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DAMAGE MODELING AND ASSESSMENT FOR BRITTLE MATERIALS

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Observed nonlinearities in frictional and brittle solids such as concrete, rocks, ceramics, and some composites arise mainly due to the nucleation and propagation of microvoids and microcracks. Microcrack formation, propagation, and coalescence damage the material and renders it more compliant. Microdefects and cracks are also usually irreversible and cause strong anisotropy in the response to loads. This paper presents a damage mechanics model to capture material anisotropy and damage under multiaxial stress states for proportional and fatigue type loadings. The theory is cast within the generally accepted principles of thermodynamics with internal variables where the dissipation inequality is invoked to develop loading surfaces. Flow rules for the onset of inelastic deformations is provided and specific damage laws are proposed. The model extension to the cyclic and fatigue type loading is presented and numerical results are provided with comparison to the available experimental data.

Keywords: Inelasticity, Concrete, Anisotropy, Thermodynamics, Flow rule.

1 INTRODUCTION

There are generally two dominant types of irreversible changes that can take place in brittle and frictional solids such as rocks, concrete, ceramics, and some composites. First is the nucleation and propagation of microcracks and microvoids that tend to destroy material bonds and render the solid damaged. This pattern usually takes place under zero or low lateral pressures. Examples where the crack formations become the main and overriding causes of material inelasticity are uniaxial, biaxial, and triaxial extension load paths. The result is a more compliant material behavior reflected in a reduced cross-sectional area to resist applied loads and a reduced stiffness due to the destruction of material bonds. The failure mode can take the form of macrocracks after the coalescence of smaller cracks forming a discrete or a fault line. The second mode of irreversible microstructural changes that could occur is when the solid is under large confining pressure such as the case under triaxial compression. The presence of confining pressure inhibits or delays the formation of microcracks, leading to observed enhancement in strength, ductility, and material stability. Permanent deformation with negligible or small loss of stiffness is observed under this scenario. This paper presents a damage mechanics approach to the modeling of brittle solid inelasticity due to the formation of microcracks and microvoids. Permanent deformations arising from inelastic flow and void closure in the present of conforming pressure is beyond the scope of this work and can be found elsewhere.

Various fields of damage mechanics have been introduced in the literature to model the effects of cracking and damage on the mechanical properties of the materials (Saboori *et al.* 2014, Wen and Yazdani 2008, Yazdani and Karnawat 1996). To provide a rational and physically relevant formulation, the present theory is cast within the general theory of continuum thermodynamics with internal variables where the dissipation inequality is provoked to obtain a bounding surface, interpreted as a damage surface. The damage surface acts in the same way as the plasticity surfaces do in the classical theories of plasticity. For the onset of inelasticity, the state of stress must be on the damage surface as the necessary condition. The sufficiency condition then is determined from the loading path requiring additional energy to the system.

2 GENERAL THEORY

Considering a stress space formulation and considering small and isothermal deformations, the Gibbs free energy, G(σ, k) can be presented as:

$$\boldsymbol{G(\sigma, k) = \frac{1}{2}\sigma : C(k) : \sigma + \sigma : \varepsilon^{i}(k) - A^{i}(k)} \quad (\mathbf{1})$$

where, the stress tensor is given by σ, k as a scalar parameter indicating the cumulative damage, C is the current compliance of the material, ε^i represents the inelastic component of the strain, and A^i signifies the inelastic free energy of microcrack formation. The dependence of the compliance (flexibility) tensor on the state of damage allows the formulation to model the continuous degradation due to crack formations. The tensor contraction operation is noted as ":" in Eq. (1). The compliance tensor can be decomposed into initial flexibility of the undamaged solid plus the added flexibility caused by damage as $C(k) = C^0 + C^C(k)$, where C^0 represent the undamaged materials compliance and C^C is the added flexibility due to energy dissipation and cracking. With this, the total strain tensor can be decomposed as shown by Eq. (2):

