Resilient Structures and Sustainable Construction
Edited by Pellicer, E., Adam, J. M., Yepes, V., Singh, A., and Yazdani, S.

ISBN: 978-0-9960437-4-8

ESTIMATION OF DYNAMIC RESPONSES ON CONTOURS OF SEVERAL ROUND CUTS AT THIN PLATE VIBRATIONS

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According to most theoretical and experimental researches, wave models quite fully and accurately describe the process of foundation vibration in the soil ground. Thus, solutions obtained for the vibration of an infinite plate with a circular cut are successfully used today to determine the amplitude-frequency characteristics of the pile foundations; however, the results of the cases with more than one cut are practically needed. In this paper, analytical expressions are found for dynamic stiffness within the wave model framework regarding the vertical vibrations of an infinitely thin plate with circular cuts, their mutual alignment being regarded. Determined expressions take into account the distance between piles located in a row or in a group. Obtained results can be used for the calculations of amplitude-frequency characteristics of pile foundations.

Keywords: Pile foundation, Characteristics of foundation vibrations, Wave model, Mutual influence of piles.

1 INTRODUCTION

According to Russian construction code, the design of the pile foundation under the machines with dynamic loadings may include the distance between pile centers within the range from 2 to 10 diameters. The standards however do not include the dependence of varying stiffness and damping on the distance between piles and their groups; thus it is advisable to have these ratios to evaluate the amplitude-frequency characteristics for engineering calculations. Note that activities to determine the amplitude-frequency characteristics of pile foundations have been started quite a long time ago, but are still unfinished. Most investigations are devoted to the interaction of a single pile with soil under the dynamic loading (Baranov 1967, Wu *et al.* 2013); attempts are undertaken to analyze the dynamic reactions of pile groups (Guz *et al.* 1974, El Naggar and Novak 1994, Nuzhdin *et al.* 2005).

This paper determines the links between the motions of the piles circular in plan and grouped, under steady vertical vibrations, and soil reaction on foundation side surfaces. The soil is simulated by an elastic inert medium that is described by manifold infinitely thin layers. Thus, the task of vertical vibrations of the infinite plate is solved, and the authors add and specify the results of Nuzhdin *et al.* (2005) and Kolesnikov *et al.* (2014) for the determination of stiffening and damping characteristics of the system, with due regard to the mutual position of circular cuts (see schematic in Figure 1), including the ones which have not been considered yet.

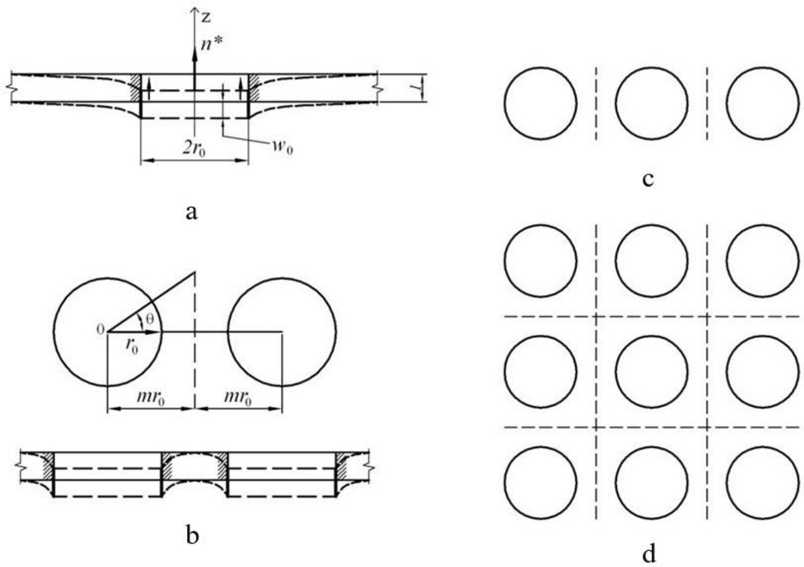


Figure 1. Schematic of position of circular cuts in the vibrating thin plate.

