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ANALYSIS OF SPACE FRAMES WITH GENERALIZED SEMI-RIGID CONNECTIONS

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A mechanical model and analytic method are proposed, in which, the axial, the shearing and the bending semi-rigid characteristics of space frames are taken into account. An independent zero-length connection element comprising six translational and rotational springs is used to simulate the beam-to-column connection. The model, namely six-spring mechanical model, has an advantage that the element number of structure does not increase. The matrix displacement method is used to analyze mechanism of the model, including element analysis and structural analysis. The stiffness matrix of the element is derived. Some reaction forces at the end of the element are obtained when it is subjected to two kinds of different loads respectively. The obtained stiffness matrix gets the characteristics of symmetry and singularity and that makes the size of total stiffness matrix for semi-rigid frame the same as that for frame with rigid joints.

Keywords: Semi-rigid joint, Spatial frame, Spatial structural analysis, Six-spring mechanical model, Stiffness matrix, Fixed-end nodal force.

1 INTRODUCTION

Steel frame structural system has been applied extensively in building projects. The researches show that most of the damage of steel frames occurred at the joints in the earthquakes. Therefore, research on the semi-rigid joint of beam-to-column connections, also concluding semi-rigid splices of columns or beams is the most important and critical work.

Most studies were focus on plane frames with semi-rigid connections. A finite element model for plane frames (Wu and Chen 1990, Wang and Duan 2011) was proposed, which can take all the moment-rotation, transverse force-displacement and axial force-displacement relationship into account. For element with semi-rigid connections, the stiffness matrix is derived, and some equivalent nodal force vectors are obtained when the element is subjected to different loads. But the research on spatial structural analysis of semi-rigid beam-to-column connections and the influence of the axial and transverse direction are rarely involved.

In this paper, a new analytic model is proposed to the space frames with general semi-rigid connections, the stiffness matrix and the equivalent nodal forces are yielded out.

2 ELEMENT MODELING

The matrix displacement method is used at present following the studies by Li (2010), Duan and Zhang (2010) for the plane frame. A space straight bar element with the sectional tension stiffness EA, torsional stiffness GI_x , bending stiffness EI_y and EI_z and a span l with general semi-rigid joints is expressed in Figure 1. We use six springs of null length to represent the semi-rigid joint at each end of the element. The three spiral springs with relative linear or nonlinear stiffness R_x, R_y and R_z are used to simulate the rotational characteristics of the joint. And another three transitional springs with relative linear or nonlinear stiffness K_x, K_y, and K_z are used to simulate the tension or compression characteristics of the joint (Nguyen and Kim 2013). Based on the material being elastic and the small deformation assumptions, the element nodal displacements can be derived into four independent groups, which about axial, torsional and bending in the two planes.

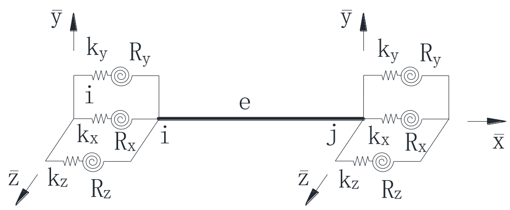


Figure 1. Six-spring model.

3 ELEMENT STIFFNESS MATRIX

In the local coordinate system, the fixed-end nodal displacements and forces can be calculated from the following equations as Eq. (1), the positive direction of the displacements and forces is the same as the positive direction of the coordinate.

$$\overline{\Delta}^e = \left[\overline{\Delta}_i \quad \overline{\Delta}_j\right]^T \qquad \overline{F}^e = \left[\overline{F}_i \quad \overline{F}_j\right]^T \tag{1}$$

in which , $\overline{\Delta}_i = \left[\overline{u}_i \quad \overline{v}_i \quad \overline{w}_i \quad \overline{\theta}_{xi} \quad \overline{\theta}_{yi} \quad \overline{\theta}_{zi}\right]^T$, $\overline{\Delta}_j = \left[\overline{u}_j \quad \overline{v}_j \quad \overline{w}_j \quad \overline{\theta}_{xj} \quad \overline{\theta}_{yj} \quad \overline{\theta}_{zj}\right]^T$,
$\overline{F}_i = \left[\overline{F}_{xi} \quad \overline{F}_{yi} \quad \overline{F}_{zi} \quad \overline{M}_{xi} \quad \overline{M}_{yi} \quad \overline{M}_{zi}\right]^T$ and $\overline{F}_j = \left[\overline{F}_{xj} \quad \overline{F}_{yj} \quad \overline{F}_{zj} \quad \overline{M}_{xj} \quad \overline{M}_{yj} \quad \overline{M}_{zj}\right]^T$.

