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BRITTLE/DUCTILE ASSESSMENT OF HYBRID REINFORCED CONCRETE BEAMS CONTAINING STEEL REBAR AND FIBERS

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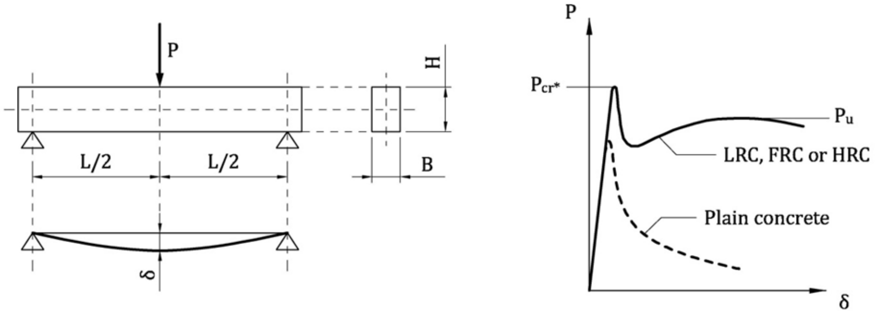
Many structures, such as precast and cast-in-situ tunnel linings, are nowadays realized with Hybrid Reinforced Concrete (HRC), where a combination of continuous steel rebar and discrete fibers is used to reinforce the cementitious matrix. Hence, the definition of a minimum amount of hybrid reinforcement (i.e., rebar and fibers), which prevents the brittle failure, is of practical interest. For predicting the brittle/ductile response of HRC beams in bending, a theoretical model is introduced and presented in this paper. It is based on the flexural response of both Lightly Reinforced Concrete (LRC) and Fiber-Reinforced Concrete (FRC) beams, separately analyzed. The numerical results of the model, and some experimental data as well, show that the minimum reinforcement of HRC beams can be determined with a new design procedure. It requires the definition of the ductility index (*DI*), which is proportional to the difference between ultimate and effective cracking load. As *DI* linearly increases with the amount of rebar and fibers, the minimum reinforcement in HRC members can be found when *DI* is equal to zero. In addition, the minimum hybrid reinforcement can be defined with a suitable linear combination of the minimum area of rebar and the minimum fiber volume fraction, related to LRC and FRC beams, respectively.

Keywords: Bending moment, Minimum reinforcement, Ductility index (*DI*).

1 INTRODUCTION

An increasing interest can be noticed on Hybrid Reinforced Concrete (HRC), where a combination of continuous rebar and discrete fibers is used to reinforce the cementitious matrix (Facconi *et al*. 2016, Liao *et al*. 2016). Specifically, HRC is often adopted in massive members, as precast and cast-in-situ tunnel linings, with the aim of reducing the amount of rebar (Chiaia *et al*. 2009, de la Fuente *et al*. 2012). In these concrete members, mainly subject to normal force and bending moment, the design actions can be smaller than the cracking load of the cementitious matrix (Chiaia *et al*. 2009, de la Fuente *et al*. 2012). Accordingly, if the bearing capacity in the post-cracking stage (i.e., the ultimate load P_u) is lower than the effective cracking load P_{cr*} (Maldague 1965), the brittle failure occurs. On the other hand, to attain the ductile failure in a concrete member in bending (Figure 1a), the following condition in Eq. (1) should be imposed to the load *P vs*. midspan deflection δ curve (Figure 1b):

$$P_u \geq P_{cr*} \tag{1}$$



(a) (b)

Figure 1. Flexural behavior of concrete beam: (a) three-point bending test; (b) load *vs*. deflection curves.

Traditionally, in Lightly Reinforced Concrete (LRC) beams, the brittle failure is avoided through the minimum reinforcement area $A_{s,min}$, defined as the amount of steel rebar which guarantees the condition $P_u = P_{cr*}$ (Bosco *et a*l. 1990, Fantilli *et al*. 2016a). The minimum reinforcement of LRC members should also ensure crack control in the serviceability stage (Levi 1985), hence building code recommendations are aimed to fulfill both the requirements (ACI 2014, CEN 2004, *fib* 2012). Similarly, in Fiber-Reinforced Concrete (FRC) beams, the minimum fiber volume fraction $V_{f,min}$ can be assumed to have the same mechanical function of $A_{s,min}$ in LRC beams (Fantilli *et al*. 2016b, Naaman 2003). Thus, by increasing the fiber content, the transition between the brittle response (called deflection-softening) and the ductile response (called deflection-hardening) occurs (Naaman 2003).

