees of freedom. For parametric investigation, we used two parameters which indicate mechanical properties of structures: shape of yield surface and yield strength. As for the shape of yield surface, it is considered that the shape is close to a rectangle when collapse mechanism of buildings is beam collapse (Ishida and Hotta 2014), and close to an ellipse in the case of column collapse (Tsumura 2003). We expressed the yield surface by equation of super ellipse, and used the exponent in that equation as parameter (N.B. A super ellipse is an ellipse when the exponent is 2.0, and becomes close to a rectangle with increase of the exponent).

2 THE MAIN TEXT AND IMPORTANT CONSIDERATIONS

2.1 Analytical Model

We used a single-mass system with two degrees of freedom for an analytical model that indicates a building. Stiffness for *x*-axis and *y*-axis of a horizontal coordinate system were set so that the natural periods were 1.0 second. Bilinear elastic-perfectly plastic type was adopted as uniaxial restoring force characteristics. The uniaxial restoring force model was expanded to two axes on the basis of flow rule of plasticity theory. The Newmark-β method (β=1/4) was applied to the numerical integration of equation of motion, and damping coefficient was 5% for each axis.

2.2 Input Seismic Motion

Fourteen seismic motions with seismic intensity of upper 5 or greater on the seven-point Japanese scale, and whose maximum ground acceleration were more than 2.0 m/s^2, were chosen for input seismic motions. To unify input condition, we coordinated conversion for observation records of seismic motions as shown in Figure 1, and we obtained *Mj* wave and *Mn* wave. The *Mj* wave is in the direction with the maximum ground acceleration, and *Mn* wave is in the direction that is perpendicular to the direction of the *Mj* wave. Table 1 shows the selected fourteen seismic motions and their maximum ground acceleration of *Mj* and *Mn* waves.

Table 1. Input seismic motions.

Seismic motion	The max. ground acc. of *Mj* wave/*Mn* wave (m/s^2)
95-HYG	8.6/4.8
03-HKD	9.7/8.2
04-NIG	3.7/3.1
05-MYG	5.6/4.1
07-ISK	9.3/7.0
07-NIG	8.1/3.4
09-SZO	5.4/5.1
11-MYG	27.7/17.0
11-SZO	10.1/6.7
11-FKS	4.4/2.8
11-IBR	5.5/3.1
12-CHB	3.9/2.0
12-MYG	6.9/4.3
13-TCG	13.0/9.8

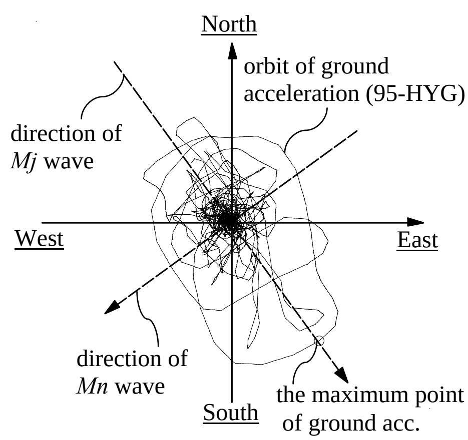


Figure 1. Method of making *Mj* wave and *Mn* wave.

2.3 Examples of Time History Response Analysis

We provide an example of time history response analysis in this section. The yield surface is circular, input seismic motion is 11-IBR, and a *Mj* wave is input in *y*-axis and *Mn* wave in *x*-axis simultaneously. Orbit of response restoring force, orbit of response displacement and relation between restoring force and displacement in *y*-axis are given in Figure 2. Restoring force is normalized by *mg* (*m*: mass, *g*: gravitational acceleration), and displacement is normalized by yield displacement. In Figure 2 (c), relation of restoring force and displacement when the *Mj* wave is input in *y*-axis one-directionally is also shown. From Figure 2 (a) and (c), it can be verified that yielding occurred on the part of yield surface other than the point which the *y*-axis intersects with the yield surface. Therefore the difference between response to bi-directional input and the one-to-one directional input was caused.

