

# PROBABILISTIC CALCULATION OF TOTAL LABOR CONSUMPTION RATE OF REINFORCED CONCRETE WORKS – WITH AND WITHOUT CORRELATIONS

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Correlations between calculation parameters directly influence the results of probabilistic calculations and their interpretation. In Monte Carlo simulations carried out to systematically integrate uncertainties into the calculation process, input parameter correlations are reflected by correlation coefficients and predominantly influence the chance/risk ratio and ranges of results. Correlations are usually determined indirectly by analyzing existing data series gathered from measurements, surveys, experiments etc. and investigated with respect to their linear relationship. Another option is to directly determine correlation coefficients based on expert surveys. In 2015-16, Kummer and Hofstadler conducted a survey at Graz University of Technology in which experts provided estimates of the coefficients corresponding to the correlations between certain parameters. These considerations concentrated on the labor intensity of reinforced concrete works expressed by the total labor consumption rate, which indicates the amount of labor required for reinforced concrete works per cubic meter of concrete. The total labor consumption rate of reinforced concrete works must consider all activities carried out within the system, such as shuttering, reinforcing and concreting. This paper presents and interprets survey results shown in violin plots, and M-estimators are derived to deliver sound recommendations for considering correlations when determining the total labor consumption rate of reinforced concrete works. An example demonstrates the influence of input parameter correlations on calculating the total labor consumption rate of reinforced concrete works.

*Keywords:* Monte Carlo simulation, Distribution function, Expert survey, Violin plot, Chance/risk ratio, Histogram.

## 1 INTRODUCTION

Probabilistic calculations usually assume uncorrelated input parameters. In the literature, this is justified by the notion that correlations are difficult to establish subjectively or that available data is often insufficient to mathematically determine correlations in construction management and economics. Furthermore, integrating correlations in simulation models requires additional effort. This paper demonstrates the influence that correlations have on the results of a Monte Carlo simulation using a worked example that calculates the total labor consumption rate of reinforced concrete works.

The total labor consumption rate of reinforced concrete works  $TCR_{A,RCW}$  [wh/m<sup>3</sup>] is calculated using Eq. (1). It is a key parameter to quantify construction cost but also to determine construction time. The total labor consumption rate reflects the number of paid working hours required to produce one cubic meter of reinforced concrete according to the contractually agreed specifications. Multiplication of the total labor consumption rate with the reinforced concrete quantity to be produced and the mean wage costs results in the direct cost of reinforced concrete works.

$$TCR_{A,RCW} = CR_{A,FW} * FR_{A,BD} + CR_{A,RW} * RR_{A,BD} + CR_{A,CW} \quad (1)$$

$TCR_{A,RCW}$	Average total labor consumption rate for reinforced concrete works [wh/m <sup>3</sup> ]
$CR_{A,FW}$	Average labor consumption rate for formwork-related activities [wh/m <sup>2</sup> ]
$FR_{A,BD}$	Average formwork ratio for the entire building [m <sup>2</sup> /m <sup>3</sup> ]
$CR_{A,RW}$	Average labor consumption rate for reinforcing works [wh/t]
$RR_{A,BD}$	Average reinforcement ratio for the entire building [t/m <sup>3</sup> ]
$CR_{A,CW}$	Average labor consumption rate for concrete works [wh/m <sup>3</sup> ]

Each of the above input parameters for the calculation of the total labor consumption rate of reinforced concrete works is associated with a varying degree of uncertainty, depending on planning accuracy and project progress. Deterministic calculations do not provide the option of accounting for these uncertainties in a systematic manner.

Monte Carlo simulations make it possible to include uncertainties of input variables in the calculation process. For this purpose, distribution functions (e.g., triangular distributions) are allocated to the calculation parameters of a deterministic computation model. For each of the iterative steps of the simulation, random values within the ranges of the specified distributions are selected (Latin Hypercube sampling is used in the case discussed in this paper). This process is repeated several thousand times, and the results of the individual iterative steps are represented in histograms (Hofstadler and Kummer 2014). The selection of a deterministic value within the calculated ranges directly determines the over- or underrun probability, and thus the chance/risk ratio relative to the output variables.

Dependencies between input parameters are modeled on the basis of correlations, which are a one-dimensional (standardized) measure of the linear dependency between two characteristics. Various types of correlation are used depending on the scaling of such characteristics. Since each construction project is unique, the amount of available data is usually insufficient, which is why estimates and expert knowledge need to be relied on. In an expert survey conducted by Kummer and Hofstadler in 2015 and 2016, the correlation coefficients between the calculation parameters for the total labor consumption rate of reinforced concrete works (Eq. 1) were estimated. Analyses were carried out using violin plots and descriptive statistics. Furthermore, M-estimators were derived that provide robust recommendations for the use of correlations in determining the total labor consumption rate of reinforced concrete works.