3.1 Considering Torsional Semi-rigid Connections

When the relative joints twist is θ_{xA} and θ_{xB}, the relationship of torsion spring relative rotation and the connection stiffness R_{xA}, R_{xB} and the element end torque M_{xA} and M_{xB} are as follows:

$$\begin{aligned} \theta_{rxA} &= M_{xA} / R_{xA} \\ \theta_{rxB} &= M_{xB} / R_{xB} \end{aligned} \tag{2}$$

The part of modified torsion stiffness matrix can be expressed as Eq. (3)

$$\left[\overline{k}\right] = \begin{bmatrix} GI_x\alpha / l & -GI_x\alpha / l \\ -GI_x\alpha / l & GI_x\alpha / l \end{bmatrix} \tag{3}$$

in which, $\alpha = \dfrac{R_{xA}R_{xB}}{R_{xA}R_{xB} + (R_{xA} + R_{xB})GI_x / l}$.

3.2 Considering Tensile Semi-rigid Connections

The modified axial stiffness matrix is as Eq. (4)

$$\left[\bar{k}\right]=\begin{bmatrix} EA\beta/l & -EA\beta/l \\ -EA\beta/l & EA\beta/l \end{bmatrix} \tag{4}$$

in which, $\beta=\dfrac{K_{xA}K_{xB}}{K_{xA}K_{xB}+(K_{xA}+K_{xB})EA/l}$.

3.3 Considering the Bending Semi-rigid Connections

In the plane $\bar{x}o\bar{y}$, the connecting stiffness of the rotating springs are R_{zA} and R_{zB} respectively, and the shear springs stiffness are K_{yA} and K_{yB} respectively. The relative lateral displacement of the semi-rigid shear is:

$$\begin{aligned} \theta_{zrA}&=\frac{M_{zA}}{R_{zA}} \qquad \theta_{zrB}=\frac{M_{zB}}{R_{zB}} \\ \Delta_{yrA}&=\frac{N_{yA}}{K_{yA}} \qquad \Delta_{yrB}=\frac{N_{yB}}{K_{yB}} \end{aligned} \tag{5}$$

When the inner force displacement of the joints is θ_{tA}, θ_{tB} the stiffness equation is:

$$\begin{aligned} M_{zA}&=i_z(\beta_{zii}\theta_A+\beta_{zij}\theta_{zB}) \\ M_{zB}&=i_z(\beta_{zji}\theta_{zA}+\beta_{zjj}\theta_{zB}) \end{aligned} \tag{6}$$

in which,

$$\begin{aligned} \beta_{zii}&=2R_{zA}(6i_zR_{zB}K_{yB}-6i_zR_{zB}K_{yA}+6i_{zA}K_{yA}K_{yB}l^2+2R_{zB}K_{yA}K_{yB}l^2)/s \\ \beta_{zij}&=\beta_{zji}=2R_{zA}R_{zB}(6i_zK_{yA}-6i_zK_{yB}+K_{yA}K_{yB}l^2)/s \\ \beta_{zjj}&=2R_{zB}(6i_zR_{zB}K_{yB}+2i_zR_{zA}K_{yA}K_{yB}l^2+6i_zK_{yA}K_{yB}l^2-6i_zR_{zA}K_{yA})/s \\ s&=12i_z^2R_{zA}K_{yB}-12i_z^2R_{zA}K_{yA}-12i_z^2R_{zB}K_{yA}+12i_z^2R_{zB}K_{yB}+12i_z^2K_{yA}K_{yB}l^2 \\ &-12i_zR_{zA}R_{zB}K_{yA}+12i_zR_{zA}R_{zB}K_{yB}+R_{zA}R_{zB}K_{yA}K_{yB}l^2+4i_zR_{zA}K_{yA}K_{yB}l^2 \\ &+4i_zR_{zB}K_{yA}K_{yB}l^2 \end{aligned}.$$

Sum up, when the relative displacement of joint is Δ_y, the stiffness equation is Eq. (7).

$$\begin{aligned} M_{zA}&=i_z\left(\beta_{zii}+\beta_{zij}\right)\Delta_y l \\ M_{zB}&=i_z\left(\beta_{zji}+\beta_{zjj}\right)\Delta_y l \end{aligned} \tag{7}$$

The modified bending stiffness matrix in $\bar{x}o\bar{y}$ plane is as follows:

$$\left[\bar{k}\right]=\begin{bmatrix} \frac{i_z}{l^2}(\beta_{zii}+2\beta_{zij}+\beta_{zjj}) & & & \\ \frac{i_z}{l}(\beta_{zii}+\beta_{zij}) & i_z\beta_{zii} & sym & \\ -\frac{i_z}{l^2}(\beta_{zii}+2\beta_{zij}+\beta_{zjj}) & -\frac{i_z}{l}(\beta_{zii}+\beta_{zij}) & \frac{i_z}{l^2}(\beta_{zii}+2\beta_{zij}+\beta_{zjj}) & \\ \frac{i_z}{l}(\beta_{zij}+\beta_{zjj}) & i_z\beta_{zij} & -\frac{i_z}{l}(\beta_{zij}+\beta_{zjj}) & i_z\beta_{zjj} \end{bmatrix}$$

In the same way, the modified bending stiffness matrix and parameters in $\bar{x}o\bar{z}$ plane can be obtained only by exchanging *y* and *z*.

