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REINFORCED CONCRETE CRACK MODEL BASED ON STIFFNESS ANALYSIS OF TENSION MEMBERS

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The average spacing of primary cracks in a reinforced concrete (RC) member greatly influences its in-service behavior, especially with regard to stiffness and average crack width. Accurate predictions of the average crack spacing are therefore crucial for satisfying serviceability requirements in RC structures. This is particularly the case when relying on analytical models that treat cracks discretely rather than in a smeared fashion. Popular code-based models for primary crack spacing are often wildly inaccurate and may lead to poor predictions of in-service behavior. In this paper, the problem or primary crack formation is approached from a stiffness perspective. The proposed model is based on the results of several experimental tension stiffening studies in the literature, as well as a previous numerical study dealing with the effect on stiffness of non-plane deformation in the neighborhood of primary cracks. The proposed model is compared to some popular code-based models and is shown to better predict average crack spacing for a wide variety of beams, slabs, and tension members.

Keywords: Tension stiffening, Bond, Serviceability, Finite element analysis.

1 INTRODUCTION

In the plane of a primary crack, any tension force is carried by the reinforcement, which develops a strain of $\varepsilon_{s2} = N/(E_s A_s)$, where N is the tension force and E_s and A_s are the elastic modulus and cross-sectional area of the reinforcement bar, respectively. Under the action of bond, some portion of the total tension force is gradually transferred to the surrounding concrete, and at a certain distance away from the plane of the crack (called the transmission length L_t), a condition of strain compatibility is achieved between the reinforcement and the concrete. At this point the strain in both materials is given by $\varepsilon_{s1} = \varepsilon_{c1} = N/(E_s A_s + E_c A_c)$, where E_c and A_c are the elastic modulus and cross-sectional area of the concrete, respectively. The distribution of bond stress τ_b over the transmission length L_t may be taken to be constant under service loads, leading to a linear variation of strain in both materials. This assumption is broadly supported by various experimental studies in the literature (Scott and Gill 1987).

Considering a segment of the tension zone of a reinforced concrete member located between adjacent primary cracks of spacing s, there are two possible scenarios that must be considered. If $s/2 \geq L_t$ (Fig. 1a), a condition of strain compatibility occurs over at least some portion of the segment and it may be shown that the average reinforcement strain ε_{sm} in the segment is given by:

$$\varepsilon_{sm} = \varepsilon_{s2} \frac{n\rho + L_t/s}{1 + n\rho} \tag{1}$$

where $n = E_s/E_c$ is the modular ratio and $\rho = A_s/A_c$ is the reinforcement ratio. On the other hand, if $s/2 < L_t$ (Fig. 1b), the average reinforcement strain in the segment is given by:

$$\varepsilon_{sm} = \varepsilon_{s2} \frac{1 + n\rho - s/(4L_t)}{1 + n\rho} \tag{2}$$

Considering Eq. (1) and (2), it is possible to determine the value of L_t for any RC tension member based on the known locations of its primary cracks, the loads at which they form, and the overall load-deformation response of the member.

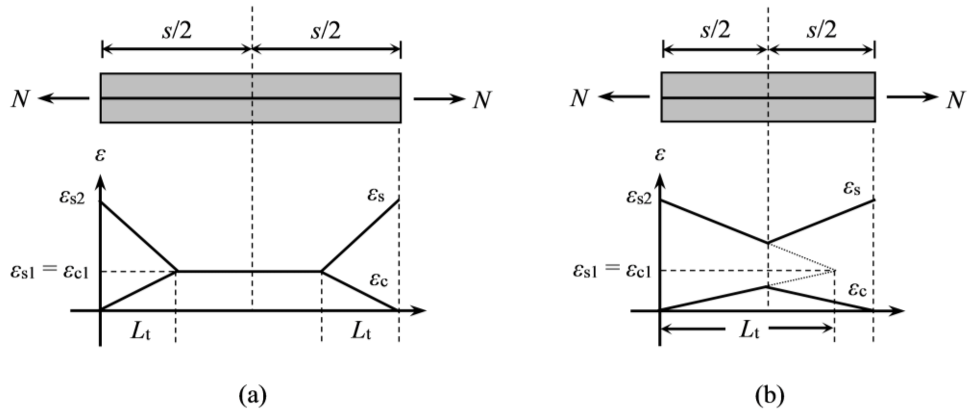

Figure 1. Reinforcement and concrete strain in (a) a long segment and (b) a short segment.