### **3 SELECTION OF EXPERTS**

It is usually impossible to interview all experts in a given field, with the exception of a few, highly specialized disciplines. This is why sampling is necessary to arrive at conclusions as to the overall population on the basis of received responses. The selection of such experts relies on fundamental questions such as: Who is in possession of relevant information? Which of these

experts are available? Whose willingness to provide information is the highest? Who is most likely in a position to provide accurate information? (Gorden 1969)

In this context, experts are defined as individuals who possess specific knowledge and intellectual skills and competencies in a clearly delineated field and who serve as a source of specific knowledge for the purpose of a survey. Expert knowledge usually comprises exceedingly large amounts of information, including simplifications, lesser known facts, rules of thumb and smart practices (i.e., heuristics) that enable efficient problem solving (Gläser and Laudel 2010, Springer Gabler 2014). As part of a preselection process, a total of about 130 experts with experience in the fields of costing, process planning, construction, final costing, and invoicing were contacted in writing and asked to participate in the survey. In total, 27 experts from Germany and Austria were recruited for the survey. They provided 25 to 27 responses with respect to correlations between the input parameters used to calculate the total labor consumption rate of reinforced concrete works. The majority of respondents (i.e., approx. 63%) worked for large companies with more than 250 employees, about 26% worked for medium-sized businesses (50 to 249 employees), and about 11% came from small businesses (10 to 49 employees). In the survey presented in this paper, experts had an average professional experience of 17.7 years; this experience ranged from 5 to 41 years. Questions were designed and developed together with social researchers (i.e., sociologists) in several revision steps, applying the principles of simplicity, clarity, impartiality, and specificity. On average, each respondent was interviewed for about 45 minutes either on the phone or in a face-to-face session to overcome ambiguities, collect missing information, or obtain background information and justifications of responses (Kummer 2015).

#### 4 ANALYSIS OF SURVEY

Expert survey respondents estimated the correlations between the individual parameters used to calculate the total labor consumption rate. This exercise was to evaluate if and to what extent experts were capable of stating and reasonably justifying correlation values. Correlations could be selected in 0.1 increments in a range from -1.0 to +1.0. A correlation of 0.0 describes complete independence between two calculation parameters, which means that any change in one of these input parameters will have no effect on the other. Furthermore, positive and negative correlations were described qualitatively to create a better understanding among expert respondents. In this context, positive correlations mean that any increase in one parameter will also lead to an increase in the other. Any change in a parameter will lead to a change in the correlated parameter in the same direction. For instance, any increase in the formwork ratio will cause a rise in the labor consumption rate for shuttering works. Correspondingly, the notion of negative correlations refers to an inverse pattern, i.e., any increase in the first parameter will result in a decrease in the correlated parameter (and vice versa). Any change to a calculation parameter thus leads to an inverse change in the second parameter. If, for example, the reinforcement ratio increases, the labor consumption rate for reinforcing works decreases. The magnitude of the relevant quantitative correlation can be described as follows, irrespective of it being positive or negative: 0.9 to 1.0 very strong; 0.7 to 0.8 strong; 0.5 to 0.6 medium; 0.3 to 0.4 weak; 0.1 to 0.2 very weak; 0.0 uncorrelated.

Figure 1 shows the violin plots of expert responses with respect to correlations between the calculation parameters used to determine  $TCR_{A,RCW}$  [wh/m<sup>3</sup>]. The diagram reveals only isolated negative correlation values whereas the majority of responses referred to positive correlations. Some experts also stated that they assumed no correlations at all for several parameters ( $r = 0$ ). Isolated extreme values and outliers are visible, which distort the mean values of correlations

accordingly. This is why it is recommended to apply a robust mean estimator (M-estimator according to Huber). Figure 2 shows the M-estimators for the individual correlations for the arrows that interconnect the relevant parameters. This key outcome of the expert survey makes it possible to systematically consider real correlations of input parameters of the total labor consumption rate of reinforced concrete works in the simulations. The relevant correlation matrix was checked for consistency.