In the case of massive HRC elements, if the computation of $A_{s,min}$ is performed without taking into account the effect of fibers, a large amount of rebar would be required (Chiaia *et al*. 2009). Hence, the definition of a new criteria for evaluating the minimum amount of hybrid reinforcement, considering both rebar and fibers, is of practical interest (Gorino *et al*. 2016). For this reason, the design-by-testing procedure recently proposed by Fantilli *et al.* (2016a, 2016b, and 2016c) for LRC and FRC beams is extended herein to HRC members.

2 NUMERICAL INVESTIGATIONS

To assess the brittle/ductile behavior of HRC beams, the results of a new theoretical model are analyzed in the following sections.

2.1 Theoretical Model

To predict the flexural behavior of HRC members, the fiber-reinforcement is modelled with an ideal tie, composed by a straight fiber and the surrounding cementitious matrix, having a single orthogonal crack in the midsection, as in Fantilli *et al*. (2016b). The pull-out mechanism of this element, evaluated with the bond-slip between fiber and concrete and the fracture mechanics of concrete in tension (Bažant and Cedolin 1991), provides the cohesive stress *vs*. crack width relationship of the cracked FRC matrix. This relationship is used to determine the response of an HRC beam, by adopting a multi-scale approach. Specifically, as already done by Barros *et al.* (2015), the beam is studied as a LRC member (Fantilli *et al.* 2016a), with a cementitious matrix defined by the previous stress *vs*. crack width relationship and a further bond-slip mechanism at the interface between rebar and FRC in tension. Moreover, the stress *vs*. strain behavior of uncracked concrete is modelled with the ascending branch of the Sargin's parabola in compression and the linear elastic relationship in tension, whereas an elastic-perfectly plastic law is adopted for steel rebar (*fib* 2012). The softening behavior of concrete in compression is neglected in the proposed model, because the minimum reinforcement, corresponding to the brittle/ductile transition in tension, cannot produce the crushing failure of the compressed zone.

2.2 Numerical Investigations

The proposed model is adopted to describe the $M - \bar{w}$ curves of 108 ideal HRC beams in three-point bending. They are divided into 36 groups of three beams, having the same geometrical and material properties, but with different amounts of rebar A_s and fibers V_f. Two beam depths H = 200 and 400 mm are assumed, whereas the width/depth ratio B/H = 1/2 and the depth/span ratio H/L = 1/6 are constant. Three compressive strengths of concrete are considered (i.e., f_c = 30, 45, and 60 MPa), whereas the same properties of steel rebar are assumed in all the groups (i.e., yielding strength f_y = 450 MPa, and Young modulus E_s = 210 GPa). Moreover, the steel fibers (with fixed length L_f = 60 mm, tensile strength f_u = 1,000 MPa, and Young modulus E_f = 210 GPa) have the aspect ratios L_f / ϕ_f = 40, 60, and 80. These geometrical and material properties are equal to those of the LRC and FRC beams previously investigated by Fantilli *et al.* (2016a) and (2016b). Hence, for each group of HRC members, $A_{s,\min}$ and $V_{f,\min}$ are already known.

As an example, Figure 2a reports the $M - \bar{w}$ curves referred to the three beams of the same group. Two stationary points, concerning the effective cracking moment (M_{cr*}) and the ultimate bending moment (M_u), are evident in each curve. Specifically, curve HRC_1 shows a brittle flexural response, because $M_u < M_{cr*}$, whereas the reinforcement of the beam HRC_2 is near to the minimum value as $M_u \cong M_{cr*}$. Finally, the $M - \bar{w}$ curve of the beam HRC_3 describes a typical ductile behavior with $M_u > M_{cr*}$.