2 CHARACTERIZING TRANSMISSION LENGTH

2.1 Basic Transmission Length

Before the formation of primary cracks, the cross-sections of a RC member remain plane under bending and axial loads, provided that the member is free of any abrupt changes in loading and geometry. After cracking, however, there is a redistribution of internal actions in the member and the highly localized bond forces acting along the reinforcement-concrete interface induce a warping effect in the cross-sections in the neighborhood of the primary cracks.

A study by Murray *et al.* (2016b) highlighted the significant influence of this non-plane deformation on the stiffness of RC tension members. Using a finite element approach, the authors introduced an *effective* reinforcement ratio ρ_{ef}, which describes the axial rigidity of a RC tension member for the special case of *perfect bond* (i.e. zero slip along the reinforcement-concrete interface) and takes into account the effect of non-plane deformation near the primary cracks.

Based on the assumption of constant bond τ_b and considering the effective reinforcement ratio ρ_{ef} defined by Murray *et al.* (2016b), it is possible to determine the transmission length associated with the special case of perfect bond, which represents with the greatest tension stiffening effect. For the case of perfect bond, this transmission length is called the *basic* transmission length and is found to be:

$$L_{tb}/d_b = (1.691 + 0.409n) - (1.135 + 1.185n)\sqrt{\rho} \tag{3}$$

where d_b is the reinforcement bar diameter.

2.2 Evaluating L_t from Tension Stiffening Experiments

In Section 2.1, the transmission length was defined for the special case of perfect bond, which is assured during the earliest stages of initial loading by chemical and micromechanical bonds at the interface between the reinforcement and the concrete. Murray *et al.* (2016b) showed that very soon after initial loading, the condition of perfect bond is violated as slip is facilitated by the formation of internal cracks, splitting cracks, and the crushing of concrete in front of the reinforcement lugs. This results in a decay of the tension stiffening effect with increasing load.

One way of modeling this behavior is to introduce a scalar damage parameter that modifies L_t according to the degree of damage at the reinforcement-concrete interface. Thus, where damage occurs, the transmission length L_t may be expressed more generally as:

$$L_t = L_{tb} / (1 - \zeta) \tag{4}$$

where ζ ($0 \le \zeta \le 1$) is a scalar damage parameter. The case of $\zeta = 0$ represents the condition of perfect bond, for which the maximum tension stiffening effect is achieved. For $\zeta = 1$, damage to the reinforcement-concrete interface is so extensive that the tension stiffening effect is exhausted.

To characterize the evolution of ζ, twenty-one tension-stiffening tests from five separate studies in the literature have been considered. For each of the tests, L_t was determined at several load levels according to Eqs. (1) and (2). Where possible, the precise positions of primary cracks at each load level were taken into account in the analysis to provide the most accurate estimate of L_t; however, in some cases the reported average crack spacing was simply used. It is then straightforward to determine corresponding values of ζ from Eq. (4) for each of the tests.

