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ANALYSIS OF EMPIRICAL COMPRESSION INDEX EQUATIONS USING THE VOID RATIO

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A new technique to assess the reliability of published compression index equations in terms of soil void ratio is presented. Several published equations pertaining to different soil types are examined in terms of accuracy and applicability. The new technique employs regression analysis to examine a substantial number of published compression data objectively. The traditional bias inherent in the selection of the number of data points and the range of void ratios for a given regression equation is eliminated. This was made possible by creating ranges for the compression index irrespective of the data set involved. This technique revealed that a strong correlation exists between the slopes and intercepts of all published equations. The slopes and intercepts of the newly developed regression equations were used to compare several well know published equations to assess accuracy and applicability. The proposed technique permits the examination of the authenticity of any published empirical equations relating to the compression index of clay to void ratio.

Keywords: Consolidation, Regression equations, Reliability, Standard error, Clay, Settlement calculations.

1 INTRODUCTION

Linear and nonlinear empirical equations have been published that relate compression index (C_c) to soil index properties. Some equations are supposed to reflect C_c of all soils while others are limited to specific soil types and/or geography (Peck and Reed 1954). Several published linear equations relate the compression index to the void ratio. Authors typically provide the correlation coefficient (R^2) as a lone measure to justify the derived expressions applicability to a wide range of soils. The number data points used and the standard error are often omitted which raises serious problems in that a few data points may produce extremely high R^2 value but lack reliability. Additionally, the methods used in data collection and data analysis were varied and introduced uncertainty to relative to the accuracy of dependability of published equations for compression index approximation. The recent publication of a substantial number of consolidation data and empirical equations warrants a thorough examination of published methods proposed for approximating C_c in terms of void ratio (e_o). Familiar equations to estimate C_c to the void ratio include Nishida (1956), Hough (1957), and Bowles (1989). Examination of 3-D models indicate that consolidation pressures and organic content significantly influence the C_c value. Al-Khafaji and Andersland (1981) demonstrated that the use of C_c in settlement analysis for organic soils is not appropriate. The normal approach of combining mineral and organic soils data is not appropriate and yield inaccurate empirical expressions for C_c vs the void ratio of mineral soils. A new technique is presented that relies on a comprehensive comparative analysis of published empirical equations and available soil index properties. A clear methodology is provided to help examine the authenticity of any empirical equation relating the compression index to the void ratio.

2 AVAILABLE EMPIRICAL COMPRESSION INDEX EQUATIONS

In engineering practice, engineers often rely on empirical equations to estimate the C_c in terms of soil index properties because they are less expensive than consolidation tests and provide meaningful initial estimates for settlement of foundations. Such estimates are only valid when dealing with normally consolidated fine-grained soils unless the pre-consolidation pressure is know. The focus of this paper is on published linear empirical equations used to estimate C_c in terms of the void ratio as shown in Table 1.

Table 1. Empirical equations for compression index approximation using void ratio.

Equation	Applicability	Reference
$C_c = 0.40(e_o – 0.25)$	All natural soils	Azzouz *et al.* 1976
$C_c = 1.15(e_o – 0.35)$	All clays	Nishida 1956
$C_c = 0.54(e_o – 0.35)$	All natural soils	Nishida 1956
$C_c = 0.75(e_o – 0.50)$	Soils of very low plasticity	Sowers 1970
$C_c = 0.156e_o + 0.0107$	All clays	Bowles 1989
$C_c = 0.43(e_o – 0.25)$	Brazilian clays	Cozzolino 1961
$C_c = 0.35(e_o – 0.5)$	Organic soils	Hough 1957

The published equations presented in Table 1 are all linear and developed using regression analysis of soil index properties. It is interesting to note that Nishida (1956) provided two equations relating to all clays and to all natural soils that are significantly different. Hough (1957) concluded that important differences exist between organic and mineral clay soils. Azzouz *et al.* (1976) proposed an expression for all natural soils. Lambe and Whitman (1969) showed that empirical expressions were not reliable, based on a linear correlation between the ratio $C_c/(1 + e_0)$ versus natural water content. Some authors have proposed expressions for specific geographic areas (Cozzolino 1961). In all cases, the slopes and/or intercepts vary significantly depending on the author and the test date used.

