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A RATIONAL METHOD FOR COMPUTATION OF CABLE STRUCTURES

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Cable structures are widely used in practical construction due to its advantages of light weight, high strength which allow to build large span structures with nice view. The classical theory cable formulated from force balance equation of single cable, that is nonlinear; therefore, to determine displacement, deformation and tension forces of the cable we need to provide cable dip or horizontal tension force and use iteration calculation. This paper presents a new method for computation of flexible cable subjected to different loading pattern including concentrated force, distributed forces, pretension force, temperature variation. By application of the Gaussian Extreme Principle method, which developed by Prof. Drs. Ha Huy Cuong, to formulate and solve nonlinear equation system of cable structures, which ensured forces balancing as well as continuity of displacements and deformations of cable structures. This method allows for simultaneous determination of displacement, deformation and tension forces of cable structure without any other additional hypothesis, which is different from present cable theory. Numerical examples with simple, flexible cables subjected to different loadings have indicated the simplicity, accuracy and stability of the proposed method.

Keywords: Nonlinear analysis, Gaussian extreme principle, Virtual displacement, Large displacement.

1 INTRODUCTION

Cables are widely used as structural components in construction of suspension bridges, cable-stayed bridges, suspension roofs, etc. Since the highly nonlinear behavior exhibits in cables due to flexibility and large deflection, the analysis of structures consisting partly or entirely of cable components is complex and has attracted numerous researchers for centuries (Pugsley 1957, Irvine 1992). Modelling of cable element is the key aspect in every computational approach for analysis of structures consist of cable components. There are different approaches for modelling of cable structures, which are published in literatures. In general, these approaches can be divided into two types: (1) the approaches based on the classical theory (inextensible cable) and (2) the approaches based on elastic theory (extensible cable).

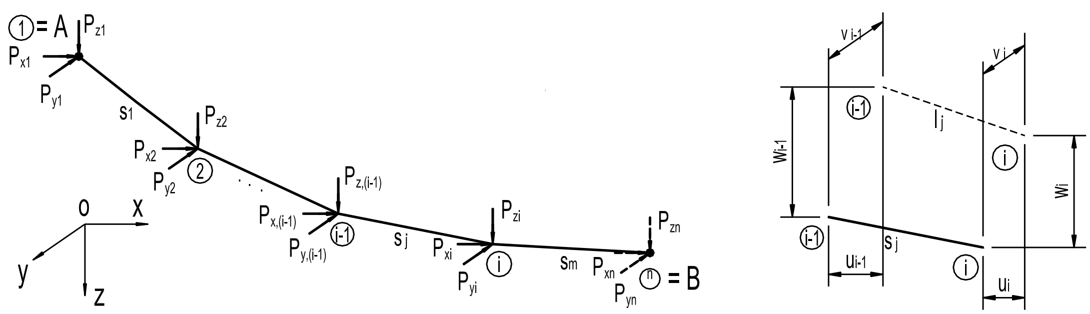
The first approaches used the classical theory of flexible cable which form the deflection shape of catenary or parabola when cable subjected to uniformly distributed load along cable length or over the span. This theory formulated from force-balanced equation of single cable, that is nonlinear; therefore, to determine displacement, deformation and tension forces of the cable we need to provide cable dip or horizontal tension force and use iteration calculation (Pugsley 1957). These approaches do not consider the sag effect due to elongation of cable under self-weight, to account for the sag effect, the elastic modulus is modified by the equivalent modulus proposed by Ernst (1965) or equivalent cross-sectional area (Petropavlovsky 1985).

The second approaches including several methods developed by different researchers using elastic theory to formulate analytical or numerical expressions of elastic cable, which accounted for sag effect of cable and realistic behavior of cable without using equivalent elastic modulus. Discussion on these approaches can be found in Irvine (1992), Andreu *et al*. (2006), and Thai (2011).

This paper presents a new method for computation of flexible cable subjected to different loading pattern including concentrated force, distributed forces, pretension force, temperature variation. This method allows for simultaneous determination of displacement, deformation and tension forces of cable structure without any other additional hypothesis, which is different from present cable theory.

2 FORMULATION FOR CABLE CALCULATION

Consider a single cable member in any cable structures, in practice, static loading pattern for cables in these types of structures can be one or combined of the following: self-weight load, concentrated loads, prestressed forces and temperature variation. In calculation, one can approximately divide cable into successive tension-only truss segments which connected by frictionless pins as shown in Figure 1; distributed self-weight load is converted to concentrated load at connected points of segments using undeformed cable length.



a) Single cable and equivalent segmented.

b) Movement of cable element.

