Resilient Structures and Sustainable Construction
Edited by Pellicer, E., Adam, J. M., Yepes, V., Singh, A., and Yazdani, S.

ISBN: 978-0-9960437-4-8

ANALYSIS OF EMPIRICAL COMPRESSION INDEX EQUATIONS USING THE WATER CONTENT

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This paper proposes a new method to evaluate the reliability of published empirical formulas in terms of accuracy and applicability to different soil types. Different empirical models are proposed to properly approximate the compression index for a wide range of water contents and soil types. They were developed using a unique technique and a substantial number of published regression equations and compression data. Familiar empirical equations were examined for their reliability in predicting the compression index of clay for any water content. A comparison was made between available and newly-proposed empirical formulas using combined regression data sets compiled independently by several authors. The newly proposed empirical compression index equations are applicable to a wide range of clay soils, and in validating other published relationships. The degree of scatter and variations in the computed compression index values are minimized for any water content.

Keywords: Compression data, Regression equations, Empirical equations, Compression index of clay, Technique.

1 INTRODUCTION

Several empirical equations have been developed to relate compression index (C_c) to soil index properties. Some equations are supposed to reflect C_c of all soils while others are limited to specific soil types and/or geography. Most authors used the correlation coefficient (R^2) as a lone measure to justify their applicability to a wide range of soils. Little or no information was provided relative to the number of data points used and/or the standard error. Further, the lack of uniformity in data collection and data interpretation makes it difficult to verify the accuracy of derived empirical equations. However, a large number of publications are now available to warrant a closer look at the validity, accuracy, and usefulness of many available empirical formulas for C_c estimation of fine-grained soils to their natural water content (w_n). The more widely used equations to estimate C_c are those developed by Bowles (1989), Peck and Reed (1954), Rendon-Herrero (1980, 1983) and Koppula (1981). Besides statistical measures, these equations seem to lack a logical and/or theoretical basis. The applicability of many of these equations to organic soils has not been established. 3-D models clearly show that consolidation pressures cannot be ignored in organic soils irrespective of the index property being used; C_c for clay sediments is actually related to consolidation pressure. Al-Khafaji and Andersland (1981) showed that the use of C_c in settlement calculations for organic soils is not justified. For a majority of practical problems, combining mineral and organic soils data is not suitable. This paper undertakes an exhaustive comparative study of available empirical equations, comparing their applicability to available published and independently-collected data. Also, additional insight is provided for future development of empirical equations.

2 AVAILABLE EMPIRICAL COMPRESSION INDEX EQUATIONS

Empirical equations to estimate the C_c are valuable because they are generally viewed as substitutes for consolidation tests. Approximate C_c values are important in preliminary settlement studies and indicate the magnitude of C_c for conducting consolidation tests. The soil index property used to estimate C_c should be easily measured in the laboratory. Some empirical formulas linearly relate C_c to natural water content (w_n) as shown in Table 1.

Table 1. Empirical equations for compression index approximation using water content.

Equation	**Applicability**	**Reference**
$C_c = 0.0115w_n$	Organic silt and clays	Bowles (1989)
$C_c = 0.01(w_n – 7.549)$	All clays	Rendon-Herrero (1983)
$C_c = 0.01w_n$	All clays	Koppula (1981)
$C_c = 0.0134(w_n – 7.034)$	River bank soil	Laskar and Pal (2012)
$C_c = 0.01(w_n – 5)$	All natural soils	Azzouz et al. (1976)
$C_c = 0.0046(w_n – 9)$	Brazilian clay	Cozzolino (1961)

The empirical expressions in Table 1 share one commonality – all are based on regression analysis of laboratory test data. Hough (1957) was the first to recognize that important differences exist between organic and mineral clay soils and suggested two different empirical equations to estimate C_c for the two types of soils. He also introduced several formulas to estimate C_c for cohesionless soils. Lambe and Whitman (1969) suggested that empirical expressions were not reliable, based in part on a graphical correlation between $C_c/(1 + e_0)$ and w_n for a number of soil samples.