3 VALIDITY OF EMPIRICAL COMPRESSION INDEX EQUATIONS

Examination of the empirical equations presented in Table 1 shows that some have similar slopes but different intercepts while others have different slopes and intercepts. This illustrates the need for an objective and dependable method to validate published equations relating the compression index to void ratio. A new method is proposed to qualitatively and quantitatively determine the validity of published empirical equations to estimate C_c in terms of void ratio. The method requires the development of large number of empirical equations based on suitable and unbiased rages for the compression index and void ratio. These include consolidation data sets published by Rendon-Herrero (1980) as shown in Table 2. Based on these newly derived empirical expressions, a linear model relating the C_c to e_o is proposed as follows:

$$C_c = \propto_e + \beta_e e_o \qquad (1)$$

α_e and β_e are the regression coefficients relating C_c to e_o for a given rage of void ratios. Regression analysis was then performed using the combined data set (76 data points). Objectivity requires that one must not be selective in the inclusion and exclusion of data points used in regression analysis. Clearly, by including and excluding test data, one can alter the correlation coefficient and standard errors associated with the derived empirical expression. Thus, the authors established various ranges of C_c and e_o to determine the data set being used in the regression analysis. These ranges were 0-1 and 0-0.5 for C_c and the corresponding regression equations were developed. The e_o was limited to ranges of 0-3, 0-2, 0-1, and 0-0.75 and the corresponding regression equations calculated. This process was applied to each of the two independent data sets reported by Rendon-Herrero (1980), using the same limits on C_c and e_o. The resulting regression intercepts α_e and slopes β_e, correlation coefficients (R^2), standard errors σ_e, average void ratio e_{avg}, and average compression index C_{cavg} are for each data range are shown in Table 2.

Table 2. Regression analysis results for compression index as a function of *in situ* void ratio (based on data reported by Rendon-Herrero 1980).

Eq. No.	C_{cavg}	e_{avg}	Limit	# Points	R^2 (%)	σ_e	α_e	β_e
R1	0.349	1.142	Full Range	76	95.6	0.0758	−0.145964	0.433883
e-1	0.259	0.951	$0 \le C_c \le 1.0$	69	92.6	0.0604	−0.107009	0.385004
e-2	0.203	0.809	$0 \le C_c \le 0.5$	63	77.3	0.0568	−0.145964	0.433883
e-3	0.293	1.023	$0 \le e \le 3.0$	72	94.4	0.0644	−0.126327	0.409942
e-4	0.208	0.821	$0 \le e \le 2.0$	64	79.3	0.0566	−0.094456	0.368087
e-5	0.153	0.682	$0 \le e \le 1.0$	48	47.0	0.0529	−0.047026	0.293533
e-6	0.119	0.576	$0 \le e \le 0.75$	30	24.3	0.0392	0.006019	0.19655

Examination of the regression equations shown in Table 2 reveals that altering the data set used by only a few data points has significant influence on the R^2 value and associated standard error. Table 2 shows that by reducing the number of data points from 76 in Eq. No. R1 (Table 2) to 30 data points in Eq. No. e-6 (Table 2) reduces the corresponding R^2 values from 95.6% to 24.3%. Typically, one should use regression equations with high correlations using large number of data points. This implies that standard errors and correlations coefficients should always be provided with the derived regression equations. Therefore, Eq. R1 is likely the most reasonable empirical expression for soils with e_o of less than 1.14. This is because it is based on 76 data points and has the highest R^2 of 94.3% with a relatively small corresponding standard error. Examination of the equations presented in Table 2 may lead to the conclusion that it is impossible to derive any substantive trends. However, consideration of the regression coefficients α_e and β_e indicates that they are linearly related irrespective of the R^2 and standard error.

Figure 1 clearly show that a linear relationship exists between the α_e and β_e regression coefficients. This relationship has a correlation coefficient of $R^2 = 0.994$ and is shown as follows:

$$\propto_e = 0.13721 - 0.64312\beta_e \tag{2}$$

Eq. (2) represents the compression index and void ratio property line and is believed to be a related to soils types being considered. The published linear empirical equations (see e-1 through e-6 listed in Table 2) relating compression index to void ratio are shown graphically in Figure 2.