2 CURRENT APPROACH FOR THE ONE CUT

First, for the following analysis of the results, let us consider the task of warping axisymmetrical vibrations of an infinite thin layer with one circular cut with radius r_0 (Figure 1a) solved in Baranov (1967). In this case, the equation of elastic medium motion, with no volumetric forces in the cylindrical system of coordinate (r, θ, t) is written as:

$$\mu\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)=\rho\frac{\partial^2 w}{\partial t^2} \tag{1}$$

with the boundary condition on the contour.

$$w\left(r_0, \theta, t\right)=w_0 e^{i\omega t} \tag{2}$$

Here, $w=w(r, \theta, t)$ is the motion along the axis z; ρ is density; μ is Lame coefficient. We assume, basing on the deformation character near the boundary, that all points remain on their rights during the vibrations (r, θ = const), and the distance between them does not change.

Only tangential stresses act on the circular cut contour.

$$\tau_{rz}=\mu\left.\frac{\partial w}{\partial r}\right|_{r=r_0} \tag{3}$$

Eq. (1) and (2) are solved by the partition method, and the solution can be presented as

$$w=e^{i\omega t}\sum_{n=0}^{\infty}(A_n\cos n\theta+B_n\sin n\theta)[C_n H_n^{(1)}(kr)+D_n H_n^{(2)}(kr)] \tag{4}$$

where $k=\omega/\sqrt{\mu/\rho}$, $H_n^{(1)}$, $H_n^{(2)}$ is the first and second kinds Hankel function, whereas A_n, B_n, C_n, D_n are the constant factors to be found.

It follows from the condition that only diverging waves take place when the plane with one cut is considered, and from the axial symmetry condition that

$$w = e^{i\omega t} D_0 H_0^{(2)}(kr) \tag{5}$$

where

$$D_0 = w_0 \big/ H_0^{(2)}(kr_0) \tag{6}$$

Since only tangential stresses Eq. (3) act on the circular cut contour, they are reduced to the resultant for the single thickness layer

$$n_0^* = -\int_0^{2\pi} \tau_{rz} \cdot 1 \cdot r_0 d\theta = \mu w_0 e^{i\omega t} \frac{2\pi k r_0}{H_0^{(2)}(kr_0)} H_1^{(2)}(kr_0)\,. \tag{7}$$

3 NEW METHOD FOR SEVERAL CUTS

In Nuzhdin *et al.* (2005), under consideration are the warping vibrations of the layer with two circular cuts with radius r_0; cut centers are within $2mr_0$ or m diameters from each other, $m > 1$ (schematic in Figure 1b), under the condition of symmetry on the line crossing the right connecting the cut centers perpendicularly in the point $r_g = mr_0$

$$\partial w(r_g / \cos\theta, \theta, t) / \partial n = 0, -\pi/2 < \theta < \pi/2 \tag{8}$$

Solution of the tasks Eq. (1), Eq. (2), Eq. (8), for example, for the left cut is found in the form of Eq. (4) in two codomains:

$$w = e^{i\omega t} D_0 H_0^{(2)}(kr) \tag{9}$$

for $r \geq r_0$, $\pi/2 < \theta < 3\pi/2$:

$$w = e^{i\omega t}\{D_0 H_0^{(2)}(kr) + 2D_0 R \sum_{n=1}^{N} [H_{2n-1}^{(2)}(kr_0) H_{2n-1}^{(1)}(kr) - H_{2n-1}^{(1)}(kr_0) H_{2n-1}^{(2)}(kr)] \cos[(2n-1)\theta]\} \tag{10}$$

$$R = H_1^{(2)}(kr_g) \Big/ \sum_{n=1}^{N} H_{2n-1}^{(2)}(kr_0)[H_{2n-2}^{(1)}(kr_g) - H_{2n}^{(1)}(kr_g)] - H_{2n-1}^{(1)}(kr_0)[H_{2n-2}^{(2)}(kr_g) - H_{2n}^{(2)}(kr_g)] \tag{11}$$

for $r \geq r_0$, $-\pi/2 < \theta < \pi/2$, D_0 coincides with Eq. (6). Here and below, opposite to Nuzhdin *et al.* (2005), where $N = 2$, the solution is considered in the general form, and later N was determined by test calculations.