$$\varepsilon = C^0 : \sigma + C^c(k) : \sigma + \varepsilon^i(k) \quad (2)$$

It is customary to distinguish between cleavage cracking mode caused by tensile stress states and those that occur under compression loads in brittle materials. The first mode is similar to the mode I cracking in fracture mechanics studies, whereas the second mode is caused by the combined action of shear sliding and crack opening. These are shown schematically in Figures 1 and 2 (Yazdani 1993). To reflect these damage modes, in an uncoupled approach, into the formulation, the added flexibility compliance is decomposed further into two parts as C^C_I and C^C_{II} such that $C^C(k) = C^C_I + C^C_{II}$. Similar to any nonlinear theory, the solution to final equilibrium state would require incremental approach. To accommodate this we assume that linear damage rule can adequately be used for the damage onset and that the resulting damage can be addressed using the damage response tensors as:

$$\dot{C}^c_I = \dot{k}R_I \qquad \dot{C}^c_{II} = \dot{k}R_{II} \quad (3)$$

where, the super-dots indicate time rate of change. Damage response tensors R_I and R_{II} are fourth-order tensors that reflect the directions of damage. If the response tensors are postulated to be isotropic, then the formulation would predict isotropic deformation. In most practical cases this is not true. Cracking is mostly preferential leading to strong anisotropic behaviour. Any comprehensive formulation should thus aim at postulating response tensors that are close to the physics of crack formation and its directionality. It is further assumed that damage is irreversible, i.e. $\dot{\boldsymbol{k}} \geq \mathbf{0}$. Using Eq. (1) through (3), and integrating, the general form of the damage surface is represented as Eq. (4):

$$\Psi(\sigma, k) = \frac{1}{2}\sigma^{+}: R_I: \sigma^{+} + \frac{1}{2}\sigma^{-}: R_{II}: \sigma^{-} + \sigma: M - \frac{1}{2}t^2(\sigma, k) \geq 0 \tag{4}$$

Here, the positive and negative cones of the stress tensors are denoted by σ^{+} and σ^{-}; respectively and where, M is the inelastic response tensor for the deformation. The scalar function, t(σ, k) is identified as the damage function and must be determined from experiments.

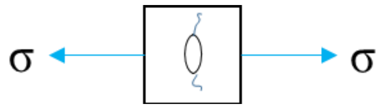

Figure 1. Microcracks nucleation pattern under uniaxial tension.

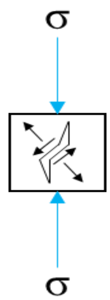

Figure 2. Microcracks nucleation pattern under uniaxial compression.

3 DAMAGE EVOLUTION AND DAMAGE FLOW RULE

For the damage to take place, the state of stress must satisfy the damage surface for which $\psi = 0$. This constitutes the necessary but not a sufficient condition. For a state where $\psi < 0$, the stress state is elastic and $\dot{k} = 0$. The condition of loading and unloading can then be formally stated below by Eq. (5):

$$\Psi = 0\,, \frac{\partial \Psi}{\partial \sigma}: \dot{\sigma} > 0 \quad \rightarrow \quad \dot{k} > 0 \tag{5}$$

$$Otherwise \quad \dot{k} = 0$$

To progress further specific forms of damage response tensors must be postulated. For isotropic processes, the R tensors should be set to be proportional to a 4^{th}-order isotropic tensor. For anisotropic formulation, we propose the following anisotropic form that could be used to capture many of the salient features of brittle solid inelasticity in multiaxial tensile stresses. Due to allocated space, only tensile stresses are considered. The proposed response tensor R_I is given as Eq. (6) (Yazdani 1993):

$$R_I = \frac{\sigma^{+} \otimes \sigma^{+}}{\sigma^{+}: \sigma^{+}} - \alpha(I - i \otimes i) \tag{6}$$