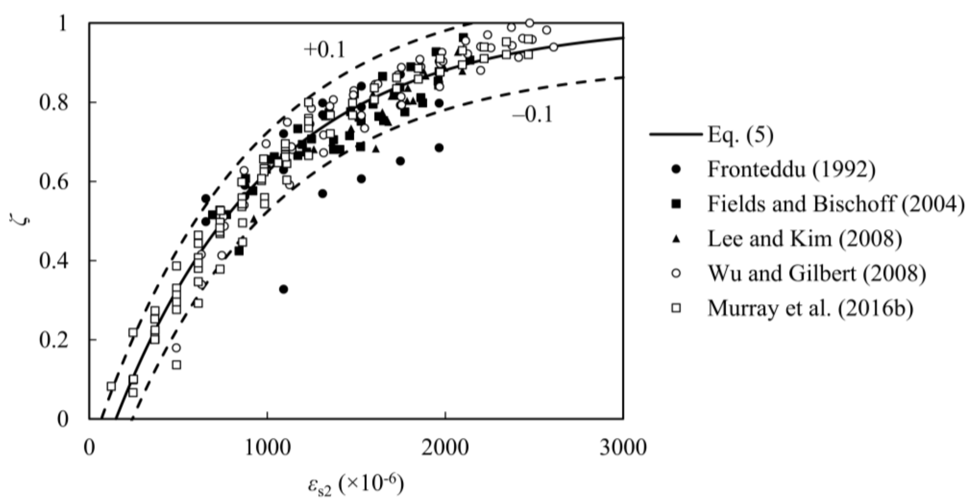

Figure 2. Evolution of scalar damage parameter ζ with increasing load.

The evolution of ζ with increasing load (characterized by ε_{s2}) is shown in Fig. 2. For ε_{s2} less than about 150×10^{-6}, no increase in the transmission length compared to the basic transmission length is detected. This is consistent with findings by Murray *et al.* (2016b), in which the onset of interface damage was shown to occur almost immediately after the first application of load. Thereafter, damage increases rapidly and is seen to gradually approach a value of $\zeta = 1$ near the yield point of the reinforcement. A curve of best fit is given in Eq. (5) for all $\varepsilon_{s2} > 150\times10^{-6}$.

$$\zeta = 1 - \exp\left[-1150\left(\varepsilon_{s2} - 150 \times 10^{-6}\right)\right] \tag{5}$$

Despite these twenty-one tests representing a reasonably large range of reinforcement ratios (ρ = 0.0099–0.0309), bar diameters (d_b = 12–19.5 mm) and concrete compressive strengths (f_{cm} = 21.6–81.0 MPa), there is little apparent influence of any of these parameters on ζ. A similar finding was made by Murray *et al.* (2016b), although the damage parameter in that study had a slightly different definition. For the tests considered in this paper, 92 percent of all data points fall within +/– 0.1 of Eq. (5).

3 DETERMINING AVERAGE CRACK SPACING

3.1 Relationship Between s_m and L_t

Primary cracks form when the concrete stress at a particular cross-section reaches the tensile strength of the concrete (i.e. $\sigma_c = f_t$). New primary cracks are therefore unlikely to occur within a distance of L_t from any existing primary crack, since the concrete stress and strain in these locations is relatively small (Fig. 1a). This behavior leads to the result that the final average crack spacing s_m must be greater than L_t and less than $2L_t$. This is because for any pair of primary cracks spaced at a distance greater than $2L_t$, a new crack can always form between them.

In the literature, the ratio s_m/L_t is often taken to be 4/3, though little justification is ever provided for this assumption. The relationship between s_m and L_t is governed by what is essentially a problem of random space-filling in one dimension. Rényi (1958) solved an analogous problem concerning cars of unit length parking in a random fashion along a street, and proved that the average density of cars should approach a value of $m \approx 0.7476$ for an infinitely long street, where m is called Rényi's parking constant. As it relates to cracks in reinforced concrete, the ratio of s_m/L_t may be shown to be $1/m \approx 1.3376$ for an infinitely long member.

For members of finite length, Rényi (1958) suggests an approximation of:

$$s_m/L_t \approx \frac{L/L_t}{m(L/L_t) + m - 1} \tag{6}$$

where L is the total length of the member within which primary cracks are likely to form, and L_t is the transmission length at the time of cracking. Values of L/L_t fall within the range of about 10–20 for the RC members analyzed in Section 4, corresponding to a ratio of about s_m/L_t = 1.37. However, L/L_t may be a small as 5 for short tension members tested in the laboratory (s_m/L_t = 1.43) and as large as about 100 (s_m/L_t = 1.34) for long structures such as pavements.

The application of Rényi's proof to the problem of cracking in reinforced concrete has (to the best of the authors' knowledge) not before been identified. The ratios given by Eq. (6) have been confirmed by the authors following several hundred thousand computer simulations for various L/L_t. Incidentally, these simulations also confirm predictions of the statistical variance of the crack spacing by Dvoretzky and Robbins (1964) and Mannion (1964).

