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YIELD STRESS BASED ON NATURAL STRAIN THEORY UNDER CYCLIC TENSILE-COMPRESSIVE LOADING AFTER LARGE UNIAXIAL TENSION

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The purpose of this study is to investigate the yield behavior under a large deformation based on the Natural Strain theory. In this study, using the test pieces made from high purity tough pitch copper, the experiments for cyclic loading of tension and compression are carried out after applying the large uniaxial tension. As a method for determining the yield stress in each cycle, the slope of the tangent at yielding in the deviatoric stress- deviatoric strain curve is used instead of the conventional proof stress. In order to reveal the change of yield stress in cyclic loads after applying large pre-deformations, the experiments are conducted with the different sizes of pre-deformation and the different strain amplitudes. Moreover, the yield behavior under cyclic loads is revealed by comparing with the results from conventional proof stress.

Keywords: Finite deformation, Cyclic loading test, Anisotropy, Slope of tangent, Ductile materials, Strain amplitude, Proof stress.

1 INTRODUCTION

The Natural Strain suggested in this study is obtained by integrating an infinitesimal strain rate on an identical line element over the whole process of the deformation path. Consequently, since the additive law of strain on an identical line element can be satisfied, the strain rate can be decomposed into the elastic component and the plastic component clearly. Moreover, the elastic component can also be decomposed into deviatoric part and volumetric part in the same manner as the conventional infinitesimal deformation theory. Therefore, the Natural Strain is an effective strain representation which can be systematically treated from infinitesimal deformation to the large deformation. On the basis of the Natural Strain theory, in a series of our previous study (Kato and Kazama 2013, Kato 2014, Kato 2016), the anisotropy of yield surface generated under large deformation has been investigated under different deformation history.

On the other hands, as for the yielding behavior under cyclic loads, there has been a lot of experimental research. However, many of these researches have been done within the range of an infinitesimal deformation. Therefore, the detailed studies on the yield behavior under cyclic loads after applying a large pre-deformation have not been fully elucidated. In the present study, the yielding phenomenon generated under cyclic loading of tension and compression after applying the large pre-deformation of uniaxial tension will be examined as the first step in this research. In general, in the case of ductile materials, the value of yield stress obtained after applying large pre-deformation shows a considerable increase as compared with the initial yield value. Moreover, the anisotropy in the material has been produced with an increase of plastic strain, and the value of yield stress on the pre-deformation side is different from the yield stress on the opposite side due to the Bauschinger effect. However, the values of these yield stresses gradually decrease and approach the steady value because of the effect of material anisotropy reduces with increase of the numbers of cycle in periodic loads.

In this research, as a method for determining the yield stress in each cycle, the slope of the tangent at yielding in the deviatoric stress- deviatoric strain curve is used instead of the conventional proof stress. Then, in order to reveal the yield behavior under cyclic loads after applying pre-deformation of large uniaxial tension, the experiments are conducted with the different sizes of pre-deformation and the different strain amplitudes.

2 THE ESTIMATION METHOD OF YIELD STRESS UNDER CYCLIC LOADING

The estimation method of yield stress in each cycle is described in this chapter. Figure.1 shows a deviatoric stress and deviatoric strain diagram when the tensile and compressive loads are subjected alternately to the test pieces after applying a pre-deformation of a large uniaxial tension. And further, Figure.2 shows an enlarged view of the deviatoric stress and deviatoric strain diagram focusing on the first cycle in the cyclic loading tests. Since the value of yield stress in the pre-deformation of large uniaxial tension has been already found from the final stress state of pre-deformation, the value of yield stress when the tensile load is applied again after pre-deformation is also already determined. And, it is indicated by a point A in Figure 2. In the neighborhood of the yield point A, the deviatoric stress and deviatoric strain diagram becomes a gentle curve, and in this study, it is formulated as a follow.

$$S_1 = a\left(1 - exp\left(b e_1\right)\right) + c e_1 + d \tag{1}$$

Here, *a*, *b*, *c* and *d* are coefficients determined by using the *Levenberg-Marquardt Method*, which is one of the non-linear least-squares methods. Moreover, in order to derive the slope of the tangent in this curve, the following equation can be derived by differentiating Eq.(1), i.e.,

$$\frac{dS_1}{de_1} = -a\,b\,exp\left(b e_1\right) + c \tag{2}$$

Hence, the slope of the tangent at the yielding point A can be specified by using Eq.(2).