3 VALIDITY OF EMPIRICAL COMPRESSION INDEX EQUATIONS

Equations published by Bowles (1989), Azzouz (1976), and Koppula (1981) have similar slopes but different intercepts. This illustrates that an objective and rational method is essential to validate empirical equations for compression index approximation. While nonlinear and multiple regression equations may be applicable in certain cases, these are not recommended due to inherently large fluctuations in approximated dependent parameters (C_c). Therefore, a new method is proposed to qualitatively and quantitatively determine the validity of linear regression equations used to estimate C_c. A number of regression equations were developed using one or more combinations of three independently compiled data sets and the linear empirical formulas listed in Table 1. A linear model relating the C_c to w_n was assumed in Eq. (1):

$$C_c = \propto_w + \beta_w w_n \tag{1}$$

α_w and β_w are the regression coefficients relating C_c to w_n for a given data set. Objectivity and unbiased analysis require that one must not be selective in choosing data points used in regression analysis. For this reason, the range of C_c was arbitrarily limited in the ranges of 0-1 and 0-0.5, and the corresponding regression equations were developed. The w_n was limited to ranges of 0-100, 0-75, and 0-50 and the corresponding regression equations determined. This process was applied to each of the two independent data sets reported by Lambe and Whitman (1969), and Rendon-Herrero (1983), using the same limits on C_c and w_n. The resulting regression coefficients α_w and β_w, correlation coefficients (R^2), standard errors σ_e, average water content w_{avg}, and average compression index C_{cavg} are shown in Table 2.

Table 2. Regression analysis results for compression index as a function of water content (Based on data reported by Lambe and Whitman 1969, and Rendon-Herrero 1983).

Based on Combined data								
Eq. No.	C_{cavg}	w_{avg}	Limit	# Points	R^2 (%)	σ_e	α_w	β_w
R1	0.313	38.24	Full Range	93	94.3	0.0803	–0.07993	0.01026
W-1	0.237	31.49	$0 \le C_c \le 1.0$	86	87.5	0.0729	–0.04771	0.00904
W-2	0.112	26.73	$0 \le C_c \le 0.5$	80	63.0	0.0685	–0.01363	0.00766
W-3	0.213	28.72	$0 \le w \le 100$	83	79.7	0.0735	–0.05385	0.00928
W-4	0.195	27.19	$0 \le w \le 75$	81	66.3	0.0682	–0.01620	0.00777
W-5	0.184	25.91	$0 \le w \le 50$	78	56.0	0.0687	–0.00742	0.00737

Careful examination reveals that reducing the data in Table 2 by only a few points has dramatic effects on R^2. This is true irrespective of the total number of data points analyzed. This makes it difficult to decide which data points to include or exclude from the analysis. Note that reducing the number of data points from 93 in equation R1 to 78 data points in Eq. (W-5) reduces the correction coefficient from 94.3% to 56%.

In general, one should use empirical formulas with high correlation and low standard error. On that basis, one may select a number of empirical equations for a given range of C_c or w_n. The implication is that no regression equation can do the job of correctly predicting the C_c over the full range of w_n values expected for soil. Hence, equation R1 is likely the most reasonable empirical expression for soils with w_n less than 100%. This is because it is based on 93 data points and has the highest R^2 of 94.3% with a relatively small corresponding standard error. Other empirical expressions may be selected for different w_n ranges. The derived empirical expressions appear to be varied and dependent on the number of data points involved. At first glance, it seems impossible to derive any substantive conclusions. Fortunately, consideration of the regression coefficients α_w and β_w shows that they are related linearly irrespective of R^2. Although regression coefficients corresponding to small correlations indicate lack of trend, the relationship between the regression coefficients α_w and β_w is strong as shown in Figure 1.