3.2 Expressions for s_m

At the time of cracking, the strain in the bare reinforcement $\varepsilon_{s2,cr}$ may be shown to be:

$$\varepsilon_{s2,cr} = \frac{f_t}{E_c}\left(1 + \frac{1}{n\rho}\right) \tag{7}$$

Then, combining Eq. (4) and (5), and assuming s_m/L_t = 1.37, an expression for s_m is found to be:

$$s_m = 1.15 \exp\left[1150 \frac{f_t}{E_c}\left(1 + \frac{1}{n\rho}\right)\right] L_{tb} \tag{8}$$

where f_t is the effective tensile strength of concrete. In the short term, provided that little shrinkage occurs prior to first loading, the term f_t/E_c may be taken as a constant value of $f_t/E_c \approx 100\times10^{-6}$, regardless of the concrete strength grade; in the long term, or where significant shrinkage occurs prior to first loading, better results are obtained by using $f_t/E_c \approx 60\times10^{-6}$.

An alternative approach is to consider the smallest possible crack spacing that will allow an additional crack to form at the center of a short segment. Based on the strain variation in Fig. 1b, an expression is determined for the concrete stress mid-way between two adjacent primary cracks. Then, considering the expressions for L_t and ζ, and again assuming that s_m/L_t = 1.37, a maximum value for the concrete stress is found by differentiating with respect to ε_{s2}. When the concrete stress is set equal to the tensile strength of the concrete, the corresponding crack spacing s must be equal to $s_{max} = 2L_t$. Thus the final average crack spacing is found to be:

$$s_m = 3600 \frac{f_t}{E_c}\left(1 + \frac{1}{n\rho}\right) L_{tb} \tag{9}$$

4 VALIDATION OF MODEL

The proposed model is validated by comparing its predictions of s_m to 83 RC members tested in the literature. The reinforcement ratio ρ used in Eqs. (8) and (9) is based on a similar effective area of concrete to that suggested by BSI Eurocode 2 (1992, 2004) and the CEB-FIP 1990 Model Code (CEB 1993). That is, the height of the effective area of concrete surrounding the reinforcement is taken to be $2.5(h - d)$. However, for widely spaced bars (such as in slabs), an additional restriction is placed on the width of the effective area of concrete, taken in this paper to be the lesser of the bar spacing s_b and $15d_b$.

Table 1 compares the average ratios of the predicted average crack spacing to the measured average crack spacing for the proposed model and three popular code-based models.

Table 1. Comparison of proposed model to three popular code-based models.

Study	**Specimen Types**	**Number of Specimens**	**Eq. (8)**	**Eq. (9)**	**EC2 (1992)**	**EC2 (2004)**	**MC90 (1993)**
Rizkalla *et al.* (1983)	UT	34	1.07	1.05	1.16	1.20	0.62
Frosch *et al.* (2003)	S	10	1.02	0.94	0.65	0.87	0.79
Gilbert and Nejadi (2004)	B, S	12	0.98	0.97	0.90	1.05	0.92
Dawood and Marzouk (2008a,b)	UT, BT	6	1.03	1.02	1.36	2.70	0.95
Castel *et al.* (2014)	B	5	1.00	0.99	0.73	0.94	0.84
Murray *et al.* (2016a)	B	8	0.92	0.90	0.67	0.88	0.75
Murray *et al.* (Unpublished)	B	8	0.94	0.90	0.63	0.89	0.74
		Average	1.02	0.99	0.95	1.17	0.75
		COV	18%	19%	32%	42%	24%

B – beams; S – slabs; UT – uniaxial tension members; BT – biaxial tension members

5 CONCLUSIONS

The proposed model is able to predict final average crack spacing s_m accurately while being comparably simple to the popular code-based models considered here. The two alternative expressions for the estimation of s_m in Eqs. (8) and (9) appear to yield almost identical results.

Compared to the three code-based models of primary crack spacing, the proposed model is considerably more accurate and consistent in its predictions. The proposed model will allow better predictions of in-service behavior of RC structures to be made.

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