3.4 Considering the General Semi-rigid Connections

Based on the independence of the three types of deformation, the deformation stiffness equation and stiffness matrix are obtained from the above parts, and finally the element stiffness matrix is formed. Thus, the element stiffness matrix of the space steel frame structure considering semi-rigid connection is presented.

$$\bar{k}^{e} = \begin{bmatrix} k_{AA}^{e} & k_{AB}^{e} \\ k_{BA}^{e} & k_{BB}^{e} \end{bmatrix} \tag{8}$$

where block k_{AA}^{e}、 k_{AB}^{e}、 k_{BA}^{e} and k_{BB}^{e} are as follows:

$$k_{AA}^{e} = \begin{bmatrix} \frac{EA}{l}\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i_z}{l^2}(\beta_{zii}+2\beta_{zij}+\beta_{zjj}) & 0 & 0 & 0 & \frac{i_z}{l}(\beta_{zii}+\beta_{zij}) \\ 0 & 0 & \frac{i_y}{l^2}(\beta_{yii}+2\beta_{zij}+\beta_{zjj}) & 0 & -\frac{i_y}{l}(\beta_{yii}+\beta_{yij}) & 0 \\ 0 & 0 & 0 & \frac{GI_x}{l}\alpha & 0 & 0 \\ 0 & 0 & -\frac{i_y}{l}(\beta_{yii}+\beta_{yij}) & 0 & i_y\beta_{yii} & 0 \\ 0 & \frac{i_z}{l}(\beta_{zii}+\beta_{zij}) & 0 & 0 & 0 & i_z\beta_{zii} \end{bmatrix}$$

$$k_{BA}^{e\,T} = k_{BA}^{e} = \begin{bmatrix} -\frac{EA}{l}\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i_z}{l^2}(\beta_{zii}+2\beta_{zij}+\beta_{zjj}) & 0 & 0 & 0 & -\frac{i_z}{l}(\beta_{zii}+\beta_{zij}) \\ 0 & 0 & -\frac{i_y}{l^2}(\beta_{yii}+2\beta_{yij}+\beta_{yjj}) & 0 & \frac{i_y}{l}(\beta_{yii}+\beta_{yij}) & 0 \\ 0 & 0 & 0 & -\frac{GI_x}{l}\alpha & 0 & 0 \\ 0 & 0 & -\frac{i_y}{l}(\beta_{yij}+\beta_{yjj}) & 0 & i_y\beta_{yij} & 0 \\ 0 & \frac{i_z}{l}(\beta_{zij}+\beta_{zjj}) & 0 & 0 & 0 & i_z\beta_{zij} \end{bmatrix}$$

$$k_{BB}^{e}=\begin{bmatrix} \frac{EA}{l}\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{i_z}{l^2}(\beta_{zii}+2\beta_{zij}+\beta_{zjj}) & 0 & 0 & 0 & -\frac{i_z}{l}(\beta_{zij}+\beta_{zjj}) \\ 0 & 0 & \frac{i_y}{l^2}(\beta_{yii}+2\beta_{yij}+\beta_{yjj}) & 0 & \frac{i_y}{l}(\beta_{yij}+\beta_{yjj}) & 0 \\ 0 & 0 & 0 & \frac{GI_x}{l}\alpha & 0 & 0 \\ 0 & 0 & \frac{i_y}{l}(\beta_{yij}+\beta_{yjj}) & 0 & i_y\beta_{yjj} & 0 \\ 0 & -\frac{i_z}{l}(\beta_{zij}+\beta_{zjj}) & 0 & 0 & 0 & i_z\beta_{zjj} \end{bmatrix}$$

4 FIXED END FORCES OF ELEMENT

4.1 Considering Tensile Semi-rigid Connections

In the plane $\bar{x}o\bar{y}$, under arbitrary loads, the axial force of the two ends of the tensile semi-rigid element are N_{xA} and N_{xB}, fixed end force of rigid connection unit are N_{xAF} and N_{xBF}.