It is now possible to examine the slopes and intercepts of published empirical equations relating the compression index to void ratio as shown in Figure 2. Clearly, the regression coefficients for equations e-1, e-2, e-3, e-4, and e-5 plot closely to the derived regression Line given by Eq. (2). Nishida's empirical formula for all clays deviates appreciably from the line. Also, equations proposed by Bowles (1989) and Hough (1957) appear to be the least accurate predictors of the compression index from all those listed in Table 1. Note that Nishida's equation proposed for all soils is accurate because it is very close to the derived regression line shown in Figure 2. Koppula (1981) found that in comparison with other well-known relationships, Nishida's equation performed very poorly in predicting compression index of cohesive soils from the province of Alberta, Canada. Also, even though, equation e-5 is close to the line, the correlation coefficients of equations with positive intercepts are not very high. Although, the various empirical formulas were based on entirely different data from that used in the derivation of the line, it is obvious that there is a definite trend. In effect, these empirical formulas verify the legitimacy of the newly developed line.

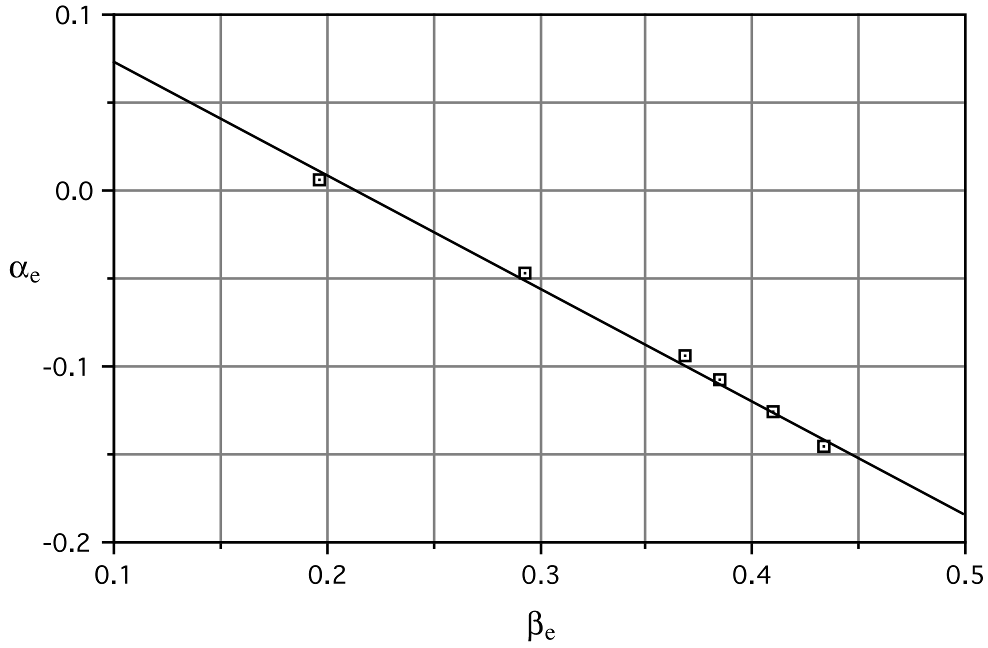

Figure 1. Relationship between regression coefficients for relationships between compression index and *in situ* void ratio.

Based on the observation that large β_e values indicate large correlation, it is possible to suggest an empirical relationship approximately within the range of the data used. Therefore, choosing equation e-3 which has a reasonably large slope, high correlation, and small standard error, a value of $\beta_e = 0.40$ is selected. Substituting into Eq. (2) gives an $\alpha_e = -0.1200$.

$$C_c = -0.12 + 0.40 = 0.40(e_o - 0.3) \tag{3}$$

Note that Eq. (3) is based on a maximum in situ void ratio of less than 3.0. This excludes the application of equation to most organic soils. Also, the empirical evaluation of compression index for soils with void ratios of less than 0.30 is not possible.