2.3 Definition of the Ductility Index and Reinforcement Ratio

As for LRC and FRC beams (Fantilli *et al.* 2016a, 2016b, and 2016c), the brittle/ductile behavior of HRC beams can be evaluated by means of the following ductility index (*DI*):

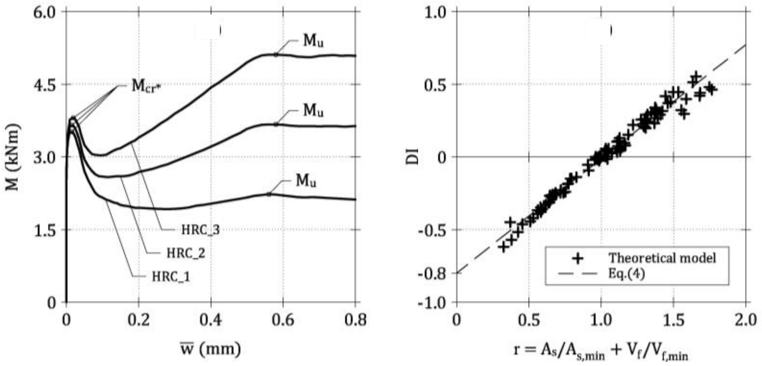
$$DI = \frac{M_u - M_{cr*}}{M_{cr*}} = \frac{P_u - P_{cr*}}{P_{cr*}} \tag{2}$$

When *DI* assumes positive values, beams show a ductile response, whereas under-reinforced concrete members exhibit $DI < 0$. Hence, the minimum amount of hybrid reinforcement (i.e., the brittle/ductile transition) can be identified by $DI = 0$.

Since both M_u (or P_u) and M_{cr*} (or P_{cr*}) depend on the amount of reinforcement, *DI* should be a function of A_s and/or V_f. As demonstrated by Falkner and Henke (2005), the effects of rebar and fibers can be superposed at ultimate limit state (i.e., in M_u). Conversely, M_{cr*} seems to be marginally affected by the reinforcement. Accordingly, the following reinforcement ratio *r* can be introduced as the parameter governing the brittle/ductile transition (Gorino *et al.* 2016):

$$r = \frac{A_s}{A_{s,\min}} + \frac{V_f}{V_{f,\min}} \tag{3}$$

Hence, the values of A_s and V_f normalized with respect to their minimum amounts $A_{s,\min}$ and $V_{f,\min}$, coming from the corresponding LRC and FRC beams, are linearly combined. Since $A_{s,\min}$ and $V_{f,\min}$ are defined for each specific type of beam, all the related parameters (e.g., concrete strength, rebar and fiber properties, beam size, etc.) are taken into account, even if the non-dimensional variable *r* appears to be independent on any geometrical and mechanical property.



(a) (b)

Figure 2. Numerical investigations: (a) M - $\overline{w}$ curves of the beams in a group, (b) general results.

If M_u is assumed to linearly increase with r, also DI should have the same dependence on the reinforcement ratio (assuming $M_{\mathrm{cr}*}$ as constant). As in the case of LRC and FRC beams (Fantilli *et al.* 2016a, 2016b, and 2016c), the existence of a unique linear function DI - r can be argued. This line should pass through the point corresponding to a beam reinforced with the minimum hybrid reinforcement (i.e., $r = 1$ and $DI = 0$), and is characterized by the slope ζ:

$$DI = \zeta \cdot (r - 1) \tag{4}$$

By reporting in the general diagram of Figure 2b all the (DI, r) couples computed for the 108 ideal HRC beams, a linear dependence appears. The slope ζ of the DI - r line, obtained with the least square approximation of the numerical results, is equal to 0.8.

It is worth noting that this value is comprised between the unit slope, typical of LRC beams, and the value ζ = 0.7 of FRC members (Fantilli *et al.* 2016a, 2016b, and 2016c). Anyway, by means of Eq. (4), the ductility index can be generally applied for all the ideal HRC beams (Figure 2b).

3 EXPERIMENTAL INVESTIGATIONS

To corroborate the accuracy of Eq. (4), the results of an experimental campaign performed by Gorino *et al.* (2016) are considered herein. Specifically, twelve un-notched concrete beams, having a length of 700 mm and a square cross-section of 150 × 150 mm, reinforced by several combinations of steel rebar and two kinds of steel fibers, have been tested. For each HRC beam, the corresponding LRC and FRC members are also analyzed.