The following formulas can be obtained by using the deformation superposition principle, and then fixed end force of element can be obtained.

$$\begin{aligned} N_{xA} &= EAN_{xA}/(lK_{xA}) - EAN_{xB}/(lK_{xB}) - F(1-a/l) \\ N_{xB} &= -EAN_{xA}/(lK_{xA}) + EAN_{xB}/(lK_{xB}) - Fa/l \end{aligned} \tag{9}$$

4.2 Considering the Bending Semi-rigid Connections

In the plane $\bar{x}o\bar{y}$, under arbitrary loads, the bending moment of the two ends of the rotation and shear semi-rigid element are M_{zA} and M_{zA}, fixed end moment of rigid connection unit are M_{zAF} and M_{zBF}.

The following formulas can be obtained by using the deformation superposition principle, and then fixed end force of element can be obtained.

$$\begin{aligned} M_{zA} &= M_{zAF} - 4i_z M_{zA}/R_{zA} - 2i_z M_{zB}/R_{zB} - 6i_z\Delta_{yr}/l \\ M_{zB} &= M_{zBF} - 4i_z M_{zB}/R_{zB} - 2i_z M_{zA}/R_{zA} - 6i_z\Delta_{yr}/l \end{aligned} \tag{10}$$

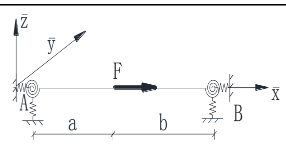
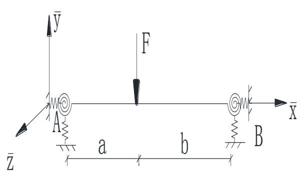
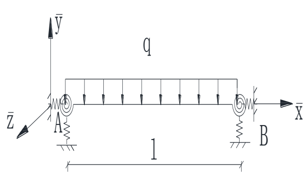
4.3 Some Examples

Some obtained fixed end forces are shown in Table 1. The other results are omitted in this paper for saving space.

5 CONCLUSIONS

The obtained stiffness matrix gets the characteristics of symmetry and singularity and that makes the size of total stiffness matrix for semi-rigid frame the same as that for frame with rigid joints. The stiffness matrix and load constants are suitable for any semi-rigid connections. The corresponding stiffness elements are same as that of the rigid connection when the *R* or *K* tend to infinite, while they are same as that of the hinged connection when the *R* or *K* tend to zero.

Table 1. Fixed end force of element with semi-rigid connections of tension, rotation, and shearing.

Type of Load	Fixed-End Force of Element		
	Reaction Force	**A**	**B**
	**N**	D_1 / D	D_2 / D
	Q	0	0
	M	0	0
	Parameter	$D_1 = FK_{xA}(aK_{xB} + EA - lK_{xB})$ $D_2 = FK_{xB}(EA - aK_{xA})$ $D = lK_{xA}K_{xB} - EAK_{xB} - EAK_{xA}$	
	**N**	0	0
	Q	$(M_{tA} + M_{tB})/l + Fb/l$	$Fa/l - (M_{tA} + M_{tB})/l$
	M	nD_1 / D	nD_2 / D
	Parameter	$D_1 = FR_{tA}(6i_tR_{tB}K_{mA}a^2b + 6i_tR_{tB}K_{mA}ab^2 - 6i_tR_{tB}K_{mB}a^2b$ $-6i_tR_{tB}K_{mB}ab^2 - R_{tB}K_{mA}K_{mB}l^2ab^2 - 4i_tK_{mA}K_{mB}l^2ab^2 -$ $2i_tK_{mA}K_{mB}l^2a^2b)$ $D_2 = FR_{tB}(-6i_tR_{tA}K_{mA}a^2b - 6i_tR_{tA}K_{mA}ab^2 + 6i_tR_{tA}K_{mB}ab^2$ $+6i_tR_{tA}K_{mB}a^2b + R_{tA}K_{mA}l^2a^2b + 2i_tK_{mA}K_{mB}l^2ab^2 +$ $4i_tK_{mA}K_{mB}l^2a^2b)$ $D = l^2s$	
	**N**	0	0
	Q	$(M_{tA} + M_{tB})/l + ql/2$	$-(M_{tA} + M_{tB})/l + ql/2$
	M	nD_1 / D	nD_2 / D
	Parameter	$D_1 = ql^2R_{tA}(12i_tR_{tB}K_{mA} - 12i_tR_{tB}K_{mB} -$ $R_{tB}K_{mA}K_{mB}l^2 - 6i_tK_{mA}K_{mB}l^2)$ $D_2 = ql^2R_{tB}(12i_tR_{tB}K_{mB} - 12i_tR_{tB}K_{mA} +$ $R_{tA}K_{mA}K_{mB}l^2 + 6i_tK_{mA}K_{mB}l^2)$ $D = 12s$	

Note: In the plane $\bar{x}o\bar{y}$, n=1, t=z, m=y; in the plane $\bar{x}o\bar{z}$, n=-1, t=y, m=z.

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