Figure 1. Modelling of single inclined cable.

In the analysis, we accept the following conditions of the cable: (1) cable is completely flexible with uniform cross-sectional area, no bending effect occurred in the cable; (2) cross-sectional area of cable remains constant during deformation; (3) during deformation stresses only develop in direction normal to cross-sectional area and uniformly distributed over this area.

According to Gaussian Extreme Principle method (Cuong 2005), constrain functional of a deformable mechanical structure is formulated to all components of external and internal forces, displacement and strain of structure. For cable structure shown in Figure 1, constrain functional can be formulated as:

$$Z = \sum_{j=1}^{m} \int_{0}^{l_j} T_j \varepsilon_j dx - \sum_{i=1}^{n} P_{xi} x_i - \sum_{i=1}^{n} P_{yi} v_i - \sum_{i=1}^{n} P_{zi} \mathrm{w}_i \rightarrow \min \quad (1)$$

where: T_j, ε_j, l_j respectively are tensile force, elongation strain and undeformed length of j^{th}

(*j=1÷m*) cable segment; P_{xi}, P_{yi}, P_{zi} and u_i, v_i, w_i respectively are components of concentrated load and displacement of cable at point i^{th} (*i=1÷n=1÷m-1*) in x, y, z directions. When cables are modelled as shown in Figure 1, approximately ones can neglect the variation of tensile force within each cable segment and Eq. (1) can be rewritten as:

$$Z = \sum_{j=1}^{m} T_j \varepsilon_j l_j - \sum_{i=1}^{n} P_{xi} u_i - \sum_{i=1}^{n} P_{yi} v_i - \sum_{i=1}^{n} P_{zi} \mathrm{w}_i \to \min \tag{2}$$

Tensile force within each cable segment can be computed via elongation strain ε_j, cross-sectional area A and elastic modulus of cable's material as:

$$T_j = E\mathrm{A}\varepsilon_j = E\mathrm{A}\left(s_j - l_j\right)\big/ l_j \tag{3}$$

where: s_j is deformed length of j^{th} cable segment, which can be computed via original coordinates and displacement components of both end points of the cable segment.

For the case when cable subjected to variation of temperature Δt, deformed length of each cable segment under the effect of temperature are equal and can be computed via initial length l_j and coefficient of thermal expansion α:

$$s_{jt} = l_j (1 + \alpha \Delta t) \tag{4}$$

Condition for minimization of constrain functional in Eq. (1) is its variation equal zero, $\delta Z = 0$. According to principle of virtual displacement, while taking variation of Eq. (1) ones consider ε_j is independent with T_j, and u_i, v_i, w_i are independent with P_{ix}, P_{yi}, P_{zi}, therefore:

$$\delta Z = \sum_{j=1}^{m} T_j l_j \delta\varepsilon_j - \sum_{i=1}^{n} P_{xi} \delta u_i - \sum_{i=1}^{n} P_{yi} \delta v_i - \sum_{i=1}^{n} P_{zi} \delta \mathrm{w}_i = 0 \tag{5}$$

Substitute Eq. (3) into Eq. (5) and taking partial derivative with respect to unknown variables u_i, v_i, w_i, ones can get:

$$\begin{aligned} \frac{\partial Z}{\partial u_i} &= \sum_{j=1}^{m} T_j l_j \frac{\partial \varepsilon_j}{\partial u_i} - \sum_{i=1}^{n} P_{xi} = 0; \\ \frac{\partial Z}{\partial v_i} &= \sum_{j=1}^{m} T_j l_j \frac{\partial \varepsilon_j}{\partial v_i} - \sum_{i=1}^{n} P_{yi} = 0; \\ \frac{\partial Z}{\partial w_i} &= \sum_{j=1}^{m} T_j l_j \frac{\partial \varepsilon_j}{\partial w_i} - \sum_{i=1}^{n} P_{zi} = 0 \end{aligned} \tag{6}$$

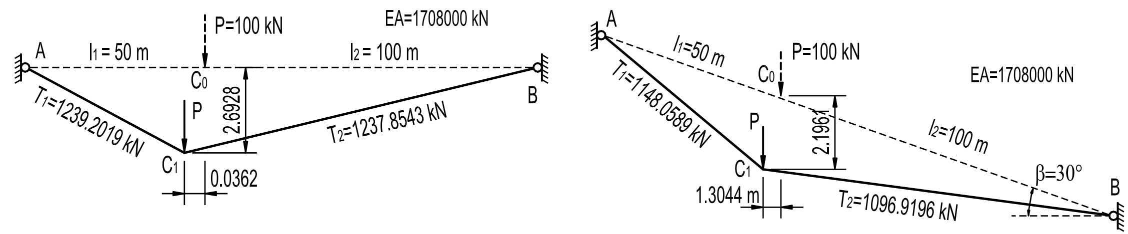
Eq. (6) leads to a system of nonlinear equations for cable with unknown variables u_i, v_i, w_i. Solving these equations for nodal displacements to determine deformed position of cables and use relations in Eq. (3) to calculate tensile forces in every cable segments.