Assuming that the stress value along the cut contour at $0 \leq \theta < \pi/2$ is measured linearly

$$\tau_{rz}(\theta) = \tau_{rz}(0) + 2[\tau_{rz}(\pi/2) - \tau_{rz}(0)]\theta/\pi \tag{12}$$

the resultant is found as

$$n_1^* = -\int_0^{2\pi} \tau_{rz} \cdot 1 \cdot r_0 d\theta = \mu w_0 e^{i\omega t} \frac{\pi k r_0}{H_0^{(2)}(kr_0)} \left\{ 2H_1^{(2)}(kr_0) - \frac{R}{2} F \right\} \tag{13}$$

$$F = \sum_{n=1}^{N} H_{2n-1}^{(2)}(kr_0)[H_{2n-2}^{(1)}(kr_0) - H_{2n}^{(1)}(kr_0)] - H_{2n-1}^{(1)}(kr_0)[H_{2n-2}^{(2)}(kr_0) - H_{2n}^{(2)}(kr_0)] \tag{14}$$

Opposite to Eq. (7), in the right part of this equation, there are additive terms that include the effect of the second cut.

Generalization of the result for the case of warping vibrations of the layer with circular cuts which centers are located on the right within $2mr_0$, $m > 1$ from each other (schematic in Figure 1c) permits describing the resultant for the inner cut by the expression

$$n_2^* = -\int_0^{2\pi} \tau_{rz} \cdot 1 \cdot r_0 d\theta = -2\int_{-\pi/2}^{\pi/2} \tau_{rz} \cdot 1 \cdot r_0 d\theta = \mu w_0 e^{i\omega t} \frac{\pi k r_0}{H_0^{(2)}(kr_0)} \left\{ 2H_1^{(2)}(kr_0) - RF \right\} \tag{15}$$

whereas in the case of the inner cut ordered in accordance with the schematic of Figure 1d, the resultant is described by the expression

$$n_3^* = -4\int_{-\pi/4}^{\pi/4} \tau_{rz} \cdot 1 \cdot r_0 d\theta = \mu w_0 e^{i\omega t} \frac{\pi k r_0}{H_0^{(2)}(kr_0)} \left\{ 2H_1^{(2)}(kr_0) - \frac{3R}{2} F \right\} \tag{16}$$

Let us add the results by the expressions describing the resultant for the boundary (not corner) cut (schematic in Figure 1d)

$$n_4^* = -\int_0^{2\pi} \tau_{rz} \cdot 1 \cdot r_0 d\theta = \mu w_0 e^{i\omega t} \frac{\pi k r_0}{H_0^{(2)}(kr_0)} \left\{ 2H_1^{(2)}(kr_0) - \frac{5R}{4} F \right\} \tag{17}$$

and for the corner cut

$$n_5^* = -\int_0^{2\pi} \tau_{rz} \cdot 1 \cdot r_0 d\theta = \mu w_0 e^{i\omega t} \frac{\pi k r_0}{H_0^{(2)}(kr_0)} \left\{ 2H_1^{(2)}(kr_0) - \frac{7R}{8} F \right\} \tag{18}$$

Generalizing the obtained results we have, according to Kolesnikov *et al.* (2014)

$$n_j^* = \mu w_0 e^{i\omega t} \pi k r_0 \left\{ \frac{2H_1^{(2)}(kr_0)}{H_0^{(2)}(kr_0)} - \frac{j}{2} \frac{H_1^{(2)}(kr_g)}{H_0^{(2)}(kr_0)} C \right\}, \quad j = 0,1,2,3 \tag{19}$$

and

$$n_4^* = \mu w_0 e^{i\omega t} \pi k r_0 \left\{ \frac{2H_1^{(2)}(kr_0)}{H_0^{(2)}(kr_0)} - \frac{5}{4} \frac{H_1^{(2)}(kr_g)}{H_0^{(2)}(kr_0)} C \right\}, \quad n_5^* = \mu w_0 e^{i\omega t} \pi k r_0 \left\{ \frac{2H_1^{(2)}(kr_0)}{H_0^{(2)}(kr_0)} - \frac{7}{8} \frac{H_1^{(2)}(kr_g)}{H_0^{(2)}(kr_0)} C \right\} \tag{20}$$

$$C = \frac{\sum_{n=1}^{N} J_{2n-1}(kr_0)[Y_{2n-2}(kr_0) - Y_{2n}(kr_0)] - Y_{2n-1}(kr_0)[J_{2n-2}(kr_0) - J_{2n}(kr_0)]}{\sum_{n=1}^{N} J_{2n-1}(kr_0)[Y_{2n-2}(kr_g) - Y_{2n}(kr_g)] - Y_{2n-1}(kr_0)[J_{2n-2}(kr_g) - J_{2n}(kr_g)]} \tag{21}$$

here J_n, Y_n is the first and second kind Bessel function.