3 EXAMINATION OF RESPONSE TO BI-DIRECTIONAL INPUT

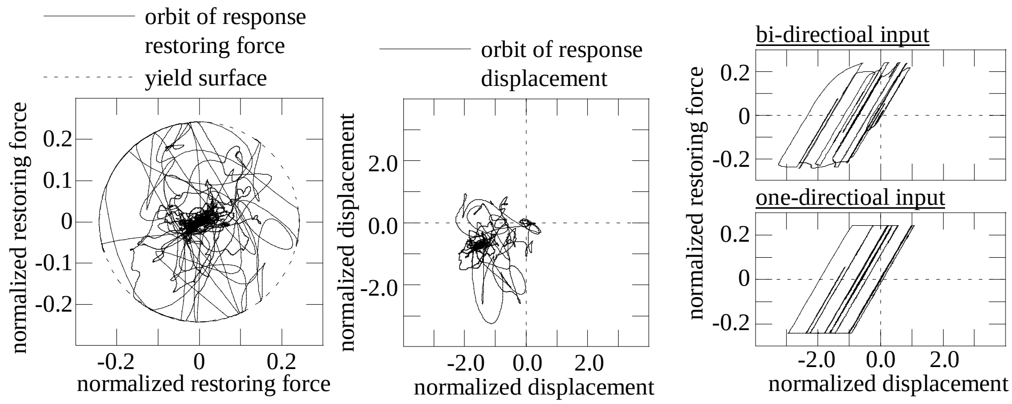
3.1 Examination Method

The esponse to bi-directional input was calculated in consideration of the directionality of seismic motion, because seismic input direction which causes maximum response displacement is not always equal to direction with maximum ground acceleration (direction of *Mj* wave in this study), and it depends on natural periods of structures. Figure 3 shows the calculation method. We set the *y*-axis as reference axis, and the angle formed by the *y*-axis and input direction of *Mj* wave is given as φ.

An example of the relationship between φ and response displacement to bi-directional input in the *y*-axis is given in Figure 4 (11-MYG, yield surface was circular, response ductility factor to one-directional input (details below) is 3.0). Response displacement was normalized by yield displacement in *y*-axis and is given as the response ductility factor. It was confirmed that the input direction that produces the maximum response ductility factor is not equal to the direction of the Mj wave. The maximum response ductility factor is abbreviated as U_2.

The effect of the bi-directional seismic input compared to the response to one-directional input. The directionality of seismic motions should also be considered when we get response displacement to one-directional input. The *y*-axis component of seismic motion was prepared according to φ and response ductility factor in *y*-axis to that uniaxial wave of *y*-axis component was calculated. The maximum response ductility factor is abbreviated as U_1, unified to 3.0 or 4.0 in each case of seismic motion by tuning yield resistance. We use U_1 as a parameter that indicates yield resistance.

In addition to the response ductility factor to one-directional input U_1, we used exponent *r* in the equation of super ellipse. This composes yield surface as a parameter which indicates the shape of yield surface. The yield surface and the equation of super ellipse is shown in Figure 5. The shape of yield surface is an ellipse when *r* is 2.0, and it becomes close to rectangular with increase of *r* from 2.0. In this study we set yield resistance in *x*-axis *Qx* equal to the one in *y*-axis *Qy*. Therefore the shape of the yield surface is circular when *r* is 2.0, and it becomes close to a square with increase of *r*.



(a) Orbit of restoring force (b) Orbit of displacement (c) Relation of restoring force and displacement

Figure 2. Result of time history response analysis.

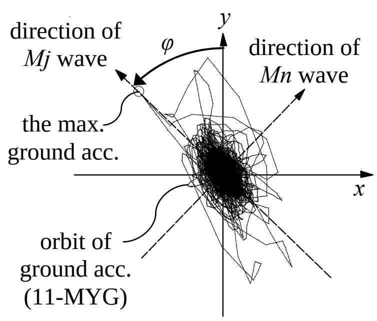


Figure 3. Angle φ between y-axis and input direction of Mj wave.

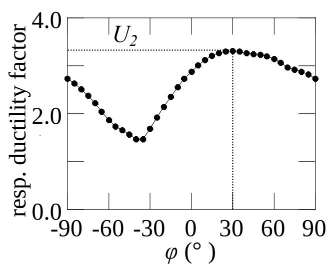


Figure 4. Relation between φ and bi-directional resp. disp.

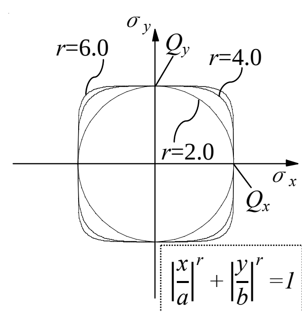


Figure 5. Shape of yield surface.

3.2 Result of Examination

Figure 6 provides the relation between exponent r and ratio of responses to biaxial input and to uniaxial input, U_2/U_1. Figure 6 (a) gives the case of U_1=3.0, and (b) gives the case of U_1=4.0.

3.2.1 *Relation between response and parameters*

Exponent of equation of super ellipse: U_2/U_1 is dispersed according to the diminishment of the exponent r, U_2/U_1 converges to 1.0 as the r gets larger. This phenomenon can be understood because the yield surface becomes close to a square with the increase of r, and yield resistance in the direction of y-axis becomes uniform. In this examination, the maximum value of U_2/U_1 is 1.25 (11-FKS, U_1=3.0, r=2.0).