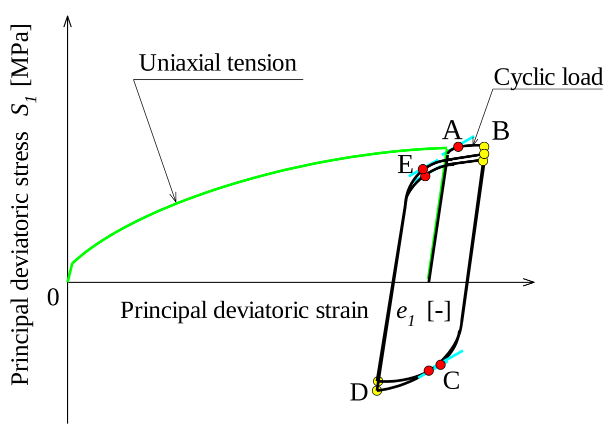


Figure 1. Cyclic loading tests for tension and compression after pre-deformation of uniaxial tension.

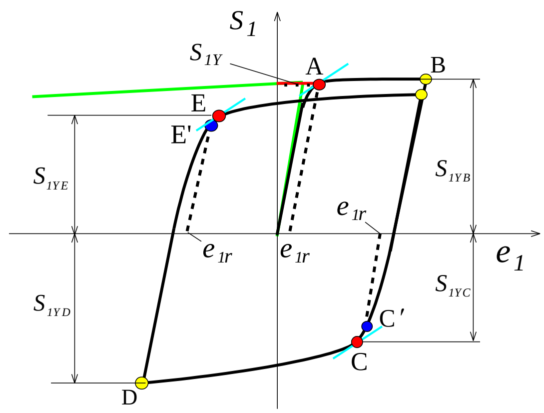


Figure 2. Deviatoric stress-deviatoric strain curve in the first cycle.

Firstly, the yield stress obtained after applying the tensile strain with specified constant strain amplitude is represented by a point B just before unloading of tension. Secondly, the yield stress in compression side is obtained by representing the gentle curve of deviatoric stress by using the Eq. (1). Then, the stress, which has the same value as the slope of tangent measured at the point A, i.e., the stress at point C in this figure, is assumed to be a yield stress. Thirdly, the yield stress obtained after applying the compressive strain with specified constant strain amplitude is represented by a point D. Finally, the yield stress in tension side, which is derived by applying a tensile load again after unloading of compression, is obtained by representing the gentle curve of deviatoric stress by using Eq.(1) once again. Similarly, the stress, which has the same value as the slope of tangent measured at the point A, i.e., the stress at point E in this figure, is assumed to be a yield stress.

On the other hand, the yield stress by proof stress is determined based on the value of the residual strain e_{1r}, which is obtained by unloading from point A. Then, as shown in point C' and E', the proof stress on the compression side and the tension side are estimated smaller compared with the results proposed in this research.

3 EXPERIMENTAL METHOD

In order to apply a large pre-deformation of uniaxial tension to the specimens, the cylindrical specimens made from high purity tough pitch copper, i.e., outer diameter 22 [mm], inner diameter 16 [mm], the gauge length 30 [mm] and purity 99.99%, are adopted in the experiments. As for the measurements of longitudinal displacements, the displacement meters equipped to the circular jig are used.

Next, as for the experimental condition, this experiment is composed of two stages. Namely, the experiments applying the pre-deformation of the large uniaxial tension to the specimens are firstly conducted. In the second place, the cyclic loading tests for tension and compression are conducted.

(1) Experimental condition of pre-deformation of large uniaxial tension

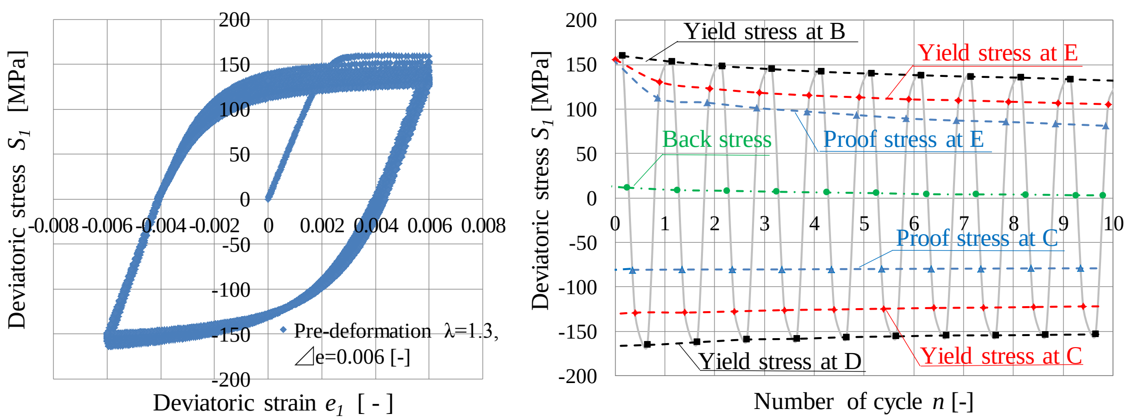
a) Three different types of stretch, i.e., values of stretch λ =1.1, 1.2, 1.3 [-], are applied to the test specimens

(2) Experimental condition of cyclic loading tests for tension and compression

b) Attaching the strain gages to the specimens, the experiments of the cyclic loads for tension and compression are conducted with the constant strain amplitude, i.e., ⊿e_1=± 0.006 [-], and in these experiments the numbers of cycles n are ten times. Then, according to the method explained in the previous chapter, the values of yield stress at points B, C, D and E are determined, and the changes in the yield stress associated with an increase of the number of cycle n are investigated. Moreover, as for the strain amplitude in the case of λ =1.2 [-], low-strain amplitude, i.e., ⊿e_1=±0.003 [-], is also examined in this research.