In general situation, cables are subjected to multiple loading pattern, final position and internal forces within the cable must be determined with consideration of loading sequence due to nonlinearity of the problem. The stage of the cable under the effect of self-weight is determined first. To solve the Eq. (6), a computer code was developed and written in MATLAB with assistance of the *fsolve* function. The following sections are numerical examples executed by the dedicated computer code to demonstrate the applicability of the proposed method.

3 NUMERICAL EXAMPLES

3.1 Single Cable under Concentrated Load

Consider a single cable subjected to a concentrated vertical load with geometrical parameters and two cable position as indicated in Figure 2, self-weight of the cable is not considered in both cases. Final stage of the cable with tensile forces in each segment are shown by continuous lines and initial stage are presented by dash lines in the Figure 2. Ones can easily check that the balance conditions are satisfied.



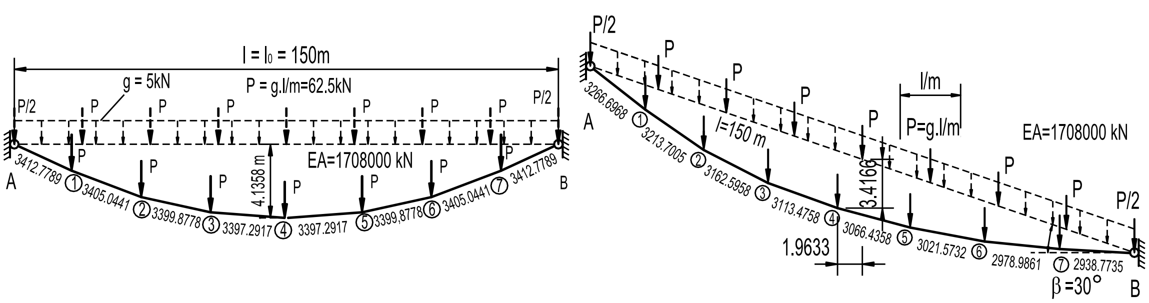
a) Horizontal cable.

b) Inclined cable.

Figure 2. Single cable subjected to concentrated load.

3.2 Single Cable under Distributed Loads

In this example, a single cable subjected to uniformly distributed load caused by self-weight is considered for two cases: horizontal supports and different level supports. Geometrical parameters and load value, including self-weight are shown in Figure 3; initial stage is shown by dash lines and final stage is shown by continuous lines. In computation, cable is divided into eight equal length segments and distributed loads, including self-weight, are converted into concentrated loads at both ends of each segment. Forces balance are well satisfied at all points.



a) Horizontal cable.

b) Inclined cable.

Figure 3. Single cable under distributed load.

3.3 Parametric Study for Effect of Segment Division

Results for the case of inclined single cable in Figure 3b were listed in Table 1 for different numbers of divided segments. Values indicated in the Table 1 are tensile forces in the cable computed at support position, and maximum deflection computed at the middle of the cable. In principle, the accuracy of computation will increase together with number of cable segments and approaching the real values of tensile forces and deflection of the cable. The results shown in Table 1 and Figure 4 indicate the convergence of computation when increase number of cable segments. This convergence is very fast and when the number of segments more than eight, the difference of computed results is less than 0.8% if double the number of segments.

Table 1. Effect of number of cable segments on computed results for single cable under distributed load.