Let us present the reactions of single-thickness layers applied to the foundation side surface in the general form for six variants under consideration

$$n_j = S_{wj}(kr_0) w_0 e^{i\omega t} = \mu w_0 e^{i\omega t} \left(S_{w1j} + iS_{w2j} \right), \; j = 0,1,...5 \tag{22}$$

where S_{w1j}, S_{w2j} are the real and imaginary dimensionless components S_{wj}, which can be presented for j = 0,1,2,3 as

$$S_{w1j}(kr_0) = 2\pi k r_0 \frac{J_0(kr_0)J_1(kr_0) + Y_0(kr_0)Y_1(kr_0)}{J_0^2(kr_0) + Y_0^2(kr_0)} - \frac{j}{2} \pi k r_0 \frac{J_0(kr_0)J_1(kr_g) + Y_0(kr_0)Y_1(kr_g)}{J_0^2(kr_0) + Y_0^2(kr_0)} C, \tag{23}$$

$$S_{w2j}(kr_0) = \frac{4}{J_0^2(kr_0) + Y_0^2(kr_0)} - \frac{j}{2} \pi k r_0 \frac{Y_0(kr_0)J_1(kr_g) - J_0(kr_0)Y_1(kr_g)}{J_0^2(kr_0) + Y_0^2(kr_0)} C \tag{24}$$

And for j = 4, 5 as

$$S_{w1j}(kr_0) = 2\pi k r_0 \frac{J_0(kr_0)J_1(kr_0) + Y_0(kr_0)Y_1(kr_0)}{J_0^2(kr_0) + Y_0^2(kr_0)} - \frac{22 - 3j}{8} \pi k r_0 \frac{J_0(kr_0)J_1(kr_g) + Y_0(kr_0)Y_1(kr_g)}{J_0^2(kr_0) + Y_0^2(kr_0)} C \tag{25}$$

$$S_{w2j}(kr_0) = \frac{4}{J_0^2(kr_0)+Y_0^2(kr_0)} - \frac{22-3j}{8}\pi kr_0 \frac{Y_0(kr_0)J_1(kr_g)-J_0(kr_0)Y_1(kr_g)}{J_0^2(kr_0)+Y_0^2(kr_0)}C \tag{26}$$

In Kolesnikov *et al*. (2014), while determining *C*, the authors considered the solutions with N = 2, which gave satisfactory results for $kr_0 << 1$ and low *m*. But $kr_0 \approx 1$ is possible, for example, in the case of the pile foundations under the machines with dynamic loadings, thus it was found experimentally that the necessary accuracy is reached at N=4, which is later used in our researches.

It follows from the presented solution of the task of the vibrating layer with one or more cuts that the dynamic stiffnesses S_{wj} are described by the complex functions depending on vibration frequency ω, sole size, as well as on medium density ρ and stiffness μ. Using asymptotic expansion S_{w1j}, S_{w2j}, we have that for the frequency $\omega \to 0$, dynamic stiffnesses tend to vanish. Reactions are ahead of respective motions by temporal ranges Δ_j, which are determined as $\Delta_j = arctg(S_{w2j}/S_{w1j})$, whereas the motion amplitudes can be evaluated from the ratio $A_j = (S_{w1j}^2 + S_{w2j}^2)^{0,5}$.

According to Baranov (1967), common practical cases are $kr_0 << 1$, thus for low kr_0, the formulas for S_{w1j}, S_{w2j} are found for j = 0,1,2,3:

$$S_{w1j} = -\frac{2\pi}{\ln(0.5kr_0)}\left[1-\frac{2j}{m(1+m^2+m^4+m^6)}\right],\quad S_{w2j} = \frac{\pi^2}{\left[\ln(0.5kr_0)\right]^2}\left[1-\frac{2j}{m(1+m^2+m^4+m^6)}\right], \tag{27}$$

which we add by the relations for the boundary and corner cuts according to the schematic in Figure 1, d (for j = 4,5)

$$S_{w1j} = -\frac{2\pi}{\ln(0.5kr_0)}\left[1-\frac{22-3j}{m(1+m^2+m^4+m^6)}\right],\quad S_{w2j} = \frac{\pi^2}{\left[\ln(0.5kr_0)\right]^2}\left[1-\frac{22-3j}{m(1+m^2+m^4+m^6)}\right], \tag{28}$$