Here the second order identity tensor is given by i, and I is the 4^{th}-order identity tensor. The symbol ⊗ reflects tensor product operation. In fatigue type processes, the damage function t (σ, k) is expressed as the product of three functions given by Eq. (7) (Saboori *et al.* 2015):

$$t(\sigma, k) = L(\sigma)F(n)q(k) \tag{7}$$

Here L(σ) reflects a strength function, F(n) signifies a strength reduction function, and q(k) identifies a shape function. The concept of loading-unloading sequence and fatigue is illustrated in Figure 3 where a schematic monotonic loading and a fatigue failure state is shown. It is known that during fatigue processes, a material could fail at much lower stress level depending on the frequency of the load, level of loading, the mean stress, and the number of cycles. To correlate with the material strength under proportional loading, the shape function presented in Eq. (7) should be set to approach unity for the monotonic limit state. The inclusion of the parameter "n", representing the number of cycles, allows modeling of fatigue behavior of the material where failure occurs at levels below limit state under monotonic loading. Different forms of function F(n) have been reported in the literature. Here we propose a simple power law as given by Eq. (8):

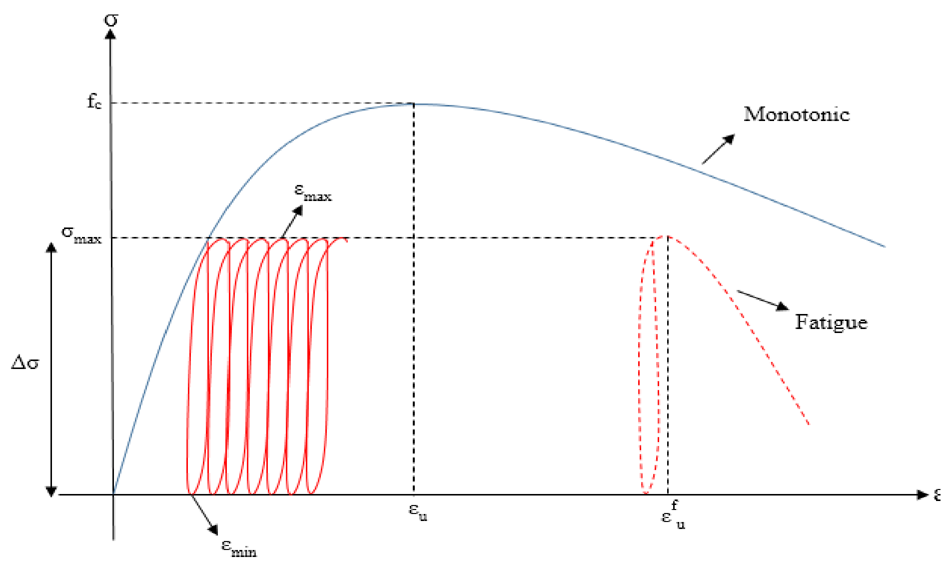

Figure 3. Schematic representation of stress-strain diagram under fatigue and monotonic loading.

$$F(n) = n^A \tag{8}$$

where, A is regarded as a material parameter. A new form of the strength function is proposed below that would provide a modeling tool for anisotropic strength reduction as Eq. (9):

$$L(\sigma) = \left[\frac{\sigma:S}{tr(\sigma)}\right]^h \tag{9}$$

where, tensor S is the preferential strength tensor, "tr" indicates the trace operator, and h is a material parameter. The Strength tensor S is expected to reflect strength anisotropy as seen in engineering materials such as composites where the strength in the direction of fibers is much higher than other directions. In concrete, due to statistically homogenous distribution of aggregates and pastes, the initial strength is usually isotropic. A possible form of the strength tensor S is suggested here as $S = Ft^1 q^1 \otimes q^1 + Ft^2 q^2 \otimes q^2 + Ft^3 q^3 \otimes q^3$, where q^i are the eigen principal vectors and Ft^i are the respective uniaxial strength in the directions of "i", with i = 1, 2, 3 (Saboori *et al.* 2015).