Number of segments	Maximum tensile force (at support)		Maximum deflection (at middle)	
	Tensile force (kN)	Difference (%)	Deflection (m)	Difference (%)
2	2920.0861	-	3.7401	-
4	3190.1892	8.47	3.4725	7.71
8	3266.6968	2.34	3.4166	1.64
16	3292.4101	0.78	3.4032	0.39
32	3302.2447	0.29	3.3999	0.09
64	3306.4136	0.17	3.4021	0.06

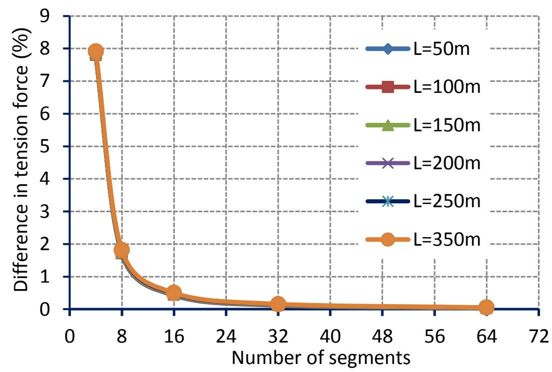


Figure 4. Convergence of computed tensile force in the cable when increase number of segments.

In order to find a reasonable number of segments for computation of a single cable under self-weight, analysis were carried out for different cable length (from 50 m to 350 m), number of segments for each case varied from 2 to 64. The Graph in Figure 4 shows the relation between number of segments with the difference of computed maximum tensile force with the exact one according to Eq. (7). According to Pugsley (1957), the maximum tensile force for a single cable under self-weight is computed at the supports as:

$$T_{\max} = \frac{g_0 l^2}{8f}\sqrt{1+\frac{16f^2}{l^2}} \tag{7}$$

From the Figure 4, the maximum tensile force in the cable is exact when number of segments is 64; the error is less than 2% and 0.5% if number of segments are 8 and 16 respectively.

3.4 Single Cable Subjected to Concentrated Load and Temperature Variation

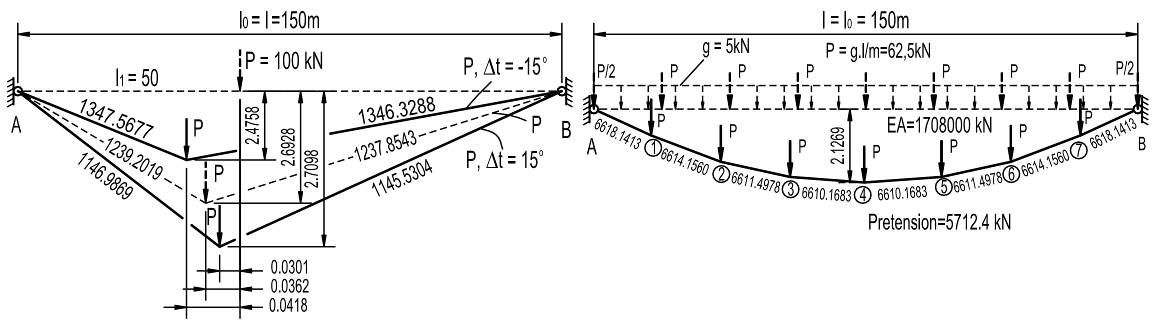
Consider the single cable in Figure 5a subjected to concentrated load and temperature variation with amplitude of ± 14°C, self-weight is negligible. Results are shown in Figure 5a and Table 2, force balance is satisfied for all cases. Raising of temperature causes tensile force to decrease and vice versa, confirming the accuracy of the computer code and correctness of proposed method.

Table 2. Tensile force and deflection of single cable in Figure 5a under temperature variation.

Δt (^oC)	Tensile in left seg. (kN)	Tensile in right seg. (kN)	Horizontal disp. (m)	Vertical disp. (m)	Horizontal balance (kN)	Vertical balance (kN)
15	1146.9867	1145.5304	0.0301	2.7098	0.42e-9	0.52e-11
0	1239.2019	1237.8542	0.0362	2.6928	0.14e-9	0.20e-10
-15	1347.5677	1346.3288	0.0418	2.4758	0.88e-10	0.31e-11

3.5 Single Cable Subjected to Self-weight and Pretension Force

The single cable in Figure 5b subjected to self-weight and pretension force simultaneously, by comparing with the results obtained in the Figure 3a, ones can see the effect of pretension force reduces the deflection of the cable but increases tensile force in every segment of the cable.



a) Effect of external load and temperature variation. b) Effect of self-weight and pretension force.

Figure 5. Single cable subjected to temperature variation and pretension force.

4 CONCLUSION

Application of the Gaussian Extreme Principle method allows the formulation and solving of nonlinear equation system of cable structures, which ensures force balancing, as well as continuity of displacements and deformations of cable structures. This method allows for simultaneous determination of displacement, deformation, and tension forces of cable structure without additional hypotheses, which is different from present cable theory. Numerical examples with simple, flexible cables subjected to different loadings have indicated the simplicity, accuracy and stability of the proposed method.

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