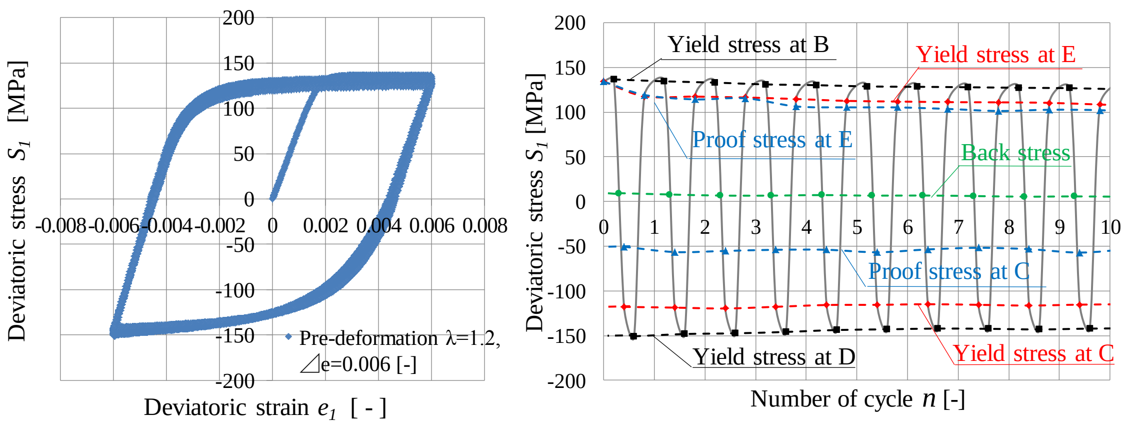
4 EXPERIMENTAL RESULTS

Figure 3, 4 and 5 show the experimental results obtained under cyclic loading tests that are conducted with the constant strain amplitude (⊿e_1=±0.006 [-]). In these figures, (a) is the deviatoric stress and deviatoric strain diagrams and (b) represents the variation in yield stress associated with an increase in the number of cycles, respectively.



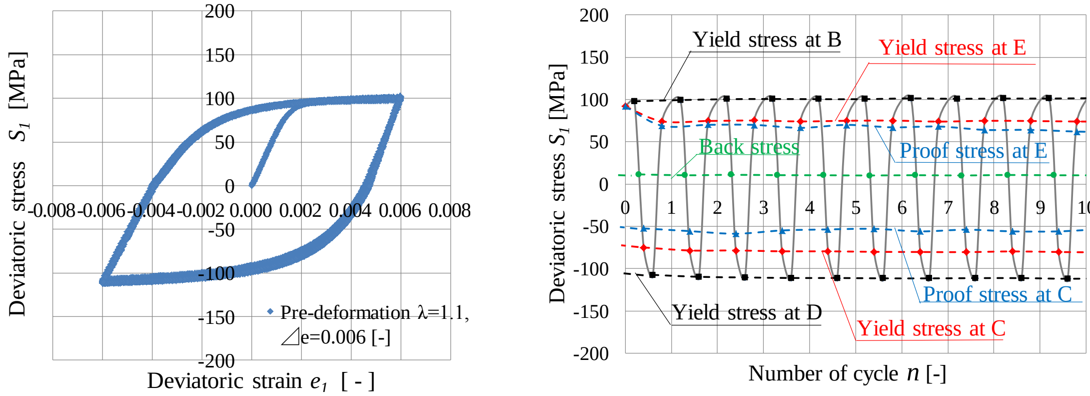
(a) Deviatoric stress and deviatoric strain. (b) Variation in yield stress with the number of cycles.

Figure 3. Variation of yield stress in cyclic loading for tension and compression (λ=1.3 , ⊿e_1 = 0.006).



(a) Deviatoric stress and deviatoric strain. (b) Variation in yield stress with the number of cycles.

Figure 4. Variation of yield stress in cyclic loading for tension and compression (λ=1.2 , ⊿e_1 = 0.006).