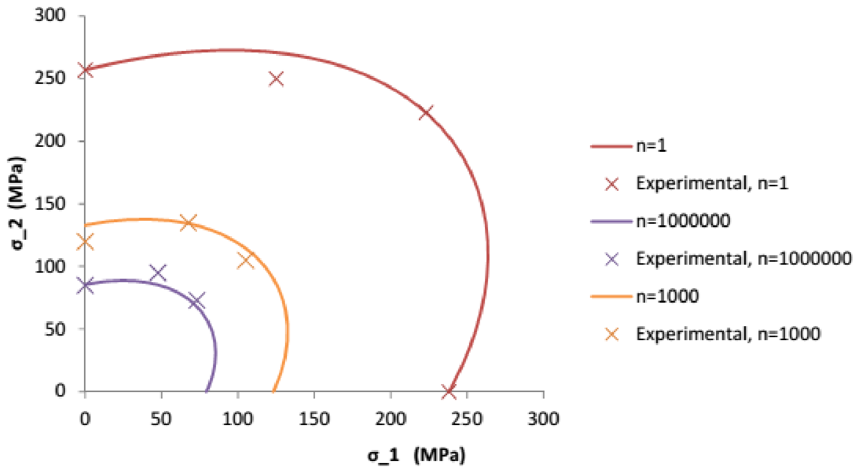

Figure 4. Failure surfaces for monotonic and fatigue loading in biaxial stress space. Data by Smith and Pascoe (1989).

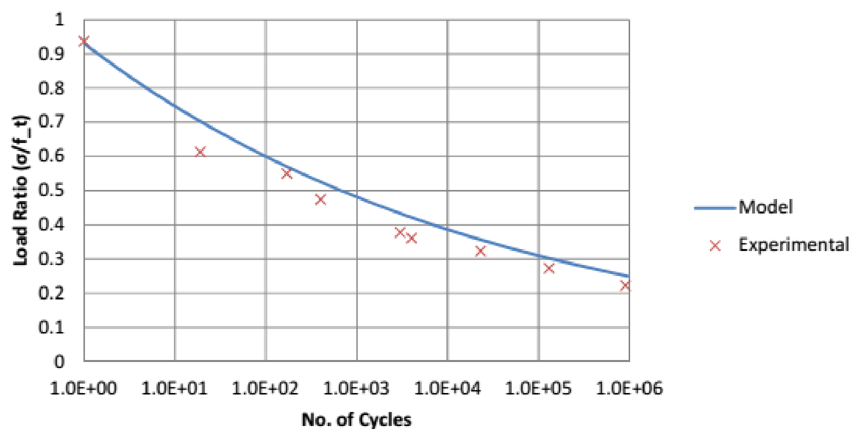

Figure 5. S-N curve for woven fabric composite under biaxial fatigue loading with stress ratio of 1. Data by Smith and Pascoe (1989).

The effect of Eq. (8) on the loading or damage surface is shown in Figure 4 where the limit surface is shown collapsing inward as the number of cycle increases. This is in agreement with experimental records. The experimental results of Smith and Pascoe (1989) are compared with the theoretical results utilizing the simple form of Eq. (8) for different values of n. Uniaxial strength of 250 MPA, A = 0.94, and h = 1 were used. In spite of the simple form of the Equation that are used, the salient features are nicely captured. The S-N curve for equal biaxial tensile stress path for woven composites is shown in Figure 5 and is compared with the experimental work of Smith and Pascoe (1989). It can also be seen that the model captures the salient feature of the behavior with reasonable agreement.

4 CONCLUSION

Damage Mechanics is relatively a new field to structural engineers. However, as is shown in this paper, the nonlinear behaviour of brittle like materials can be modelled utilizing the first principles of mechanics and thermodynamics and therefore eliminating curve fitting approach for the constitutive modelling. Assuming small deformations, that is appropriate for brittle solids, strain decomposition into elastic, damage, and inelastic strains is carried out. The dependence of the compliance tensor on accumulated damage allows the theory to model the nonlinear and anisotropic response of the material. The general form of the damage surface for monotonic loading is provided. To extend the model for fatigue like loading, the monotonic damage function was altered into the product of three functions that included a strength reduction function, a softening function, and a shape function. A power law for the softening function was proposed and a new form of anisotropic strength function was given. The model prediction was compared with experimental results of woven composites with reasonable arrangement.

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