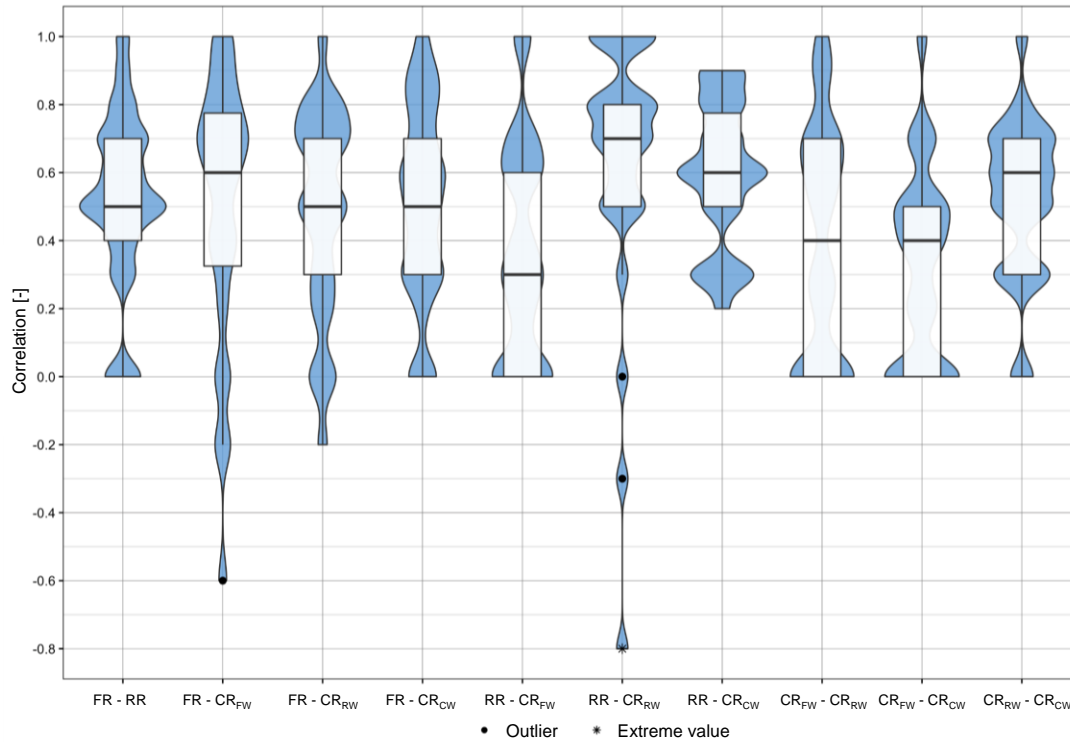


Figure 1. Correlation between the parameters of the total labor consumption rate of reinforced concrete works (Kummer 2015).

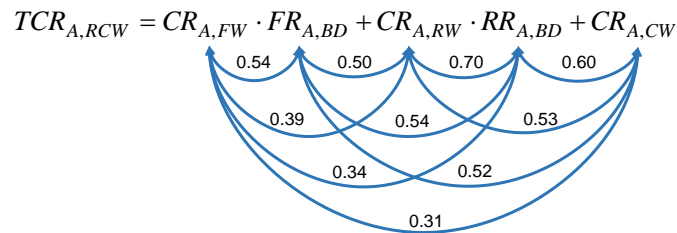


Figure 2. Correlation between the parameters of the total labor consumption rate of reinforced concrete works – M-estimators.

## 5 WORKED EXAMPLE

A worked example applies the M-estimators derived for the correlations between the input parameters to calculate  $TCR_{A,RCW}$  [wh/m<sup>3</sup>] in order to demonstrate the influence that these correlations have on the output of a Monte Carlo simulation (Latin Hypercube sampling with

50,000 iterative steps) compared to uncorrelated parameters. To simplify the process, triangular distributions are allocated to the individual input parameters. The following input data (i.e., minimum, expected and maximum values) were applied to each of the five input parameters:

- $CR_{A,FW}$  MIN: 1.00 | EXP: 1.20 | MAX: 1.60 wh/m<sup>2</sup>
- $FR_{A,BD}$  MIN: 2.70 | EXP: 3.00 | MAX: 3.30 m<sup>2</sup>/m<sup>3</sup>
- $CR_{A,RW}$  MIN: 12.00 | EXP: 13.00 | MAX: 16.00 wh/t
- $RR_{A,BD}$  MIN: 100.00 | EXP: 115.00 | MAX: 140.00 t/m<sup>3</sup>
- $CR_{A,CW}$  MIN: 0.90 | EXP: 1.00 | MAX: 1.30 wh/m<sup>3</sup>

The equation used to calculate  $TCR_{A,RCW}$  (see Eq. (1)) is used as a deterministic calculation model for the simulation. Calculations are performed both with uncorrelated parameters ( $r = 0$ ) and the correlation coefficients that were derived as M-estimators as part of the expert survey (see Figure 2). Figure 3 shows that the spread of the total labor consumption rate becomes wider when considering the correlations. This is justified by the fact that only positive correlations are applied and that the deterministic calculation model (see Eq. (1)) exclusively uses addition and multiplication. Combined with positive correlations, these two operations result in an increased range of simulation results for identical input parameters.