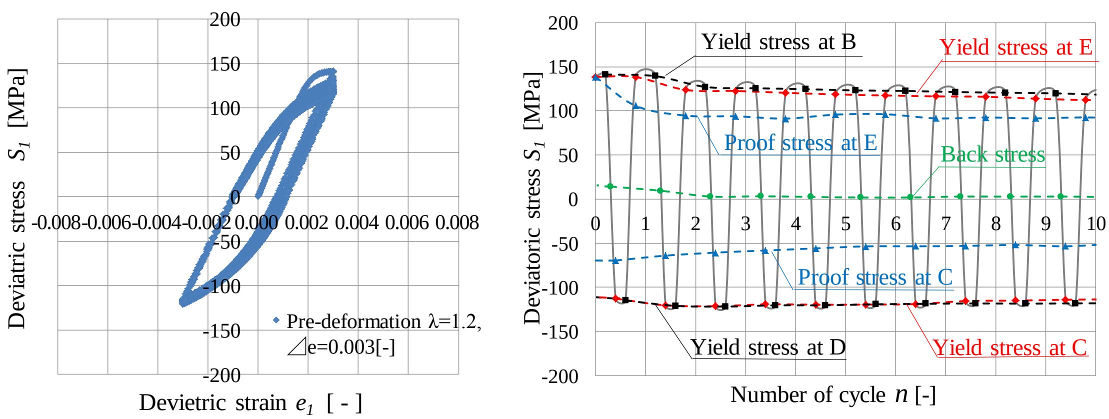


(a) Deviatoric stress and deviatoric strain. (b) Variation in yield stress with the number of cycles.

Figure 5. Variation of yield stress in cyclic loading for tension and compression (λ=1.1 , $\angle e_1$ = 0.006).

In these figures, Figure 3 shows the experimental results when the value of preliminary deformation is the largest, i.e. λ=1.3[-], among them. As obviously from this figure, it is found that the yield stress on the compression side and the tensile side, i.e., yield stress at C and D (see S_{1yC} and S_{1yD} in Figure 2) and yield stress at E and B (see S_{1yE} and S_{1yB} in Figure 2) have a decreasing tendency as the number of cycles increase (see red or black broken lines). In particular, in this condition, which is affected largely by the pre-deformation of tension, the decreasing tendency on the tension side appears remarkably compared with the decreasing tendency on the compression side. Furthermore, as for the back stress on the tension side, which is indicated by the green broken line, a decreasing tendency is observed as the number of cycle increases.

On the other hand, among these three figures, Figure 5 shows the result for the smallest pre-deformation, i.e. λ=1.1[-]. In this condition, the yield stress tends to increase slightly during the early stage in the cyclic loading due to the strain hardening. However, as the number of cycles increases, the increasing tendency and the decreasing tendency in the yield stress cancel each other. As a result, the yield stress tends to approach asymptotically to the constant value.



(a) Deviatoric stress and deviatoric strain. (b) Variation in yield stress with the number of cycles.

Figure 6. Variation of yield stress in cyclic loading for tension and compression (λ=1.2 , $\angle e_1$ = 0.003).

Moreover, Figure 4 shows the experimental result when the pre-deformation of tension is the intermediate value of these above-mentioned conditions, namely, λ=1.2[-]. On the other hand, Figure 6 shows the experimental results obtained under a small strain amplitude, i.e., $\triangle e_1 = \pm$ 0.003 [-]. Here, in order to compare the effects of strain amplitude only, this experiment is conducted under the same pre-deformation condition, i.e., λ=1.2[-]. From the comparison of Figure 4 and Figure 6, it can be seen that the decreasing tendency of yield stress in Figure 6 is more remarkable as compared with the result in Figure 4, because the increase of stress due to strain hardening is small.

On the other hands, the results of the yield stress by conventional proof stress are represented by blue broken lines in Figure 3,4,5, and 6. In all of these figures, it can be confirmed that the results by proof stress are estimated smaller compared with the yield stress estimated in this study at both the tensile side and the compression side.

5 CONCLUSIONS

In this study, in order to examine the yield behavior under cyclic loads of tension and compression after applying a large pre-deformation of uniaxial tension, the yield stress in each cycle was estimated by examining the slope of tangent in the deviatoric stress - deviatoric strain curve based on the Natural Strain theory. As the result, the following conclusions are obtained.

(1) The yield stress at the tension side and at the compression side have a decreasing tendency as the number of cycles increase. However, the decreasing tendency of these yield stresses reduces as the number of cycles increase and they approach to the steady value.

(2) In the case that the value of the pre-deformation of tension is large, the decrease tendency of the yield stress on the tension side become larger compared with the compression side that is the opposite direction to the pre-deformation.

(3) On the other hand, in the case that the value of the pre-deformation of tension is small, it could be confirmed that the yield stress slightly increase during the early stage of cycle due to strain hardening.

(4) Moreover, the yield stress estimated in this study was compared with the conventional proof stress. Consequently, the yield stress by proof stress becomes smaller compared with the yield stress estimated in this study at both the tension side and the compression side.

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