According to Fantilli *et al.* (2016c), the minimum reinforcement of both LRC and FRC beams can be determined by applying a unified design-by-testing approach to some of the tested beams. In particular, the following formulae (Eqs. (5a) and (5b)) can be used:

$$A_{\mathrm{s,min}} = \frac{\zeta \cdot A_\mathrm{s}}{DI + \zeta} \tag{5a}$$

$$V_{\mathrm{f,min}} = \frac{\zeta \cdot V_\mathrm{f}}{DI + \zeta} \tag{5b}$$

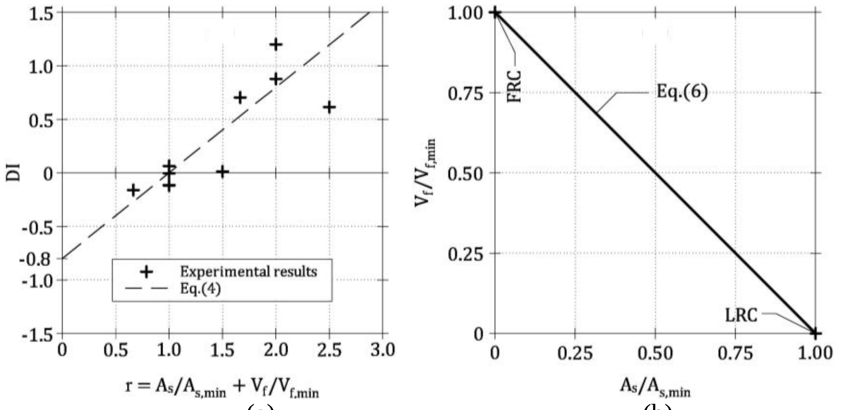
where A_s and V_f are the amounts of rebar and fibers in the tested beam, and $\zeta = 0.8$ is assumed for both LRC and FRC beams. Then, from the values of $A_{s,min}$ and $V_{f,min}$, determined for the LRC and FRC beams associated to an HRC member, the evaluation of r is possible with Eq. (3).

The experimental values of DI obtained from the tests of Gorino *et al.* (2016) are plotted, as a function of r, in the diagram depicted in Figure 3a. In the same diagram, the comparison with Eq. (4) is also performed. The proposed linear relationship shows an agreement with the experimental points, and the brittle/ductile transition (i.e., $DI = 0$ in Eq. (2)) is occurs for $r \cong 1$.

Accordingly, since the theoretical and experimental results of Figure 3a provides the minimum hybrid reinforcement by imposing $r = 1$ into Eq. (3), the minimum amount of rebar and fibers used to reinforce HRC members is given by the linear combination of $A_{s,min}$ and $V_{f,min}$, as represented in Figure 3b (Gorino *et al*. 2016) and Eq. 6:

$$\frac{A_s}{A_{s,min}} + \frac{V_f}{V_{f,min}} = 1 \qquad (6)$$

This finding matches up with the results of other models (Chiaia *et al.* 2009, Liao *et al.* 2016, Mobasher *et al.* 2015) and with the recent recommendations of Model Code 2010 (*fib* 2012).



(a) (b)

Figure 3. Experimental investigations: (a) general results, (b) minimum reinforcement of HRC beams.

4 CONCLUSIONS

According to the theoretical and experimental investigations previously described, the following conclusions can be drawn:

(i) The reinforcement of HRC beams can be quantified by means of the reinforcement ratio r, i.e., a linear combination of the area of rebar A_s and of the fiber content V_f, normalized with respect to their minimum amounts $A_{s,min}$ and $V_{f,min}$, respectively (Eq. (3)).

(ii) The brittle/ductile flexural response of such beams can be described by the ductility index DI (Eq. (2)), which is proportional to the difference between the ultimate load, P_u, and the effective cracking load, P_{cr*}. Both theoretical and experimental results suggest the existence of a linear relationship between DI and r (Eq. (4)).

(iii) The minimum hybrid reinforcement which satisfy $DI = 0$ is defined by any linear combination of $A_{s,min}$ and $V_{f,min}$ (Eq. (6)). Thus, the minimum reinforcement traditionally required by building codes for LRC beams can be reduced.

Further theoretical and experimental studies should be developed to extend the present approach for evaluating the brittle/ductile flexural response to statically indeterminate structures (e.g., slabs on ground, frames, etc.).

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