It is evident that additional empirical equations can be suggested which correspond to those listed in Tables 1 and 2. Thus, using the predefined ranges of independent variables along with Eq. (2), it should be possible to refine these formulas. However, it is important to only know that such expressions will be associated with fewer data points than used in the derivation of Eq. (3). Consequently, caution must be exercised when using expressions that are based on limited experimental data.

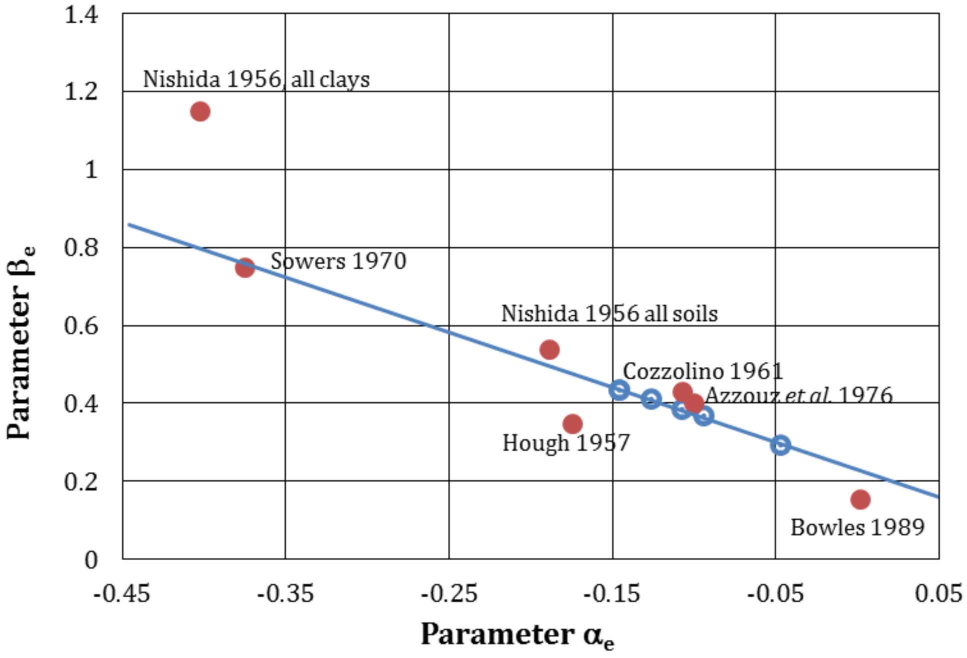

Figure 2. Relationship between regression coefficients developed between the compression index and the *in situ* void ratio.

4 SUMMARY AND CONCLUSIONS

Published empirical expressions used to estimate compression index of soils in terms of void ratio have been developed using linear regression analysis of laboratory data. The number of data points used and standard errors are often excluded which makes it difficult to assess their validity. Furthermore, the variability of soil parameters, soil types, and machine and operator errors makes it impossible to suggest a unified approach to compression index estimation in terms of the void ratio. In some cases, organic soils test data is included in the derivation of regression equations. However, organic soils properties change under constant effective consolidation pressure (Al-Khafaji and Andersland 1981) and should not have been used. Therefore, empirical expressions relating C_c to the void ratio should be limited to mineral soils. A new technique proposed that permits the examination of the validity of published linear empirical equations relating C_c to e_o. The new technique demonstrates that several published empirical equations are not reliable. Although some published empirical equations are limited to mineral soils, others are purported to apply to all soils including organic soils. Use of these formulas is often legitimized based on the R^2 value without proper examination of the associated standard errors and number of data points used. Consideration of a number of widely-known empirical compression equations revealed significant correlations between the slopes and intercepts of derived empirical equations. Examination of data scatter reveals that high values of e_o are generally associated with organic soils and should not be included with data for mineral soils. The inclusion of such data points in derivations of empirical expressions alters the validity of these equations when dealing with clay soils. This paper presents a new technique that permits the assessment of the validity of published equations relating the compression index to void ratio. The new technique reveals that the slopes and intercepts of published regression equations are strongly related.

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This paper is dedicated to Dr. and Mrs. Orlando Andersland.

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