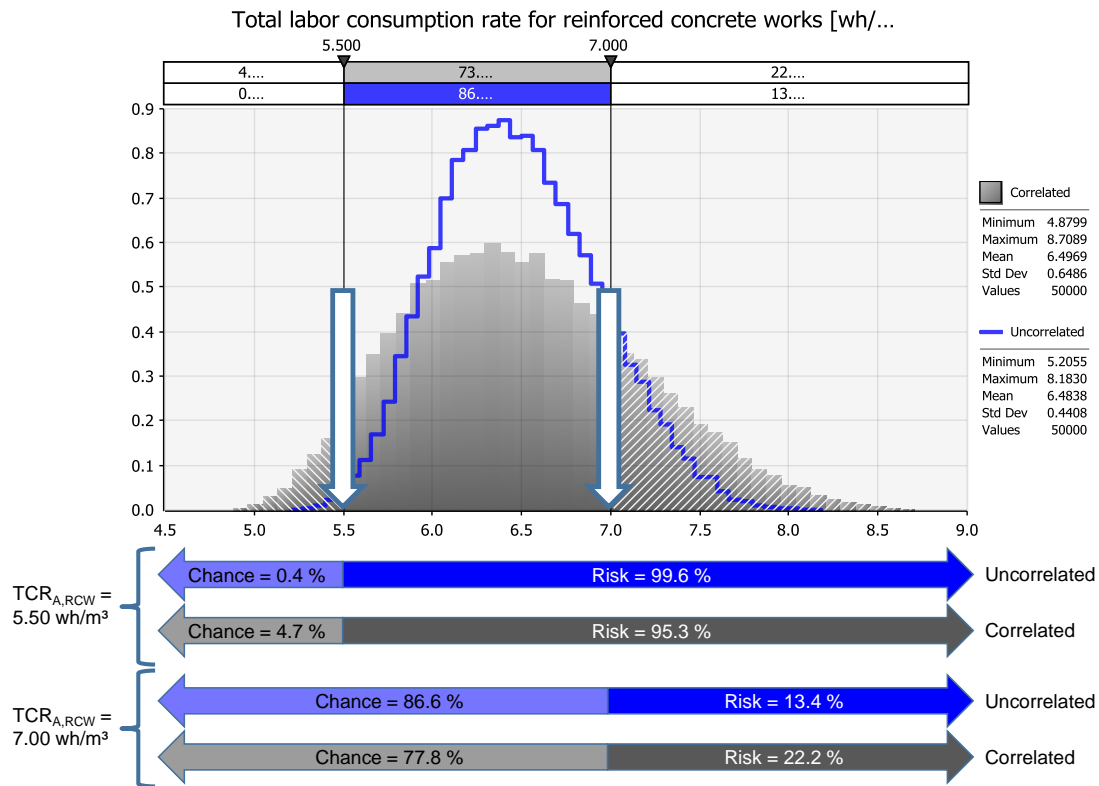


Figure 3. Comparison of the total labor consumption rate of reinforced concrete works using uncorrelated and correlated input parameters.

Compared to a simulation with correlated input parameters, an uncorrelated analysis would assign insufficient weighting to high and low labor consumption rate values and thus distort any subsequent calculations. Overall, correlated input parameters reveal a wider range of possible results.

Neglecting correlations and using uncorrelated input parameters leads to a narrower range of results, and thus an underestimation of existing uncertainties. If no correlations between input parameters can be defined or estimated, sensitivity analyses help capture their effects on simulation results, particularly on their ranges, and support their consideration in subsequent analyses, calculations, and decisions.

Various over- or underrun chance/risk ratios will result from the deterministic value selected within the calculated histogram ranges. If a labor consumption rate of 5.50 wh/m<sup>3</sup> is chosen, the chance of underrun will amount to about 0.4% in the uncorrelated scenario, whereas correlated parameters will result in a percentage of approximately 4.7% (see Figure 3). Conversely, the risk for a selected value of, say, 7.00 wh/m<sup>3</sup> is estimated to amount to 13.4% in the case of uncorrelated input parameters, whereas the correlated scenario leads to a risk estimated at 22.2% (i.e., about 8.8% greater).

Calculations were carried out using the @Risk software as an add-in to MS Excel. A value of 1 was assigned to the 'seed' so as to be able to reproduce simulation results if and when required.

## 6 SUMMARY

Although the above derivation of correlation coefficients from expert surveys should not be preferred over a mathematical calculation based on a solid database, it can indeed deliver useful indications if no data is available or if data collection is impossible. Users of this method should always bear in mind the fundamental principles of applying correlations to Monte Carlo simulations, and check results for plausibility. For comparison, a simulation with uncorrelated parameters can be carried out to determine the influence on output parameters. A worked example in which the total labor consumption rate of reinforced concrete works was calculated illustrates this type of comparison.

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