Examination of the regression coefficients indicates that all α_w-values are negative and all β_w-values are positive. Also, the larger β_w-values correspond to larger correlation coefficients. A linear relationship was determined between the α_w and β_w regression coefficients. The equation has a correlation coefficient of $R^2 = 0.999$ and is given by:

$$\propto_w = 0.17873 - 25.119\,\beta_w \tag{2}$$

Eq. (2) is referred to as the compression water content property line and is believed to relate to soil type. It is important to note that since higher β_W-values correspond to higher correlation, it is also possible to judge the quality of these formulas. It is now possible to examine published linear empirical equations, which relate C_c to w_n. Thus, the validity of the published equations presented in Table 1 is clearly illustrated in Figure 2. It appears that Rendon-Herrero (1983) and Azzouz (1976) equations are most accurate and that Cozzolino (1961) and Bowles (1989) equations are the least accurate in predicting the compression index of clay soils.

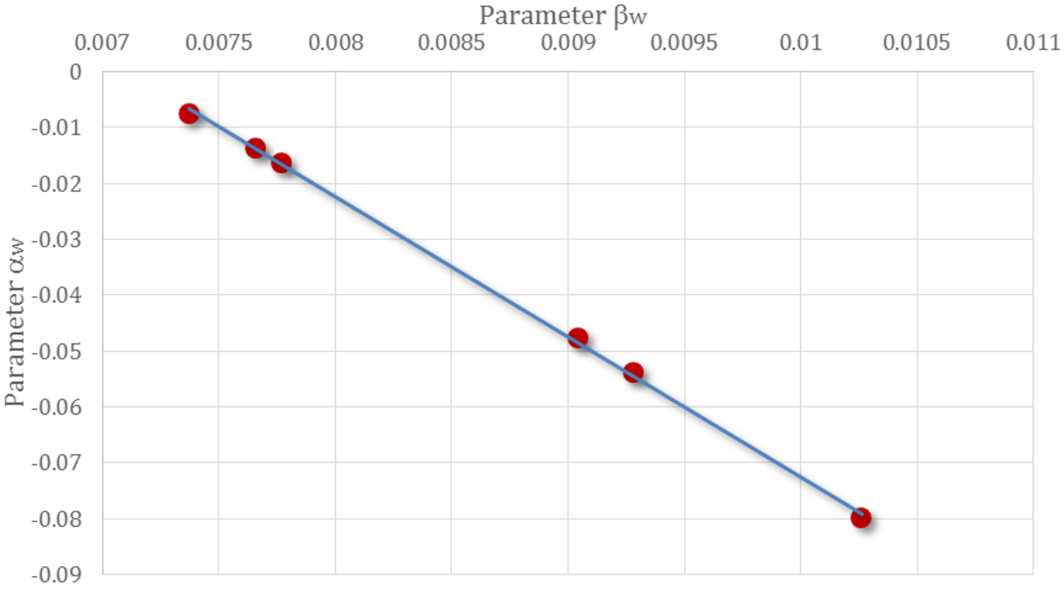

Figure 1. Relationship between α_w and β_w based on the compression index vs the natural water content.