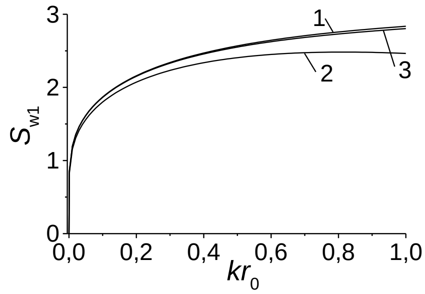
and then

$$\Delta_j = arctg\left[\frac{\pi}{2\ln(0,5kr_0)}\right],\ j = 0,1,...,5, \tag{29}$$

$$A_j = \frac{2\pi}{\ln(0.5kr_0)}\sqrt{1+\left[\frac{\pi}{2\ln(0.5kr_0)}\right]^2}\left[1-\frac{2j}{m(1+m^2+m^4+m^6)}\right],\ j = 0,1,2,3, \tag{30}$$

$$A_j = \frac{2\pi}{\ln(0.5kr_0)}\sqrt{1+\left[\frac{\pi}{2\ln(0.5kr_0)}\right]^2}\left[1-\frac{22-3j}{2m(1+m^2+m^4+m^6)}\right],\ j = 4,5. \tag{31}$$

It follows from the presented results that, independently on the distances between cut centers, the reactions for different kr_0 are ahead of the respective motions by practically the same temporal range; their values coincide with the ones obtained for one cut case. Analysis of the formulas for the amplitudes gives that the increasing distance between the cuts (*m* is rising) results in the fast reduction of their mutual influence. For example, in the case of the inner cut, ordered as is shown in Figure 1, d, which is the most influenced by the neighboring ones, amplitude variation is about 6% at *m* = 2, whereas at *m* = 3 it is only 0.25 % as compared to one cut. Comparing the reactions on the cut contours regarding their mutual position, we find that at *m* = 2, the boundary cut amplitude is higher than the inner cut amplitude by 1 %, whereas the amplitude of the corner cut is 3 % higher.



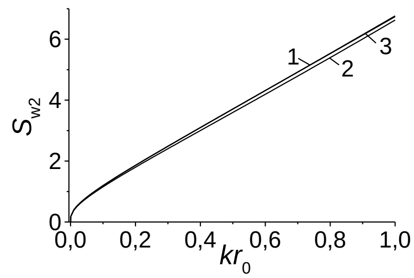


Figure 2. S_{w1} (a) and S_{w2} (b) for one (line 1) and inner cuts at m = 2 (line 2), m = 3 (line 3).

Illustrating varying S_{w1} and S_{w2} within the range $0 \le kr_0 \le 1$, Figure 2 shows the graphs of real and imaginary dimensionless components of the dynamic stiffnesses S_w at the vibration of the layer with one cut and inner cut located in the group as shown in Figure 1,d for $m = 2$ and $m = 3$.

It follows from the presented data that, when the piles are grouped, the maximal mutual influence is observed at $m = 2$ – variation of S_{w13} as compared to the values obtained for one cut may reach 14 % at $kr_0 \approx 1$; but as $m = 3$ – this value reduced down to 1.5 % within the whole range of kr_0. The difference between S_{w20} and S_{w23} at $m = 2$ does not exceed 1.5 %, but at $m = 3$, it is practically absent.

Note in conclusion that analytical expressions were found when the task of vertical vibrations of the infinite plate with circular cuts was solved, which permit performing engineering calculations to determine the stiffness and damping of the pile foundation, as well as the coefficients of variation of the dynamic stiffnesses regarding the pile position. It is demonstrated that as the piles are distanced essentially, their mutual influence is minimal or zero, which permits using the amplitude-frequency characteristics of an individual pile. The results are in good agreement with the data presented in (Guz *et al.* 1974, Pyatetzkij *et al.* 1993) and permit specifying the evaluations of the pile foundation characteristics under dynamic loadings.

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