Yield resistance: Although it cannot be said that the number of input seismic motions is sufficiently large, the standard deviation of U_2/U_1 for each exponent r, abbreviated as S, was calculated to discuss the relationshihp between the r and scattering of U_2/U_1 obtained from these analyses, and given in Figure 7. The mean value of any case is about 1.0. S in the case of U_1=4.0 and around r=6.0 was larger than the one in the case of U_1=3.0, therefore it is verified that convergence to 1.0 in the case of larger yield resistance was earlier than that of smaller one. This is because the response tends to receive more effect of the shape of yield surface in the case where the yield resistance is small in comparison to the scale of response.

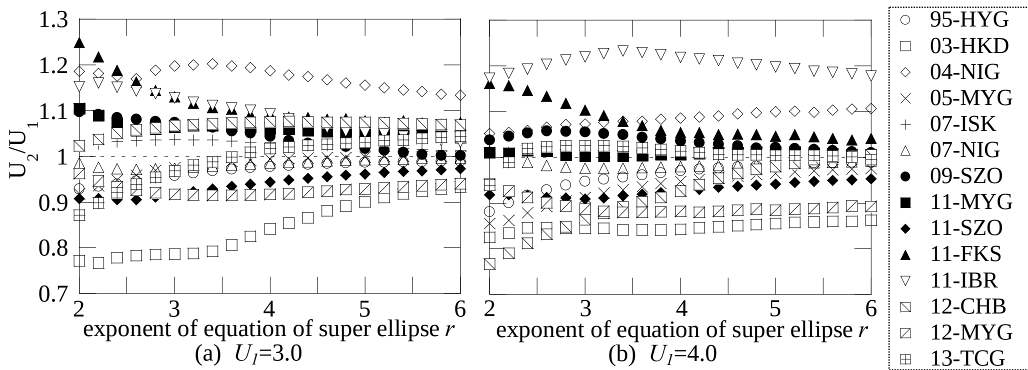


Figure 6. Relation between r and U_2/U_1.

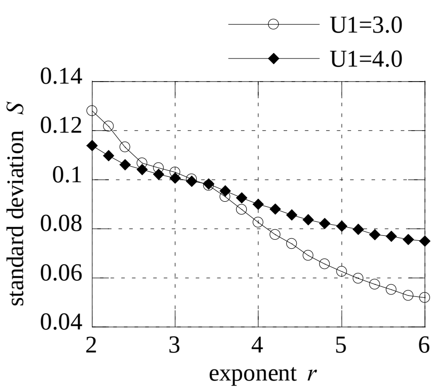


Figure 7. Relation between r and standard deviation S.

Table 2. Oblateness of elastic response displacement.

Seismic motion	$_eU_s/_eU_p$
95-HYG	0.25
03-HKD	0.92
04-NIG	0.73
05-MYG	0.48
07-ISK	0.42
07-NIG	0.24
09-SZO	0.88
11-MYG	0.70
11-SZO	0.72
11-FKS	0.53
11-IBR	0.90
12-CHB	0.27
12-MYG	0.73
13-TCG	0.88

3.2.2 *Relation between response displacement to bi-directional input and oblateness of elastic response displacement*

Wada and Hirose (1989) examined the oblateness of elastic response displacement on the horizontal plane, and fixed the direction with the maximum response displacement as the principal axis and the direction perpendicular to the principal axis as the secondary axis. They indicated that the input magnification of seismic motion in the secondary axis affected the response behavior of structures. Accordingly, we calculated the oblateness of elastic response displacement $_eU_s/_eU_p$, where $_eU_p$ and $_eU_s$ indicate maximum response displacement in the principal and secondary axis. ${}_eU_s/{}_eU_p$ of each input seismic motion is given in Table 2. It was found from U_2/U_1, shown in Figure 6, that the cases whose U_2/U_1 is relatively large have the value of ${}_eU_s/{}_eU_p$ which is larger than about 0.5 and have a large bi-directionality of response.

4 CONCLUSIONS

1) When seismic motion is input bi-directionally to the analytical model, yielding occurs on the part of yield surface other than the point where the axis for response evaluation intersects with the yield surface. The model yields before reaching yield resistance in that axis. Therefore, difference of response state from the case of one-directional input is caused.

2) With regards to the exponent of equation of super ellipse which composes yield surface, r, it is verified that the ratio of response displacement to bi-directional input to the one to one-directional input, U_2/U_1, is dispersed according to diminishment of r, and converges to 1.0 with increase of r. The maximum value of U_2/U_1 is 1.25 (11-FKS, U_1=3.0, r=2.0).

3) As for the response ductility factor to one-directional input U_1 which indicates yield resistance, it is found that the convergence of U_2/U_1 to 1.0 with increase of r caused by the larger U_1 delays than the one by the smaller U_1.

4) From the aspect of oblateness of bi-directional elastic response displacement, it is verified that the response displacement to bi-directional input is often relatively much greater than one-directional response, when the oblateness of elastic response is larger than about 0.5 and bi-directionality of response is strong.

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