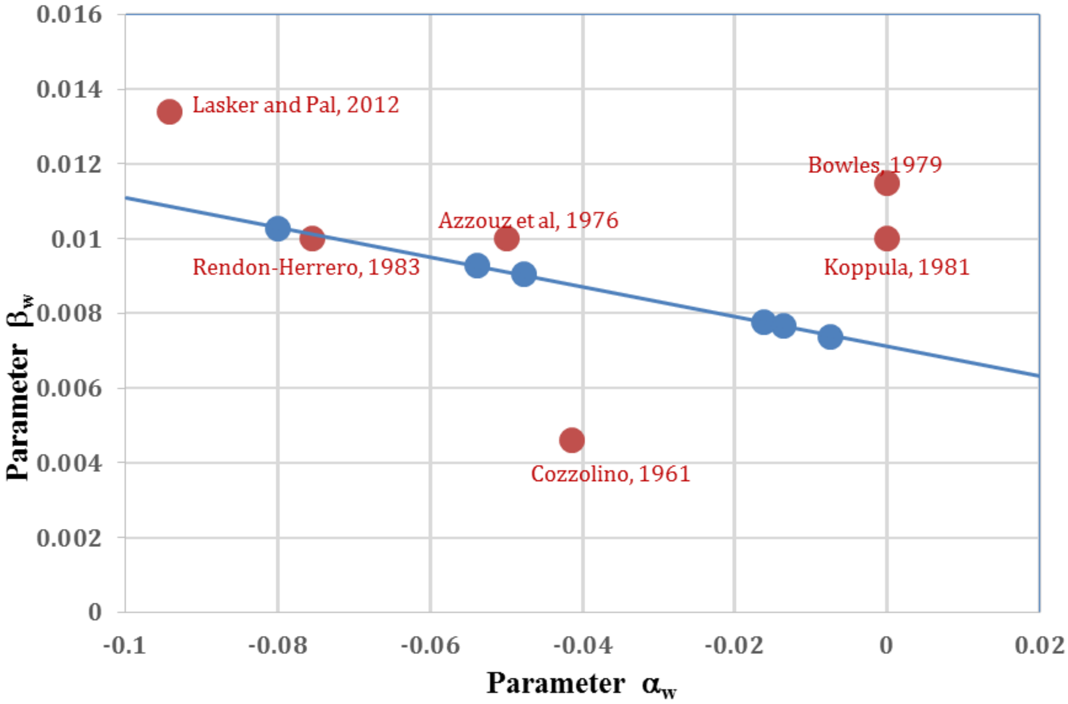

Figure 2. Property line for the compression index and water content.

Now we can produce a regression equation, which is most reliable in computing the compression index using Eq. (2). Noting that within the range of water content values, higher β_w -values produce high correlations, then substituting $\beta_w = 0.011$ into Eq. (2) yields $\alpha_w = -0.0976$.

Hence, it is recommended that the following empirical expression be used to estimate compression index in terms of water content:

$$C_c = -00976 + 0.011 w_n = 0.011(w_n - 8.9) \quad (3)$$

It is important to note that Eq. (3) is based on a maximum water content of 147.4%. Therefore, it is suggested that it be used for water contents of less than 150%. Clearly, the computed compression index for soils with a natural water content of less than 8.9% will be meaningless.

4 SUMMARY AND CONCLUSIONS

Most empirical equations used to estimate compression index of soils in terms of soil index properties have been developed using data for disturbed as well as undisturbed soils. The variability of soil parameters, soil types, and machine- and operator- errors makes it impossible to suggest a unified approach to compression index estimation. Unlike mineral soils, organic soils are highly compressible and their index properties change under constant effective consolidation pressure. Consequently, prediction of C_c should be limited to mineral soils. Most empirical formulas to estimate C_c are based on liquid limit, water content, and void ratio assuming linear relationships restricted to one independent variable. While some of these empirical equations are restricted to specific soils, others are supposedly applicable to all soils. Use of these equations is often legitimized based on the R^2 value but no attempt has been made to examine their applicability to independently compiled data and the standard error. Consideration of the data associated several widely-known empirical compression equations revealed interesting and useful trends. Examination of data scatter reveals that high values of w_n are generally associated with organic and volcanic soils. The inclusion of such data points in derivations of empirical equations could alter the applicability of many of these equations to mineral soils. The variability of C_c relating to organic soils is well documented. In fact, Al-Khafaji and Andersland (1981) have shown that the use of C_c in settlement calculations of organic soils is not justified. Based on work presented in this paper (Figure 1), it may become possible to define regions of applicability of empirical Cc equations to a variety of soils.

Acknowledgements

This paper is dedicated to Dr. and Mrs. Orlando Andersland.

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